Gravity with Granularity*

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Abstract

We evaluate the consequences of oligopolistic behavior for the estimation of gravity equations for trade flows. With oligopolistic competition, firm-level gravity equations based on a standard CES demand framework need to be augmented by markup terms that are functions of firms' market shares. At the aggregate level, the additional term takes the form of the exporting country's market share in the destination country multiplied by an exporter-destination-specific Herfindahl-Hirschman index. For both cases, we show how to construct appropriate correction terms that can be used to avoid problems of omitted variable bias. We illustrate the quantitative importance of our results for combined French and Chinese firm-level export data as well as for a sample of product-level imports by European countries. Our results show that correcting for oligopoly bias can lead to substantial changes in the coefficients on standard gravity regressors such as distance or the impact of currency unions.

Keywords: Gravity Equation, Oligopoly, CES Demand, Aggregative Game

Journal of Economic Literature Classification: F12, F14, L13

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1 Introduction

Gravity equations have been the predominant tool for analyzing the determinants of bilateral trade flows since their introduction by Tinbergen (1962) over 50 years ago. In their most basic form, gravity equations predict that trade between countries is a log-linear function of the economic mass of the two trading partners and bilateral frictions such as distance or tariffs. Even in this simple form, gravity equations have substantial explanatory power, often explaining in excess of 70-80% of the variation in the trade flows between countries. Starting with Anderson (1979), researchers have shown that gravity equations can be derived from a number of mainstream theoretical frameworks, allowing a tight link to economic welfare analysis. Not surprisingly then, gravity equations have become the workhorse tool for evaluating trade-related economic policies, such as trade agreements, WTO membership or currency unions.

Despite the rapid progress research on gravity equations has made over the past decades, existing approaches remain at odds with a key stylized fact about international trade, however: much of world trade is dominated by a small number of large firms. The classic example is the market for wide-bodied passenger aircraft which comprises just two firms (Airbus and Boeing); but the markets of many other tradable goods such as cars, mobile phones or television sets are also dominated by a handful of large producers. That is, in the language of Gaubert and Itskhoki (forthcoming), trade flows are "granular". Given their size, it seems likely that such "granular" firms enjoy substantial market power and have incentives to internalize the effects of their actions on aggregate market outcomes. In this paper, we evaluate the consequences of oligopolistic behavior for the estimation of gravity equations and propose modifications to existing frameworks necessary to reconcile the two.

We start our analysis by deriving firm-level gravity equations from a standard CES demand framework. Instead of the usual assumption of monopolistically competitive firms, however, we introduce oligopolistic competition.[1] This leads to the inclusion of variable markup terms in firm-level gravity equations that depend on firms' market shares. Market shares in turn are determined by both firm- and destination-specific variables as well as bilateral trade frictions, so that omitting oligopoly markup terms will bias coefficient estimates on all other included regressors. Since the relevant markup variation is at the firm-product-destination-time level, standard approaches such as the inclusion of combinations of fixed effects are not feasible.

[1] We focus on quantity competition in the main body of the paper and present results for price competition in a separate appendix.

Instead, we show how to adjust trade flows by a suitable correction term to eliminate markup bias, requiring only information on market shares and demand and supply elasticities. To estimate the latter, we propose an extension of the well-known Feenstra-Broda-Weinstein estimation procedure (see Feenstra, 1994; Broda and Weinstein, 2006) that accounts for market power and can be implemented using data on firm-level market shares and unit values.

We also analyze the consequences of oligopolistic behavior for the estimation of gravity equations at more aggregate levels, such as on sector-level trade flows.[2] We show that the presence of firm-level markup terms leads to bias here as well but that a suitable correction term can again be constructed. This time, the correction takes the form of a destination-specific Herfindahl-Hirschman index (HHI) for exports multiplied by the aggregate market share of the exporting country. Intuitively, what matters for the market power of the exporting country is both its overall market share and how that share is distributed among individual exporters. For example, oligopolistic behavior will be much more pronounced if the overall market share is accounted for by just one firm rather than a large number of small producers.

Having shown theoretically how the presence of oligopolistic competition impacts the estimation of gravity equations, we next analyze the quantitative importance of the resulting bias using both firm- and sector-level data. We pool French and Chinese firm-level export data to allow the separate identification of bilateral variables such as distance from destination-specific multilateral resistance and absorption terms. To measure market shares, we combine our firm-level export data with information on product-level absorption for European countries from the PRODCOM database. We demonstrate that adjusting trade flows by our correction term can lead to substantial changes in the coefficient on standard bilateral gravity regressors such as distance or the impact of currency unions, particularly in settings where individual French and Chinese exporters have significant market shares.

Moving on to the estimation using sectoral data, we show how to combine standard sources for trade data with information on exporter-specific HHIs from the World Bank's Exporter Dynamics Database (EDD) to construct theory-consistent correction terms for the exports of around 50 countries to European markets. Again, correcting trade flows by these terms can lead to significant changes in gravity equation estimates, although the effects are less pronounced than at the firm level.

[2] Throughout the paper, we use the terms 'sector' and 'product' interchangeably. While our empirical analysis is based on product-level data classified according to the Harmonized System (HS), all of our theoretical results apply to both sector- and product-level data.

Our paper contributes to several strands of the literature on gravity equations. Anderson (1979), Eaton and Kortum (2002), Anderson and van Wincoop (2003), Chaney (2008) and Melitz and Ottaviano (2008) show how to derive gravity equations from a number of different theoretical frameworks. For example, Chaney (2008) derives firm and aggregate gravity equations from a CES demand framework under monopolistic competition. We demonstrate how allowing for oligopolistic competition adds additional markup-based terms to otherwise identical equations. Melitz and Ottaviano (2008) use a setting with monopolistic competition and linear demand to generate an aggregate-level gravity equation. Similar to our approach, this framework generates variable destination-specific markups although it does not yield a firm-level gravity equation. We prefer to work with the more standard CES demand system as this allows a clean separation of the effects of oligopolistic behavior from markup variability arising from the shape of the demand function.[3]

We also contribute to part of the gravity literature that is concerned with obtaining consistent estimates of parameters of interest, such as distance elasticities. For example, Anderson and van Wincoop (2003) point out the need to control for multilateral resistance terms in gravity equations and Redding and Venables (2004) propose to include exporter and importer fixed effects to this end. Santos Silva and Tenreyro (2006) advocate the use of Poisson pseudo maximum likelihood (PPML) estimation techniques to address bias arising from heteroscedasticity in log-linearized models and to allow the inclusion of zero trade flows. Helpman, Melitz, and Rubinstein (2008) show how to account for the self-selection of firms into export markets when estimating aggregate gravity equations. We contribute to this literature by showing how to correct parameter bias arising from oligopolistic behavior by exporting firms at different levels of aggregation.

Third, within the last decade there has been revived interest in integrating oligopolistic competition into models of international trade, partially building on earlier contributions by the strategic trade policy literature (see Brander, 1995). For example, Edmond, Midrigan, and Xu (2015) study the gains from trade in the oligopolistic competition model of Atkeson and Burstein (2008). Eckel and Neary (2010) model the consequences of market integration in a setting with Cournot competition between multi-product firms. Parenti (2018) looks at the impact of trade liberalization in a model of imperfect competition where a few oligopolistic firms coexist with a monopolistically competitive fringe. Head and Mayer (2019) compare counterfactual predictions for the effects of freer trade across a number of modeling frameworks, including CES demand with monopolistic and oligopolistic competition and random

[3] CES demand also seems a more natural starting point as it generates gravity for both individual firms and at more aggregate levels.

coefficients discrete choice models with oligopolistic price setting. None of these papers investigates the consequences of oligopolistic behavior for the estimation of gravity equation, which is our key contribution.

Finally, in work concurrent to and independent of ours, Heid and Staehler (2020) propose an extension of Arkolakis, Costinot, and Rodriguez-Clare (2012)'s formula to evaluate the gains from trade under oligopoly. To consistently estimate parameters necessary for the quantification of their model, they derive and estimate an aggregate gravity equation in oligopoly under the assumption that all industries are symmetric and each country hosts one firm per industry. Moreover, they have to take key parameters (such as price elasticities) from the existing literature, although the underlying estimation procedures are inconsistent with oligopolistic competition. By contrast, the firm- and industry-level gravity equations that we derive and estimate allow industries to differ in an arbitrary way and each country to host multiple (heterogeneous) firms. Moreover, we propose an adaptation of existing estimation procedures to obtain key parameter estimates in a way consistent with oligopolistic behavior.

The rest of this paper is organized as follows. In Section 2, we derive a firm-level gravity equation from a CES-demand framework with oligopolistic quantity competition. Section 3 shows how to modify the Feenstra-Broda-Weinstein estimation procedure to account for oligopolistic behavior and obtain demand and supply elasticity estimates. In Section 4, we derive our correction term for aggregate product-level trade flows. Section 5 discusses the challenges facing the empirical implementation of our methods, describes the data sources and presents results for our firm and sector level gravity estimations. Section 6 concludes. Appendix A collects proofs of our theoretical results. Results obtained when assuming price instead of quantity competition are presented in Appendix B. Appendix C contains lists of the countries present in our datasets.

2 Firm-Level Gravity in Oligopoly

We consider a multi-country world with a continuum of sectors, indexed by z. The representative consumer in country n maximizes

$$U_n = \int_{z \in Z} \alpha_n(z) \log \left(\sum_{j \in \mathcal{J}_n(z)} a_{jn}^{\frac{1}{\sigma(z)}} q_{jn}^{\frac{\sigma(z)-1}{\sigma(z)}} \right)^{\frac{\sigma(z)}{\sigma(z)-1}} dz,$$

where $\alpha_n(z)$ denotes the sector-z expenditure share in country n, $\mathcal{J}_n(z)$ is the set of products available in sector z and country n, and $\sigma(z)$ denotes the elasticity of substitution between products in sector z. Consumption of product j in country n is denoted q_{jn}. The utility shifter a_{jn} captures quality differences or other factors such as brand appeal.

Given these preferences, the direct and inverse demands for product $i \in \mathcal{J}_n(z)$ in country n are given by:

$$q_{in} = a_{in} p_{in}^{-\sigma(z)} P_n(z)^{\sigma(z)-1} \alpha_n(z) E_n \quad \text{and} \quad p_{in} = a_{in}^{\frac{1}{\sigma(z)}} q_{in}^{-\frac{1}{\sigma(z)}} Q_n(z)^{-\frac{\sigma(z)-1}{\sigma(z)}} \alpha_n(z) E_n, \tag{1}$$

where E_n is total expenditure in country n, and

$$P_n(z) \equiv \left(\sum_{j \in \mathcal{J}_n(z)} a_{jn} p_{jn}^{1-\sigma(z)} \right)^{\frac{1}{1-\sigma(z)}} \quad \text{and} \quad Q_n(z) \equiv \left(\sum_{j \in \mathcal{J}_n(z)} a_{jn}^{\frac{1}{\sigma(z)}} q_{jn}^{\frac{\sigma(z)-1}{\sigma(z)}} \right)^{\frac{\sigma(z)}{\sigma(z)-1}}$$

are the sector-z CES price index and composite commodity in country n, respectively. From now on, we focus on a single sector and drop the index z.

Each product $j \in \mathcal{J}_n$ is offered by a different firm, which may be either a domestic or foreign producer. Firms compete in quantities in each market n, being able to segment markets perfectly.[4] The profit of the firm offering product i from selling in destination n is

$$\pi_{in} = p_{in} q_{in} - C_{in}(q_{in}),$$

where $C_{in}(q_{in})$ is the firm's cost of producing and selling output q_{in}. We allow for variable returns to scale and assume a functional form for costs of

$$C_{in}(q_{in}) = \frac{1}{1+\gamma} c_{in} (\tilde{\tau}_{in} q_{in})^{1+\gamma} = \frac{1}{1+\gamma} c_{in} \tau_{in} q_{in}^{1+\gamma},$$

where c_{in} is a firm-destination-specific cost shifter and $\tilde{\tau}_{in}$ a firm-destination-specific trade cost that takes the usual iceberg form.[5]

We assume throughout that the returns-to-scale parameters γ satisfies $\gamma > -1/\sigma$, which means that the marginal cost of production should not decrease too fast with output. This (weak) assumption guarantees that all the profit functions we consider will be unimodal.

Unlike in monopolistically competitive markets, firms take into account the impact of

[4] We focus on quantity competition here and present results for price competition in Appendix B.

[5] For one unit of the output to arrive in destination n, the firm needs to ship $\tilde{\tau}_{in}$. Note that we define $\tau_{in} = \tilde{\tau}_{in}^{1+\gamma}$ to ease the subsequent notation.

their actions on the CES-composite, Q_n, when setting quantities. For what follows, it is useful to generalize further the degree of strategic interaction between firms by introducing a conduct parameter, λ (see Bresnahan, 1989): When firm i increases its output q_{in} by an infinitesimal amount, it perceives the induced effect on Q_n to be equal to $\lambda \partial Q_n / \partial q_{in}$. Under monopolistic competition, the conduct parameter λ takes the value of zero, whereas it is equal to one under Cournot competition. The first-order condition of profit maximization of firm i in destination n is given by

$$\begin{aligned} 0 = \frac{\partial \pi_{in}}{\partial q_{in}} &= \frac{\alpha_n E_n}{Q_n^{\frac{\sigma-1}{\sigma}}} a_{in}^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} q_{in}^{-\frac{1}{\sigma}} - \frac{\sigma-1}{\sigma} \lambda \frac{\partial Q_n}{\partial q_{in}} \frac{\alpha_n E_n a_{in}^{\frac{1}{\sigma}} q_{in}^{\frac{\sigma-1}{\sigma}}}{Q_n^{\frac{\sigma-1}{\sigma}+1}} - C'_{in}(q_{in}) \\ &= \frac{\sigma-1}{\sigma} p_{in} \left(1 - \lambda s_{in}\right) - C'_{in}(q_{in}), \end{aligned} \tag{2}$$

where

$$s_{in} \equiv \frac{a_{in}^{\frac{1}{\sigma}} q_{in}^{\frac{\sigma-1}{\sigma}}}{\sum_{j \in \mathcal{J}_n} a_{jn}^{\frac{1}{\sigma}} q_{jn}^{\frac{\sigma-1}{\sigma}}} \tag{3}$$

is the market share of firm i in destination n.

Rearranging terms in equation (2) yields firm i's optimal markup in destination n:

$$\mu_{in} = \frac{1}{\sigma} + \lambda \frac{\sigma-1}{\sigma} s_{in} \tag{4}$$

where $\mu_{in} \equiv \left(p_{in} - C'_{in}(q_{in})\right)/p_{in}$ is the Lerner index of product i in country n. Under monopolistic competition conduct ($\lambda = 0$), the usual constant markup $1/\sigma$ obtains. If instead $\lambda > 0$, then markups are no longer constant and depend positively on market shares. We will make use of the additional flexibility afforded by the conduct parameter λ in Section 4, but for now, we assume Cournot conduct and set $\lambda = 1$.

Given the optimal markup in equation (4), firm i's price is $p_{in} = c_{in} \tau_{in} q_{in}^{\gamma} / (1 - \mu_{in})$ and the value of its sales in market n can be written as:

$$r_{in} = p_{in} q_{in} = \left(\frac{c_{in} \tau_{in}}{1 - \mu_{in}} \right)^{\frac{1-\sigma}{1+\sigma\gamma}} \left(a_{in} P_n^{\sigma-1} E_n \right)^{\frac{1+\gamma}{1+\sigma\gamma}} \tag{5}$$

So far, we have not imposed any structure on trade costs, τ_{in} or the taste and cost shock terms, a_{in} and c_{in}. For comparison with the existing literature and to facilitate the exposition of our identifying assumptions, we now assume that the two shock terms can be decomposed log-linearly as $\log a_{in} = \varepsilon_i^a + \varepsilon_n^a + \varepsilon_{in}^a$ and $\log c_{in} = \varepsilon_i^c + \varepsilon_n^c + \varepsilon_{in}^c$, respectively. We further assume that trade costs can be decomposed as $\log \tau_{in} = \beta X_{in} + \varepsilon_{in}^{\tau}$ where the X_{in} include variables with bilateral variation such as (log) distance, common language or dummies for the presence of trade agreements or currency unions. Obtaining consistent estimates of the coefficients on these bilateral terms (β) is a key objective of much of gravity equation-based research.[6] Finally, we again assume a three-way decomposition of the trade cost error term, $\varepsilon_{in}^{\tau} = \varepsilon_{i}^{\tau} + \varepsilon_{n}^{\tau} + \eta_{in}^{\tau}$.

Taking the logarithm of equation (5) yields a firm-level gravity equation of the form

$$\log r_{in} = \xi_n + \zeta_i + \beta \frac{1-\sigma}{1+\sigma\gamma} X_{in} + \frac{\sigma - 1}{1+\sigma\gamma} \log\left(1 - \mu_{in}\right) + \varepsilon_{in} \tag{6}$$

where ξ_n and ζ_i summarize destination- and firm-specific terms and

$$\varepsilon_{in} = \frac{1}{1+\sigma\gamma}\Big[\left(1+\gamma\right)\varepsilon_{in}^{a} + \left(1-\sigma\right)\varepsilon_{in}^{c} + \left(1-\sigma\right)\eta_{in}\Big].$$

Note that under the assumption of monopolistic competition, the markup term involving μ would be constant and could be subsumed in ζ_i. In that case, estimation of (6) would yield consistent estimates of the coefficient on X_{in} provided that we control for firm and destination fixed effects (ζ_i and ξ_n) and that the usual orthogonality assumptions (explicitly or implicitly) made in the gravity literature hold.[7]

In the presence of strategic interaction between firms, however, the markup term will depend on firms' market shares and will thus be correlated with the regressors of interest, X_{in}; not including this term will lead to an omitted variable bias. For example, we would expect firms to have lower market shares in more distant markets, ceteris paribus, and hence to charge lower markups there. This implies that $\log(1-\mu_{in})$ will be higher in such markets, leading to a positive correlation between distance and the omitted variable.

Note that this problem is qualitatively different from those arising from other hard-to-observe gravity components such as expenditure (E_n), price indices (P_n) or firm-level marginal costs because these components can be controlled for by firm or destination fixed effects. By contrast, markups vary at the firm-destination level and the inclusion of bilateral fixed effects

[6] See, for example, Baier and Bergstrand (2007) and Rose (2000) on the effects of free trade agreements and currency unions, respectively, on trade flows.

[7] Specifically, for least-squares estimation of the log-linearized gravity equation, the orthogonality conditions are $\mathbb{E}\left(\eta_{in}^{\tau}|X_{in}\right) = \mathbb{E}\left(\varepsilon_{in}^{a}|X_{in}\right) = \mathbb{E}\left(\varepsilon_{in}^{c}|X_{in}\right) = 0$. Note that these assumptions allow for non-zero correlations between the bilateral variables and taste, production and trade cost shocks working through the firm- and destination-level-specific components (ε_{i}^{a}, ε_{n}^{a}, ε_{i}^{c}, ε_{n}^{c}, ε_{i}^{τ} and ε_{n}^{τ}). This is not a problem for consistent estimation as these components can be controlled for through firm and destination fixed effects. If the data used to estimate equation (6) contain a time dimension, it is also possible to allow for time-invariant bilateral elements in the error term which can be captured through bilateral fixed effects as is standard, for example, in the literature on the trade effects of preferential trade agreements (e.g., Baier and Bergstrand, 2007)

would make it impossible to identify separately the effect of key regressors of interest such as distance, tariffs or dummy variables for trade agreement.[8]

Instead, we propose to solve the omitted variable problem by constructing a proxy for the markup term in (6). Specifically, if we had estimates for σ and γ and data for s_{in}, we could compute

$$\widehat{\mu}_{in} = \frac{1}{\widehat{\sigma}} + \frac{\widehat{\sigma}-1}{\widehat{\sigma}} s_{in}$$

and estimate

$$\log \widetilde{r}_{in} \equiv \log r_{in} - \frac{\widehat{\sigma}-1}{1+\widehat{\sigma}\widehat{\gamma}} \log\left(1-\widehat{\mu}_{in}\right) = \xi_n + \zeta_i + \beta \frac{1-\sigma}{1+\sigma\gamma} X_{in} + \varepsilon_{in}. \tag{7}$$

Given our earlier orthogonality assumptions, using $\log \widetilde{r}_{in}$ instead of $\log r_{in}$ as the dependent variable would yield a consistent estimate of $\beta \frac{1-\sigma}{1+\sigma\gamma}$. Using our estimates for σ and γ would then allow recovering the parameter of interest, β.[9] This approach raises the question of how to estimate σ and γ. In the next section, we show how to adapt the estimation procedure by Feenstra (1994) and Broda and Weinstein (2006) to our setting with firm-level data and oligopolistic competition.

3 Estimation of Supply and Demand Elasticities

Feenstra (1994) and Broda and Weinstein (2006) propose estimators for the elasticity of substitution, σ, based on the key identifying assumption that shocks over time to import demand and export supply for a given product are uncorrelated. The equivalent condition in our context is that $\mathbb{E}\left(\varepsilon^a_{in}\varepsilon^{\tau c}_{i'n'}\right) = 0$ for all i, i' and n, n', where $\varepsilon^{\tau c}_{in} = \varepsilon^{\tau}_{in} + \varepsilon^{c}_{in}$. That is, we assume that the firm-destination-level elements of taste and cost shocks are uncorrelated across firms and markets.

Note that this assumption is consistent with non-zero correlations between overall taste and cost shocks (i.e., $\mathbb{E}\left(a_{in}c_{in}\right) \neq 0$ is allowed). In particular, our method allows for a positive correlation between firm-level costs and quality (ε^a_i and ε^c_i) which is to be expected if the production costs of firms producing high-quality products are higher. Likewise, our results are robust to a positive correlation between destination market quality and cost shocks (ε^a_n and ε^c_n). For example, such a correlation could arise if firms sell higher-quality goods to

[8] Having a time dimension in the data would not help either because markups would then vary by firm, destination and time.

[9] Note the parallel to the literature on trade and quality which uses a similar approach to correct export values or quantities (e.g., Khandelwal, Schott, and Wei, 2013).

high-income markets and incur positive costs of doing so.

We start our derivation by expressing firm-level revenues of firm i in market n in terms of expenditure shares. From equation (1),

$$\log s_{in} = \log\left(\frac{p_{in}q_{in}}{E_n}\right) = \log a_{in} + (1-\sigma)\, p_{in} + (\sigma - 1)\log P_n.$$

Now assume that we observe another firm i' selling to the same market n. We can then subtract the logged market share of that firm to eliminate the price index:[10]

$$\Delta^f \log s_{in} = \log s_{in} - \log s_{i'n} = \log a_{in} - \log a_{i'n} + (1-\sigma)\left(\log p_{in} - \log p_{i'n}\right)$$

If we observe the same two firms in another destination n', we can compute a double difference across the two markets as

$$\Delta^d \Delta^f \log s_{in} = (1-\sigma)\, \Delta^d \Delta^f \log p_{in} + \Delta^d \Delta^f \log a_{in},$$

where Δ^f and Δ^d denote log differences across firms and destinations, respectively. Note that double differencing only leaves the firm-destination-specific parts of the taste shocks:

$$\Delta^d \Delta^f \log a_{in} = \left(\varepsilon^a_{in} - \varepsilon^a_{i'n}\right) - \left(\varepsilon^a_{in'} - \varepsilon^a_{i'n'}\right).$$

We next derive a similar supply-side equation. We start by rewriting firm i's price in market n as $p_{in}^{1+\gamma} = \left(\frac{c_{in}\tau_{in}}{1-\mu_{in}}\right)(s_{in}E_n)^\gamma$. Taking logs yields

$$(1+\gamma)\log p_{in} = \log\left(c_{in}\tau_{in}\right) - \log\left(1-\mu_{in}\right) + \gamma \log s_{in} + \gamma \log E_n.$$

Double differencing across firms and markets as above, we obtain

$$(1+\gamma)\, \Delta^d \Delta^f \log p_{in} = \Delta^d \Delta^f \log\left(c_{in}\tau_{in}\right) - \Delta^d \Delta^f \log\left(1-\mu_{in}\right) + \gamma \Delta^d \Delta^f \log s_{in},$$

where the double-differenced cost shock again only contains the parts of production and trade costs that are firm-destination specific:

$$\Delta^d \Delta^f \log\left(c_{in}\tau_{in}\right) = \left(\varepsilon^{\tau c}_{in} - \varepsilon^{\tau c}_{i'n}\right) - \left(\varepsilon^{\tau c}_{in'} - \varepsilon^{\tau c}_{i'n'}\right).$$

[10] In principle, we could also subtract the average across all firms active in market n. However, we will argue below that taking differences across individual firms with high market shares is better suited to dealing with selection problems.

Note that as per our identifying assumption, the double-differenced cost and taste shocks are uncorrelated, yielding the following moment condition:

$$\mathbb{E}\left(\Delta^d\Delta^f \log a_{in} \times \Delta^d\Delta^f \log\left(c_{in}\tau_{in}\right)\right) = 0.$$

For given σ and γ, we can construct the sample analogues from data on export prices and market shares:

$$\Delta^d\Delta^f \widehat{\log\left(c_{in}\tau_{in}\right)} = \left(1+\gamma\right)\Delta^d\Delta^f \log p_{in} + \Delta^d\Delta^f \log\left(1-\mu_{in}\right) - \gamma\Delta^d\Delta^f \log s_{in}$$

and

$$\Delta^d\widehat{\Delta^f \log} a_{in} = \Delta^d\Delta^f \log s_{in} - \left(1-\sigma\right)\Delta^d\Delta^f \log p_{in}.$$

The sample analogue of our moment condition is then given by

$$\Psi\left(\sigma,\gamma\right) = \frac{1}{|\mathcal{J}_{nn'}|}\sum_{j\in\mathcal{J}_{nn'}} \Delta^d\widehat{\Delta^f \log} a_{in} \times \Delta^d\Delta^f \widehat{\log\left(c_{in}\tau_{in}\right)},$$

where $\mathcal{J}_{nn'}$ denotes the set of firms active in the same two markets. Notice that we obtain one moment condition per country pair. Stacking these up allows to implement a standard GMM estimator of σ and γ.[11]

4 Sector-Level Gravity in Oligopoly

In this section, we study sector-level trade flows in the oligopoly model of Section 2. We first analyze the equilibrium in a given market using an aggregative games approach (Nocke and Schutz, 2018b; Anderson, Erkal, and Piccinin, 2020). We then leverage Nocke and Schutz (2018a)'s approximation techniques to derive a sector-level gravity equation that accounts for oligopolistic behavior.

Oligopoly analysis in a given destination market. Consider sector z in destination n. Dropping reference to both z and n to ease notation, we define the market-level aggregator H as

$$H \equiv Q^{\frac{\sigma-1}{\sigma}} = \sum_{j\in\mathcal{J}} a_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}}$$

[11] In practice, this means that we need to observe a sufficiently large number of firms selling in the same sector in at least three different markets.

and firm i's type T_i as

$$T_i \equiv a_i^{\frac{1}{\sigma}} \left(\frac{\alpha E}{c_i \tau_i} \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma-1}{\sigma(1+\gamma)}}. \tag{8}$$

Plugging these definitions into equation (2), making use of equation (3), and rearranging, we obtain:

$$1 - \lambda s_i = s_i^{\frac{1+\sigma\gamma}{\sigma-1}} \left(\frac{H}{T_i} \right)^{\frac{\sigma(1+\gamma)}{\sigma-1}}, \tag{9}$$

where λ is the conduct parameter introduced in Section 2. As the left-hand side is strictly decreasing in s_i and the right-hand side is strictly increasing in s_i, the equation has a unique solution in s_i, which we denote $S(T_i/H, \lambda)$—the *market-share fitting-in function*. It can easily be verified that $S(\cdot, \cdot)$ is strictly increasing in its first argument and strictly decreasing in its second.

The equilibrium level of the aggregator, $H^*(\lambda)$, is pinned down by market shares having to add up to unity:

$$\sum_{i \in \mathcal{J}} S\left(\frac{T_i}{H}, \lambda \right) = 1. \tag{10}$$

The uniqueness of the solution follows by the strict monotonicity of the market-share fitting-in function.

To summarize:

Proposition 1. *In each destination market, and for any conduct parameter λ, there exists a unique equilibrium in quantities. The equilibrium aggregator level $H^*(\lambda)$ is the unique solution to equation* (10)*. Each firm i's equilibrium market share is $s_i^*(\lambda) = S(T_i/H^*(\lambda), \lambda)$, where $S(T_i/H^*(\lambda), \lambda)$ is the unique solution to equation* (9)*. From equation* (3)*, firm i's equilibrium output is given by*

$$q_i^*(\lambda) = a_i^{-\frac{1}{\sigma-1}} \left(s_i^*(\lambda) H^*(\lambda) \right)^{\frac{\sigma}{\sigma-1}}.$$

Proof. See Appendix A.1 □

The first-order approach to sector-level gravity. Let $\mathcal{E} \subsetneq \mathcal{J}$ denote the subset of exporters in country e that sell in the destination market n. The aggregate exports of those firms to market n are given by

$$\underbrace{\sum_{i \in \mathcal{E}} s_i^*(\lambda)}_{\equiv s_e^*(\lambda)} \quad \times \quad \alpha E.$$

We are interested in these aggregate exports when firms compete in a Cournot fashion, i.e., when $\lambda = 1$. Unfortunately, there is no closed-form solution to $s_e^*(1)$. Our approach therefore entails approximating $s_e^*(1)$ at the first order.

As we show in the following, the approximation relies on two versions of the Herfindahl-Hirschman index (HHI), namely the HHI of all firms selling in the destination market,

$$\text{HHI}(\lambda) \equiv \sum_{j \in \mathcal{J}} \left(s_j^*(\lambda)\right)^2,$$

and the (normalized) HHI of all those exporters in country e that sell in the destination market,

$$\text{HHI}_e(\lambda) \equiv \sum_{j \in \mathcal{E}} \left(\frac{s_j^*(\lambda)}{s_e^*(\lambda)}\right)^2.$$

We obtain:

Proposition 2. *At the first order, in the neighborhood of $\lambda = 0$ (monopolistic competition conduct), the logged joint market share in destination n of the firms from export country e is given by*

$$\log s_e^*(\lambda) = \log s_e^*(0) + \frac{\sigma - 1}{1 + \sigma\gamma}\Big[\text{HHI}(\lambda) - s_e^*(\lambda)\,\text{HHI}_e(\lambda)\Big]\lambda + o(\lambda).$$

Proof. See Appendix A.2. □

The proposition shows that the logged joint market share of the exporters from country e differs from the one that would obtain under monopolistic competition by a market power term that takes account of both the overall concentration in the destination market as well as the concentration among the country-e exporters.

This result motivates the following approximation:

$$\log s_e^*(1) \simeq \log s_e^*(0) + \frac{\sigma - 1}{1 + \sigma\gamma}\Big[\text{HHI}(1) - s_e^*(1)\,\text{HHI}_e(1)\Big]. \tag{11}$$

Empirical specification of sector-level gravity in oligopoly. In Cournot oligopoly, the logged sector-level exports from country e to destination market n are given by

$$\begin{aligned}\log r_{en} &= \log(\alpha_n E_n) + \log s_{en}^*(1)\\ &\simeq \log(\alpha_n E_n) + \log s_{en}^*(0) + \frac{\sigma - 1}{1 + \sigma\gamma}\Big[\text{HHI}_n(1) - s_{en}^*(1)\,\text{HHI}_{en}(1)\Big],\end{aligned} \tag{12}$$

where the second line follows from the approximation in equation (11). The expression for the (counterfactual) joint market share of the country-e exporters in market n under monopolistic competition can be derived from equations (8) and (9), setting $\lambda = 0$. We obtain:

$$\log s^*_{en}(0) = \frac{\sigma-1}{1+\sigma\gamma}\log\left(\frac{\sigma-1}{\sigma}\alpha_n E_n\right) - \frac{\sigma(1+\gamma)}{1+\sigma\gamma}\log H_n + \log\left(\sum_{i\in\mathcal{E}_e}\frac{a_{in}^{\frac{1+\gamma}{1+\sigma\gamma}}}{(c_{in}\tau_{in})^{\frac{\sigma-1}{1+\sigma\gamma}}}\right), \tag{13}$$

where $\mathcal{E}_e$ is the set of country-e exporters, which we assume here to be the same in all destinations n.[12]

We impose the following structure on the shocks to quality, marginal costs and trade costs:

$$\begin{aligned}\log a_{in} &= \log a_i + \varepsilon^a_n + \varepsilon^a_{en},\\ \log c_{in} &= \log c_i + \varepsilon^c_n + \varepsilon^c_{en},\\ \log \tau_{in} &= \log \tau_i + \beta X_{en} + \varepsilon^\tau_n + \varepsilon^\tau_{en}.\end{aligned}$$

Define

$$\begin{aligned}\zeta_e \equiv\ & \log\sum_{i\in\mathcal{E}_e}\frac{a_i^{\frac{1+\gamma}{1+\sigma\gamma}}}{(c_i\tau_i)^{\frac{\sigma-1}{1+\sigma\gamma}}},\\ \xi_n \equiv\ & \log(\alpha_n E_n) + \frac{\sigma-1}{1+\sigma\gamma}\log \mathrm{HHI}_n + \frac{\sigma-1}{1+\sigma\gamma}\log\left(\frac{\sigma-1}{\sigma}\alpha_n E_n\right) - \frac{\sigma(1+\gamma)}{1+\sigma\gamma}\log H_n\\ & + \frac{1+\gamma}{1+\sigma\gamma}\log\varepsilon^a_n - \frac{\sigma-1}{1+\sigma\gamma}\left(\log\varepsilon^c_n + \varepsilon^\tau_n\right),\\ \eta_{en} \equiv\ & \frac{1+\gamma}{1+\sigma\gamma}\log\varepsilon^a_{en} - \frac{\sigma-1}{1+\sigma\gamma}\left(\log\varepsilon^c_{en} + \varepsilon^\tau_{en}\right).\end{aligned}$$

Combining equations (12) and (13), yields the sector-level gravity equation

$$\log\widetilde{r}_{en} = \zeta_e + \xi_n + \beta\frac{1-\sigma}{1+\sigma\gamma}X_{en} + \eta_{en}, \tag{14}$$

where

$$\log\widetilde{r}_{en} \equiv \log r_{en} + \frac{\sigma-1}{1+\sigma\gamma}s_{en}\,\mathrm{HHI}_{en} \tag{15}$$

is the value of the export flows from e to n, purged from oligopolistic market power effects, and s_{en} is the market share of country-e exporters in destination n.

[12] In principle, our approach for correcting for oligopoly bias could be combined with existing approaches addressing selection and intensive margin effects in aggregate-level gravity equations (e.g., Helpman, Melitz, and Rubinstein, 2008). We leave this for future work and focus on the oligopoly aspect here.

5 Empirical Implementation

In this section, we show how to implement our methods for firm- and sector-level gravity estimations empirically. We start by reviewing a number of challenges we face in terms of data and estimation issues. We then present basic descriptive statistics on our data and the GMM estimates for σ and γ. Finally, we run firm and sector level regressions with and without oligopoly correction terms and investigate if and under which circumstances ignoring oligopolistic behavior can lead to quantitatively important coefficient bias.

5.1 Estimation Challenges

Recall that our aim is to obtain consistent estimates of the coefficients on bilateral variables using either firm or sector level data. In Section 2 we showed that after subtracting a markup correction term from firm export values, we could estimate a standard gravity equation with a set of firm-product-year and destination-product-year fixed effects as well as the bilateral variables of interest.[13]

A first issue that immediately arises is how to control for destination-specific fixed effects in a setting with firm-level export data. If we only have data for exports from a single country, it is immediately clear that we can no longer separate the impact of bilateral variables from the fixed effects. For example, if we use information on the exports of French firms only, standard bilateral variables such as common language become destination-specific as France is the only origin country in our data. Intuitively, we will not be able to distinguish whether firms' exports to a given destination are high because France and the country in question share a common language or because of other destination-specific factors such as a high price index or expenditure level. In order to address this issue, we follow Bas, Mayer, and Thoenig (2017) by combining two datasets on the exports of French and Chinese firms, respectively. This ensures that there is within-destination variation in the bilateral regressors of interest, enabling the use of destination fixed effects.

Secondly, we have so far ignored selection issues by not formally modeling the export market entry decisions of firms. In practice, however, most firms only export to a small subset of possible destinations for any given product. When estimating (7) in log-linear form, firm-product-destination observations with zero trade flows drop out. This creates a selection

[13] Recall that we dropped the sector/product index (z) for most of our derivations and also ignored the time dimension to ease exposition. But these dimensions are of course present in our data, and hence price indices and expenditure levels will vary by destination, product and year, requiring the use of fixed effects at that level.

problem, as firms will be more likely to export positive amounts to a given destination if they experience a positive taste, production or trade cost shock for that destination, potentially creating a non-zero correlation with the regressors of interest. For example, firms selling in more distant foreign markets will be more likely to have received a positive shock, allowing them to operate in this more difficult environment. A partial solution to this problem is to include zeros in our left-hand side variable and estimate (7) in multiplicative form, using Poisson pseudo-maximum likelihood (PPML) estimation techniques (see Santos Silva and Tenreyro, 2006). Recent computational advances in PPML estimation (e.g., Correia, Guimaraes, and Zylkin, 2019) make it possible to include the large number of fixed effects required in our setting.[14]

However, this still leaves us with a potential selection problem in our GMM estimation procedure for σ and γ in Section 3. Here, we adapt an approach proposed by Bas, Mayer, and Thoenig (2017) and restrict our estimation sample to the largest three French and Chinese firm in each product category as measured by *overall* product-specific exports. The basic idea is that these firms have high overall exports because they are very productive, produce high-quality products in general (high ε_i^a or ε_i^c) or have access to low-cost market access technologies (low ε_i^τ). Such firms will tend to serve all or at least most available markets, making the destination-specific shocks less important for market entry decisions. We acknowledge that this is an imperfect solution but simulation evidence by Bas, Mayer, and Thoenig (2017) shows that focusing on top exporters does indeed substantially reduce selection bias. Finally, in order to obtain a sufficiently large number of observations for the computation of moments in our GMM estimation, we restrict the estimates of σ and γ to be identical within 2-digit HS products.

5.2 Data Sources

As discussed, we use firm-level export data for French and Chinese exporters provided by the two countries' customs authorities. In each dataset, we observe the products and destinations to which a firm exports, as well as the quantity and value of the underlying flow. Both datasets record export data at the 8-digit level but we aggregate this information up to the 6-digit level of the Harmonised System (HS) which is the lowest level at which the two

[14] We include zero trade flows when estimating (7) on a product-by-product basis in Section 5.4. Unfortunately, memory constraints prevent us from using a fully rectangularized dataset (with all possible firm-destination-product-year combinations) when pooling across products, as the resulting data would have over 140 billion observations. Nevertheless, PPML estimation is still preferable to OLS in this context as it also addresses problems arising from heteroscedasticity in log-linearized models as discussed by Santos Silva and Tenreyro (2006).

national classifications are comparable with each other. Because we observe both values and quantities, we can compute unit values which are a commonly used proxy for prices in the trade literature.

A final challenge for our firm-level analysis is to obtain information on market shares at a level of disaggregation that is sufficient to capture meaningful strategic interaction between firms. To our knowledge, the only suitable database here is Eurostat's PRODCOM database which allows computation of absorption at a level at, or close to, HS 6-digit.[15] Together with the information on the value of destination-product-level exports by individual French and Chinese firms, this allows the computation of market shares at the HS-6digit "plus" level, where "plus" means that some products have to be aggregated further to make the classifications of the trade and production data consistent with each other. The downside of using PRODCOM is that absorption data is only available for approximately thirty European countries. After combining our data sources, we end up with information on export values and quantities as well as market shares for 29 European destination markets, approximately 1,800 products and 250,000 exporters for the period 2000-2010.[16]

For our aggregate gravity regressions, we require product-level data on the value of bilateral exports, absorption data for the computation of market shares and exporter-destination-product-specific HHIs. We again use PRODCOM for absorption data and combine them with trade data from Eurostat's COMEXT database. For HHIs, we use the World Bank's Exporter Dynamics Database (EDD) which provides destination-specific Herfindahl indices computed from firm-level export data for 48 exporting countries at the HS 2-digit level in 2010. We thus also need to aggregate exports and absorption data to the HS 2-digit level.[17] Given that this is a relatively high degree of aggregation (90 aggregated manufacturing prod-

[15] Absorption is defined as domestic production + imports - exports and thus is the theoretical equivalent to E_n in our model. In principle, this information is available at the HS 6-digit level but issues such as classification changes over time often require aggregation to higher levels. The original classification of the PRODCOM data is the 8-digit CN classification, which changes almost every year. We apply the procedure developed by Van Beveren, Bernard, and Vandenbussche (2012) to map the CN classification to an artificial HS classification, "HS 6digit plus", that is comparable over time and also compatible with the 6-digit HS classification. The idea is to aggregate both trade and PRODCOM data as little as possible and as much as required to guarantee a 1-to-1 mapping between them. See their paper for an in-depth discussion of the procedure.

[16] Possibly because of measurement issues in PRODCOM, we occasionally observe cases where absorption is smaller than a firm's exports to a given market, resulting in market shares larger than one; in such cases, we winsorize market shares to 0.95.

[17] As before, we also require estimates for γ and σ. Here, we simply set the parameters to their standard values used in the literature, $\sigma = 5$ and $\gamma = 0$. A promising question for future research is how to obtain these estimates from product-level data, so that no firm-level data is needed for the estimation of these parameters. This would require extending the standard Feenstra-Broda-Weinstein approach to allow for heterogeneous firms and oligopolistic behavior.

ucts), we alternatively use French and Chinese firm-level data to compute exporter HHIs at the 6-digit level and estimate aggregate gravity regressions at that level. We source information on bilateral variables (distance, dummy for Euro membership of both trading partners, and a dummy for common official language) from CEPII.

5.3 Descriptive Statistics

The key determinants of our oligopoly correction term are firm-level market shares as well as estimates for demand elasticities (σ) and returns to scale (γ). The first line of Table 1 presents information on the market shares for the French and Chinese exporters in our firm-level dataset. The average market share across the approximately 8.3 million firm-destination-product-year combinations in our data is small at 0.4% and the median is even smaller (around 0.01%). Clearly, the average firm in our data does not enjoy much market power.

However, this does not necessarily imply that correcting firm-level exports for oligopoly forces will not matter quantitatively, as estimation results could be substantially biased by a small number of exporters with high market shares. The remaining lines of Table 1 focuses on such outliers. The second line begins by showing descriptive statistics for the top exporters in a given market (i.e., the French or Chinese firm with the highest market share). The average top-exporter market share is around 8%, substantially larger than the average exporter's share. In the third line we show the cumulative market of the top 3 exporters, which has a mean of around 14%. The remaining lines further zoom in on the largest exporters by restricting the sample to top exporters that account for at least 10%, 50% or 90% of a given market. The table shows that there are over 21,000 top exporters that have at least a 10% market share, around 6,500 exporters with at least a 50% share and still close to 500 top exporters with a share of over 90%. Clearly, there are a substantial number of markets in our data in which firms enjoy sizable market power, potentially leading to quantitatively important oligopoly bias in gravity coefficient estimates.

In Table 2, we instead zoom in on the concentration of exports and compute the share of French and Chinese total exports to each market accounted for by their top exporters. In the average market the top exporter accounts for around half of each country's exports to a given market and the top 3 exporters account for more than 70 percent.

Table 3 presents summary statistics on market shares of each exporter country for our aggregate HS 2-digit product-level sample. The average market share of the 48 countries exporting to the European destinations contained in the PRODCOM data is only around

Table 1: Summary Statistics for Firm-Level Market Shares

	Market Shares		
	Mean	Median	# obs
All firms	0.40%	0.01%	8,332,962
Top exporters	8.18%	1.80%	133,917
Top 3 exporters	13.89%	3.03%	131,201
Top exporters (at least 10% market share)	39.20%	23.32%	21,765
Top exporters (at least 50% market share)	84.54%	90.00%	6,412
Top exporters (at least 90% market share)	95.26%	95.47%	492

Note: 'Top exporter' denotes the French or Chinese exporter with the largest market share in a given product-destination-year combination.

Table 2: Summary Statistics for French and Chinese Export Shares to each Market

	Share in Market-specific Exports		
	Mean	Median	# obs
Top exporters	50.36%	45.73%	131,201
Top 3 exporters	73.62%	80.20%	131,201

Note: 'Top exporter' denotes the French or Chinese exporter with the largest market share in a given product-destination-year combination.

1.2%. Again, however, there is substantial variation around this mean, with 100 exporter-importer-product combinations where the exporter has a market share of 20% or more and still 42 combinations with market shares of at least 50%.[18]

Table 3: Summary Statistics for Aggregate Product-Level Market Shares

	Market Shares			
	Mean	Median	Std. Dev.	# obs
All 2-digit products	1.19%	0.03%	5.89%	11,605
Market share $\geq$ 20%	53.8%	44.0%	26.2%	100
Market share $\geq$ 50%	80.1%	86.7%	18.9%	42

Note: Table shows descriptive statistics for the aggregate destination-specific market shares of countries at the HS 2-digit level.

Table 4 shows descriptive statistics for our estimates of σ and γ. As discussed, we constrain coefficient estimates to be identical within 2-digit HS codes to guarantee a sufficient number of observations underlying each estimate. For the average and median sector, we estimate mildly decreasing returns to scale of $\gamma = 0.34$ and $\gamma = 0.19$, respectively. For our price elasticity estimates, we find a mean of 5.39 and a median of 3.74. Reassuringly, these

[18]The small average market shares reported in Table 3 also reflects the fact that HHI data are mainly available for smaller exporting countries in the EDD. See Appendix C for a list of exporting and importing countries in our data.

numbers are very similar to estimates at comparable levels of aggregation estimated in the literature (e.g., Broda and Weinstein, 2006).

Table 4: Price Elasticities and Returns-to-Scale Estimates – Cournot Competition

	σ	γ
Mean	5.39	0.34
25th Percentile	2.22	0.03
Median	3.74	0.10
75th Percentile	7.50	0.30
Min	1.01	-0.13
Max	26.07	4.46
Standard Deviation	4.07	0.69
HS 2-digit products	78	78

Note: Table shows descriptive statistics for estimates of σ and γ. Estimates computed using 6-digit HS firm-level information but constrained to be identical within 2-digit HS products.

5.4 Firm-Level Estimation Results

We now turn to the estimation of our firm-level gravity equations with and without correction for oligopoly bias (i.e., equations (6) and (7)). As a first step, we pool across all firms in our data and estimate equations (6) and (7) via PPML using a full set of firm-product-year and destination-product-year fixed effects. We include three regressors commonly used in gravity equations: bilateral distance, a dummy variable for the presence of a common official language in the exporting and importing country and a dummy variable taking a value of one if both countries were a member of the Eurozone in a given year. Note that because our destination countries are all in Europe, there is insufficient variation to include other commonly used indicators such as dummies for contiguity or membership in a free trade agreement.[19]

Table 5 presents the results using estimated values of σ and γ to construct the correction term. It shows that when pooling across all firms in our data, a setting where the median firm only has a negligible market share (see Table 1), there is not much bias in the coefficient estimates. The point estimates do not change much when correcting for oligopoly bias, with

[19]The destination countries in our sample were either already EU member states or had implemented free trade agreements with the EU before 2000. By contrast, China did not have any FTAs with countries in our sample before 2010. Thus, there would be no variation in the FTA dummy that is separate from our firm-product-year dummies. Likewise, there is insufficient variation to include an indicator for contiguity as it would largely overlap with our common official language dummy, which is one only for the pair France-Belgium annd France-Switzerland.

Table 5: Firm-Level Gravity Estimates, Estimated σ and γ.

Regressor	With correction	Without correction	Bias (%)
ln(distance)	-0.302***	-0.316***	-4.6%
	(0.0707)	(0.0698)	
Euro	0.334***	0.332***	-0.6%
	(0.0370)	(0.0358)	
Common language	-0.058	-0.025	56.3%
	(0.061)	(0.058)	
Observations	10,471,193	10,471,193	

Note: Table shows results for PPML estimation of equations (6) and (7). Standard errors in brackets, clustered at the destination-year level. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

Table 6: Firm-Level Gravity Estimates, $\sigma = 5$, $\gamma = 0$.

Regressor	With correction	Without correction	Bias (%)
ln(distance)	–0.312***	-0.299***	4.1%
	(0.0708)	(0.0694)	
Euro	0.343***	0.340***	-0.8%
	(0.0380)	(0.0364)	
Common language	-0.034	-0.011	36.3%
	(0.072)	(0.062)	
Observations	10,462,038	10,462,038	

Note: Table shows results for PPML estimation of equations (6) and (7). Standard errors in brackets, clustered at the destination-year level. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

the exception of the effect of common language, which is, however, estimated very imprecisely in our sample. [20] In Table 6 we report results for the case where the correction term has been constructed assuming that $\sigma = 5$ and $\gamma = 0$ in all sectors. Again the point estimates do not change much when controlling for the correction term.

We now turn to an alternative setting where considerations of market power are likely to be important in practice: the estimation of firm-level gravity equations at the sectoral or product level. While firms might not have noticeable market power for most products, there is a significant minority of highly concentrated sectors where firms do have large market shares. We illustrate the importance of oligopoly bias at the product level by estimating firm-level gravity equations separately for each HS 6-digit product in our data. Table 7 reports descriptive statistics for the absolute percentage difference in coefficient estimates with and without a correction for oligopoly bias.[21] For example, line 1 reports that whereas for the median HS 6-digit product in our data, the oligopoly bias on the ln(*distance*) coefficient is only around 13%, this increases to 38% at the 75th percentile and to 74% at the 90th percentile. That is, not correcting for market power would bias the distance coefficient by over 74% in 10% of sectors.[22] This bias is of comparable magnitude for our Eurozone dummy and for the common language dummy.[23]

Table 7: Firm-Level Gravity Estimates by Product, Estimated σ and γ

	Absolute Percentage bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	#obs
ln(distance)	13%	38%	74%	747
Eurozone	14%	44%	110%	746
Common language	11%	38%	95%	739

Note: Table shows results based on firm-level regressions by product. Bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

[20] In order for our results not to be driven by extreme outliers in the oligopoly correction term, the results presented here exclude observations where the correction term is larger than 100. Choosing other cutoffs or dropping the top and bottom 1% of observations in terms of the correction term instead yields quantitatively similar results.

[21] Note that these regressions are based on fully rectangularized product-level datasets. That is, if a firm does not report exports for a given destination-year combination, we set these exports to zero.

[22] We had to exclude a large share of the 1,824 products in our sample because we could not estimate sector-specific values for σ and γ due to a lack of observations, because the PPML estimation did not converge or because the bilateral coefficients were not identified due to only French or only Chinese exporters being present for a given product. We chose to exclude a further small set of of products where the distance coefficient took on implausible values (positive or smaller than -20).

[23] The number of underlying product-level regressions varies across coefficients because there is not always sufficient variation in the data to identify the coefficients on the Eurozone and common language dummies. For example, the common language coefficient is not identified if the French firms in a given sector/product do not export to Belgium or Switzerland, or if they only export to these two countries.

As a robustness check, we also report product-level results for the importance of oligopoly bias when we set $\sigma = 5$ and $\gamma = 0$ and and recompute our correction terms. As seen in Table 8, this approach substantially increases the estimated oligopoly bias. Intuitively, there are now considerably fewer sectors with values of $\sigma < 5$ and hence fewer correction terms of relatively small magnitude.

Table 8: Firm-Level Gravity Estimates by Product, $\sigma = 5$ and $\gamma = 0$

	Absolute percentage bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	#obs
ln(distance)	36%	70 %	91%	676
Eurozone	33%	84%	123%	675
Common language	33%	85%	131%	670

Note: Table shows results based on firm-level regressions by product. Bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

5.5 Product Level Estimation Results

While firm-level trade data are increasingly becoming available to researchers, in many cases gravity estimations are still based on more aggregate types of data, such as product-level trade flows. Of course, the issue of coefficient bias due to oligopolistic behavior does not go away at this level as aggregate exports are simply the sum of underlying firm-level exports. In this section, we use the HHI-based correction term proposed in Section 4 to investigate the quantitative importance of oligopoly bias for product-level regressions.

We first present results for the pooled regressions using the correction terms based on the HHI information from the World Bank's EDD. We work at the 2-digit level of the HS classification which is the most disaggregated level for bilateral HHI data available in the EDD. Table 9 shows results for a regression in which we pool all 2-digit products and regress bilateral product level exports on a set of bilateral gravity regressors as well as exporter-product and importer-product fixed effects.

The results in Table 9 demonstrate that when pooling across all sectors, the oligopoly bias is again relatively small. Table 10 shows that, when we run our regressions product by product, there are a number of 2-digit products where oligopoly bias is substantial for all regressors, even though sectors are quite aggregate. For example, for 10% of products, the bias of the distance coefficient is larger than 15%. Moreover, for 10% of products we find a bias on the Euro dummy of at least 138%. For the common language dummy, coefficient estimates display a bias of 50% or more for 10% of products.

Table 9: Product-Level Gravity Estimates, All Products, by 2-digit HS, $\sigma = 5$ and $\gamma = 0$

Regressor	Without correction	With correction	Bias (%)
ln(distance)	-1.693***	-1.715***	0.3%
	(0.172)	(0.174)	
Euro	0.475**	0.582***	-18.4%
	(0.184)	(0.186)	
Common Language	0.589	0.551	6.9%
	(0.252)	(0.262)	
Observations	35,049	35,049	

Note: Table shows results for PPML estimation of equation (14) in exponential form with exporter-product and importer-product fixed effects, pooling across all 2-digit product in our data. Standard errors in brackets, clustered by country pair. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

In Table 11, we present results estimated at the finer 6-digit HS level, where the HHIs are constructed from our firm-level data as previously explained. We find that the magnitude of the bias becomes substantially larger when regressions are estimated at this more disaggregated level, because firms have more market power in more finely defined markets. For example, for 10% of sectors the bias of the distance coefficient is now larger than 73%.

Table 10: Product-Level Gravity Estimates by 2-digit HS Product, $\sigma = 5$ and $\gamma = 0$

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	1.8%	4.8%	15.6%	72
Euro	5.4%	25.3%	138.1%	70
Common Language	3.0%	11.7%	52.3%	72

Note: Table shows results based on product-level regressions estimated separately for each HS 2-digit product in our data. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

Table 11: Product-Level Gravity Estimates by 6-digit HS Product, $\sigma = 5$ and $\gamma = 0$

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	20.9%	45.8%	73.7%	728
Euro	22.8%	55.9%	112.8%	728
Common Language	22.2%	55.3%	125.6%	728

Note: Table shows results based on product-level regressions estimated separately for each HS 6-digit product in our data. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

6 Conclusions

In this paper, we have evaluated the consequences of oligopolistic behavior for the estimation of gravity equations for trade flows. We showed that with oligopolistic competition, firm-level gravity equations based on a standard CES demand framework need to be augmented by markup terms that are functions of firms' market shares. At the aggregate level, the additional term takes the form of the exporting country's market share in the destination country multiplied by an exporter-destination-specific Herfindahl-Hirschman index. We showed how to construct appropriate correction terms for both cases that can be used to avoid problems of omitted variable bias. Using combined French and Chinese firm-level export data as well as a sample of product-level imports by European countries, we showed that correcting for oligopolistic behavior can lead to substantial changes in the coefficients on standard gravity regressors.

Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. To complete the proof of the proposition, we need to: (a) Show that the function S is well defined, and study its monotonicity properties as well as its limits; (b) show that the equilibrium condition (10) has a unique solution; (c) show that, at $\lambda = 1$, the first-order conditions of profit maximization are sufficient for global optimality, so that the profile of quantities $(q_j^*(1))_{j\in\mathcal{J}}$ does constitute a Nash equilibrium of the Cournot game. We do so below.

(a) As $1 + \sigma\gamma > 0$, the right-hand of equation (9) is strictly increasing in s_i, whereas the left-hand side is non-increasing in s_i. It follows that equation (9) has at most one solution. As s_i tends to 0, the left-hand side of that equation tends to 1, whereas the right-hand side tends to 0. As s_i tends to ∞, the left-hand side tends to 1 or $-\infty$, and the right-hand side tends to $+\infty$. The equation therefore has a unique solution, $S(T_i/H, \lambda) \in (0, 1/\lambda)$, where $1/\lambda \equiv \infty$ when $\lambda = 0$.

It is easily checked that $S(\cdot,\cdot)$ is strictly increasing in its first argument and strictly decreasing in its second argument. By monotonicity, $S(\cdot, \lambda)$ has limits at 0 and ∞. Clearly, those limits are equal to 0 and $1/\lambda$, respectively.

(b) The results in part (a) of the proof imply that the left-hand side of equation (10) is strictly decreasing in H, and has limits 0 and $|\mathcal{J}|/\lambda$ as H tends to ∞ and 0, respectively. It follows that equation (10) has a unique solution, $H^*(\lambda)$.

(c) Rewriting equation (2) with $\lambda = 1$ and rearranging terms yields:

$$\frac{\partial \pi_i}{\partial q_i} = q_i^{\gamma} \left[\frac{\sigma - 1}{\sigma} \alpha E \frac{a_i^{\frac{1}{\sigma}} q_i^{-\frac{1+\sigma\gamma}{\sigma}}}{\sum_{j \in \mathcal{J}} a_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}}} \left(1 - \frac{a_i^{\frac{1}{\sigma}} q_i^{\frac{\sigma-1}{\sigma}}}{\sum_{j \in \mathcal{J}} a_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}}} \right) - c_i \tau_i \right],$$

where we have dropped the destination subscript for ease of notation. As $1 + \sigma\gamma > 0$, the term inside square brackets is strictly decreasing in q_i. Moreover, that terms tends to $+\infty$ and $-\tau_i c_i$ as q_i tends to 0 and $+\infty$, respectively. It follows that q_i maximizes firm i's profit if and only if firm i's first-order condition holds at q_i. □

A.2 Proof of Proposition 2

Proof. To apply Taylor's theorem, we require the value of $s_e^{*\prime}(0)$. This requires computing the partial derivatives of $S(\cdot, \cdot)$ at $\lambda = 0$ as well as $H^{*\prime}(0)$. Differentiating equation (9) with respect to s_i, λ, and $t_i \equiv T_i/H$ at $\lambda = 0$ yields

$$-s_i d\lambda = \frac{1 + \sigma\gamma}{\sigma - 1} \frac{ds_i}{s_i} - \frac{\sigma(1+\gamma)}{\sigma - 1} \frac{dt_i}{t_i}.$$

It follows that[24]

$$t_i \partial_1 \log S(t_i, 0) = \frac{\sigma(1+\gamma)}{1 + \sigma\gamma} \quad \text{and} \quad \partial_2 \log S(t_i, 0) = -\frac{\sigma - 1}{1 + \sigma\gamma} S(t_i, 0).$$

Next, we differentiate equation (10) with respect to λ and H:

$$\sum_{j \in \mathcal{J}} \left[-\frac{T_j}{H} \partial_1 S\left(\frac{T_j}{H}, \lambda\right) \frac{dH}{H} + \partial_2 S\left(\frac{T_j}{H}, \lambda\right) d\lambda \right] = 0.$$

[24]Notation: $\partial_k S$ is the partial derivative of S with respect to its kth argument.

Setting $\lambda = 0$ and plugging in the values of the partial derivatives of S, we obtain:

$$\sum_{j \in \mathcal{J}} \left[-\frac{\sigma(1+\gamma)}{1+\sigma\gamma} s_j^*(0) \frac{dH}{H} - \frac{\sigma - 1}{1+\sigma\gamma} \left(s_j^*(0) \right)^2 d\lambda \right] = 0.$$

Making use of the definition of HHI(0) and of the fact that market shares add up to unity, we obtain:

$$\frac{H^{*\prime}(0)}{H^*(0)} = -\frac{\sigma - 1}{\sigma(1+\gamma)} \,\mathrm{HHI}(0).$$

We can now compute $s_i^{*\prime}(0)$:

$$\begin{aligned}
s_i^{*\prime}(0) &= \frac{\partial}{\partial \lambda} S\left(\frac{T_i}{H^*(\lambda)}, \lambda \right) \Bigg|_{\lambda=0} \\
&= -\frac{T_i}{H^*(0)} \partial_1 S \left(\frac{T_i}{H^*(0)}, 0 \right) \frac{H^{*\prime}(0)}{H^*(0)} + \partial_2 S \left(\frac{T_i}{H^*(0)}, 0 \right) \\
&= \frac{\sigma - 1}{1+\sigma\gamma} \left[s_i^*(0) \,\mathrm{HHI}(0) - (s_i^*(0))^2 \right].
\end{aligned}$$

It follows that

$$\begin{aligned}
\frac{s_e^{*\prime}(0)}{s_e^*(0)} &= \frac{\sigma - 1}{1+\sigma\gamma} \frac{1}{s_e^*(0)} \sum_{j \in \mathcal{E}} \left[s_j^*(0) \,\mathrm{HHI}(0) - \left(s_j^*(0) \right)^2 \right] \\
&= \frac{\sigma - 1}{1+\sigma\gamma} \left[\mathrm{HHI}(0) - s_e^*(0) \sum_{j \in \mathcal{E}} \left(\frac{s_j^*(0)}{s_e^*(0)} \right)^2 \right] \\
&= \frac{\sigma - 1}{1+\sigma\gamma} \left[\mathrm{HHI}(0) - s_e^*(0) \,\mathrm{HHI}_e(0) \right].
\end{aligned}$$

Applying Taylor's theorem at the first order in the neighborhood of $\lambda = 0$ yields:

$$\begin{aligned}
\log s_e^*(\lambda) &= \log s_e^*(0) + \frac{d}{d\lambda} \log s_e^*(\lambda) \Bigg|_{\lambda=0} \lambda + o(\lambda) \\
&= \log s_e^*(0) + \frac{\sigma - 1}{1+\sigma\gamma} \left[\mathrm{HHI}(0) - s_e^*(0) \,\mathrm{HHI}_e(0) \right] \lambda + o(\lambda) \\
&= \log s_e^*(0) + \frac{\sigma - 1}{1+\sigma\gamma} \left[\mathrm{HHI}(\lambda) - s_e^*(\lambda) \,\mathrm{HHI}_e(\lambda) \right] \lambda + o(\lambda),
\end{aligned}$$

where the last line follows from the fact that $\mathrm{HHI}(\lambda) - \mathrm{HHI}(0)$ and $s_e^*(\lambda) \,\mathrm{HHI}_e(\lambda) - s_e^*(0) \,\mathrm{HHI}_e(0)$ are at most first order. □

B Price Competition

B.1 Theoretical Results

Under price competition, the profit of firm i when selling in destination n is:

$$\pi_{in} = p_{in} a_{in} p_{in}^{-\sigma} P_n^{\sigma-1} \alpha_n E_n - C_{in}\left(a_{in} p_{in}^{-\sigma} P_n^{\sigma-1} \alpha_n E_n\right),$$

where we have dropped the sector index z for ease of notation.

The degree of strategic interactions between firms continues to be governed by the conduct parameter $\lambda \in [0,1]$: When firm i increases its price by an infinitesimal amount, it perceives the induced effect on P_n to be equal to $\lambda \partial P_n / \partial p_{in}$. It is still the case that monopolistic competition arises when $\lambda = 0$, whereas Bertrand competition arises when $\lambda = 1$. The first-order condition of profit maximization of firm i in destination n is given by

$$\begin{aligned} 0 = \frac{\partial \pi_{in}}{\partial p_{in}} &= a_{in} p_{in}^{-\sigma} P_n^{\sigma-1} \alpha_n E_n + \left(p_{in} - C'_{in}(q_{in})\right) \left[-\frac{\sigma}{p_{in}} + \frac{\sigma-1}{P_n} \lambda \frac{\partial P_n}{\partial p_{in}} \right] \alpha_n E_n a_{in} p_{in}^{-\sigma} P_n^{\sigma-1} \\ &= q_{in} \left(1 - \frac{p_{in} - C'_{in}(q_{in})}{p_{in}} \left[\sigma - \lambda(\sigma-1) s_{in}\right] \right), \end{aligned} \tag{16}$$

where

$$s_{in} \equiv \frac{a_{in} p_{in}^{1-\sigma}}{\sum_{j \in \mathcal{J}} a_{jn} p_{jn}^{1-\sigma}} \tag{17}$$

continues to be the market share of firm i in destination n.

Equation (16) pins down firm i's optimal markup under price competition:

$$\mu_{in} = \frac{1}{\sigma - \lambda\left(\sigma - 1\right) s_{in}},$$

where $\mu_{in} = \frac{p_{in} - C'_{in}(q_{in})}{p_{in}}$ is firm i's Lerner index. Apart from this change in the expression for the firm's optimal markup, all other firm-level results go through as before.

We now turn our attention to the sector-level results. As in Section 4, we begin by employing an aggregative games approach to analyze the equilibrium in a given market, dropping the market subscript n to ease notation. The market-level aggregator H is now defined as

$$H \equiv P^{1-\sigma} = \sum_{j \in \mathcal{J}} a_j p_j^{1-\sigma}$$

and firm i's type as

$$T_i \equiv a_i \left(\alpha E\right)^{\frac{\gamma(1-\sigma)}{1+\gamma}} \left(c_i \tau_i\right)^{\frac{1-\sigma}{1+\gamma}}.$$

Plugging these definitions into equation (16), making use of equation (17), and rearranging, we obtain:

$$\left(1 - s_i^{\frac{1+\sigma\gamma}{\sigma-1}} \left(\frac{H}{T_i}\right)^{\frac{1+\gamma}{\sigma-1}}\right)(\sigma - \lambda(\sigma-1)s_i) = 1. \tag{18}$$

Note that the left-hand side of equation (18) is strictly decreasing on the interval

$$\left(0, \min\left\{\frac{\sigma}{\lambda(\sigma-1)}, \left(\frac{T_i}{H}\right)^{\frac{1+\gamma}{1+\sigma\gamma}}\right\}\right)$$

and tends to σ and 0 as s_i tends to the lower and upper endpoints of that interval, respectively. Equation (18) therefore has a unique solution on the above interval, denoted $S(t_i, \lambda)$ with $t_i \equiv T_i/H$. (Solutions outside that interval necessarily give rise to strictly negative markups and are thus suboptimal.)

It is easily checked that S is strictly increasing in its first argument, strictly decreasing in its second argument, and tends to 0 and $1/\lambda$ as t_i tends to 0 and ∞, respectively.

As before, the equilibrium condition is that market shares must add up to unity:

$$\sum_{j \in \mathcal{J}} S\left(\frac{T_i}{H}, \lambda\right) = 1. \tag{19}$$

The properties of the function S, described above, imply that this equation has a unique solution, $H^*(\lambda)$.

To summarize:

Proposition A. *In each destination market, and for any conduct parameter λ, there exists a unique equilibrium in prices. The equilibrium aggregator level $H^*(\lambda)$ is the unique solution to equation* (19)*. Each firm i's equilibrium market share is $s_i^*(\lambda) = S(T_i/H^*(\lambda), \lambda)$, where $S(T_i/H^*(\lambda), \lambda)$ is the unique solution to equation* (18)*. From equation* (17)*, firm i's equilibrium price is given by*

$$p_i^*(\lambda) = \left(\frac{s_i^*(\lambda) H^*(\lambda)}{a_i}\right)^{\frac{1}{1-\sigma}}.$$

Proof. All that is left to do is check that first-order conditions are sufficient for optimality when $\lambda = 1$. Combining equations (16) and (18) yields:

$$\frac{\partial \pi_i}{\partial p_i} = q_i \left[1 - \chi(p_i)\phi(p_i)\right],$$

where

$$\chi(p_i) \equiv 1 - \left(\frac{a_i p_i^{1-\sigma}}{\sum_j a_j p_j^{1-\sigma}}\right)^{\frac{1+\sigma\gamma}{\sigma-1}} \left(\frac{\sum_j a_j p_j^{1-\sigma}}{T_i}\right)^{\frac{1+\gamma}{\sigma-1}} \text{ and } \phi(p_i) \equiv \sigma - (\sigma - 1)\frac{a_i p_i^{1-\sigma}}{\sum_j a_j p_j^{1-\sigma}}.$$

As $1 + \sigma\gamma > 0$, the functions χ and ϕ are strictly increasing. Moreover, $\phi(p_i) > 0$ for every p_i, whereas there exists $\widetilde{p}_i > 0$ such that $\chi(p_i) > 0$ if $p_i > \widetilde{p}_i$ and $\chi(p_i) < 0$ if $p_i < \widetilde{p}_i$. Hence, π_i is strictly increasing on the interval $(0, \widetilde{p}_i)$, and firm i's first-order condition holds nowhere on that interval. The fact that $\lim_{p_i \to \infty} \chi(p_i) = 1$ and $\lim_{p_i \to \infty} \phi(p_i) = \sigma$ and the monotonicity properties of χ and ϕ on $(\widetilde{p}_i, \infty)$ imply the existence of a unique $\widehat{p}_i$ at which firm i's first-order condition holds. Moreover, π_i is strictly increasing on $(\widetilde{p}_i, \widehat{p}_i)$ and strictly decreasing on $(\widehat{p}_i, \infty)$. First-order conditions are therefore sufficient for optimality. □

Having characterized the equilibrium in a given destination, we now adapt the first-order approach to sector-level gravity to the case of price competition. As in Section 4, let $\mathcal{E} \subsetneq \mathcal{J}$ denote the subset of exporters in country e that sell in the destination market n. The combined market share of those exporters in market n is given by

$$s_e^*(\lambda) \equiv \sum_{i \in \mathcal{E}} s_i^*(\lambda).$$

As before, we approximate $s_e^*(1)$ at the first order. The definitions of HHI and HHI_e are as in Section 4.

We obtain:

Proposition B. *At the first order, in the neighborhood of $\lambda = 0$, the logged joint market share in destination n of the firms from export country e is given by*

$$\log s_e^*(\lambda) = \log s_e^*(0) + \frac{\sigma - 1}{\sigma(1 + \sigma\gamma)} \left[\text{HHI}(\lambda) - s_e^*(\lambda)\, \text{HHI}_e(\lambda)\right] \lambda + o(\lambda).$$

Proof. The proof follows the same developments as the proof of Proposition 2. We begin by computing the partial derivatives of S at $\lambda = 0$. It is useful to rewrite first equation (18) as

$$s_i = t_i^{\frac{1+\gamma}{1+\sigma\gamma}} \left(1 - \frac{1}{\sigma - \lambda(\sigma-1)s_i}\right)^{\frac{\sigma-1}{1+\sigma\gamma}}. \tag{20}$$

Taking the logarithm and totally differentiating the equation at $\lambda = 0$ yields:

$$\frac{ds_i}{s_i} = \frac{1+\gamma}{1+\sigma\gamma}\frac{dt_i}{t_i} - \frac{\sigma-1}{\sigma(1+\sigma\gamma)} s_i d\lambda.$$

The partial derivatives of S are thus given by

$$t_i \partial_1 \log S(t_i, 0) = \frac{1+\gamma}{1+\sigma\gamma} \quad \text{and} \quad \partial_2 \log S(t_i, 0) = -\frac{\sigma-1}{\sigma(1+\sigma\gamma)} S(t_i, 0).$$

To obtain $H^{*\prime}(0)$, we differentiate equation (19):

$$\sum_{j\in\mathcal{J}} \left[-\frac{T_j}{H}\partial_1 S\left(\frac{T_j}{H}, \lambda\right)\frac{dH}{H} + \partial_2 S\left(\frac{T_j}{H}, \lambda\right) d\lambda \right] = 0.$$

Setting $\lambda = 0$, plugging in the values of the partial derivatives of S, and using the fact that market shares add up to unity, we obtain:

$$\frac{H^{*\prime}(0)}{H^*(0)} = -\frac{\sigma-1}{\sigma(1+\gamma)} \text{HHI}(0).$$

Next, we compute $s_i^{*\prime}(0)$:

$$\begin{aligned} s_i^{*\prime}(0) &= -\frac{T_i}{H^*(0)}\partial_1 S\left(\frac{T_i}{H^*(0)}, 0\right)\frac{H^{*\prime}(0)}{H^*(0)} + \partial_2 S\left(\frac{T_i}{H^*(0)}, 0\right) \\ &= \frac{\sigma-1}{\sigma(1+\sigma\gamma)} \left[s_i^*(0)\,\text{HHI}(0) - (s_i^*(0))^2 \right]. \end{aligned}$$

Adding up and dividing by $s_e^*(0)$ yields:

$$s_e^{*\prime}(0) = \frac{\sigma-1}{\sigma(1+\sigma\gamma)} \left[\text{HHI}(0) - s_e^*(0)\,\text{HHI}_e(0) \right].$$

As in the proof of Proposition 2, we can then apply Taylor's theorem to obtain the result. □

Proposition B motivates the following approximation:

$$\log s_e^*(1) \simeq \log s_e^*(0) + \frac{\sigma-1}{\sigma(1+\sigma\gamma)} \left[\text{HHI}(1) - s_e^*(1)\,\text{HHI}_e(1) \right].$$

As in Section 4, this approximation can then be used to derive the sector-level gravity regression

$$\log \widetilde{r}_{en} = \zeta_e + \xi_n + \beta \frac{1-\sigma}{1+\sigma\gamma} X_{en} + \eta_{en}$$

where

$$\log \widetilde{r}_{en} \equiv \log r_{en} + \frac{\sigma - 1}{\sigma(1+\sigma\gamma)} s_{en} \, \mathrm{HHI}_{en}$$

is the value of export flows from e to n, purged from oligopolistic market power effects. Note that the correction term under price competition is equal to the one under quantity competition divided by σ.

B.2 Empirical Results

Table 12 presents results for our estimates of σ and γ using the estimation procedure from Section 3 but replacing the Cournot markup formula with its Bertrand equivalent. This only leads to minor changes in coefficient estimates.

Table 12: Price Elasticities and Returns-to-Scale Estimates – Price Competition

	σ	γ
Mean	4.96	0.31
25th Percentile	2.06	0.02
Median	3.27	0.10
75th Percentile	5.22	0.28
Min	1.01	-0.11
Max	26.03	4.5
Standard Deviation	4.89	0.67
HS 2-digit products	78	78

Note: Table shows descriptive statistics for estimates of σ and γ. Estimates computed using 6-digit HS firm-level information but constrained to be identical within 2-digit HS products.

Tables 13 and 14 show results for the pooled firm-level regressions, first with the estimated values for σ and γ from Table 12 and then setting $\sigma = 5$ and $\gamma = 0$. In both specifications, the point estimates do not change much when correcting for oligopoly bias (except for the common-language coefficient in Table 14, which is however estimated very imprecisely), as expected from such pooled regressions.

Tables 15 and 16 present results for our firm-level regressions run product by product, first with the estimated values for σ and γ from Table 12 and then setting $\sigma = 5$ and $\gamma = 0$. As before, there is always a significant minority of sectors for each of the three regressors were the oligopoly bias is substantial.

Table 13: Firm-Level Gravity Estimates with Price Competition, Estimated σ and γ,

Regressor	Without correction	With correction	Bias (%)
ln(distance)	-0.318***	-0.314***	1.2%
	(0.0692)	(0.0703)	
Eurozone	0.322***	0.324***	-0.6%
	(0.0366)	(0.0358)	
Common Language	-0.029	-0.028	3.5%
	(0.538)	(0.529)	
Observations	10,474,220	10,474,220	

Note: Table shows results for PPML estimation of equations (6) and (7) in exponential form with firm-product-year and destination-product-year fixed effects, pooling across all firms in our data. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

Table 14: Firm-Level Gravity Estimates with Price Competition, $\sigma = 5$, $\gamma = 0$

Regressor	Without correction	With correction	Bias (%)
ln(distance)	-0.294** *	-0.296***	-0.6%
	(0.0690)	(0.0691)	
Eurozone	0.331***	0.330***	0.5%
	(0.0365)	(0.0362)	
Common Language	-0.026	-0.016	37.2%
	(0.528)	(0.531)	
Observations	10,471,286	10,471,286	

Note: Table shows results for PPML estimation of equations (6) and (7) in exponential form with firm-product-year and destination-product-year fixed effects, pooling across all firms in our data. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

We now turn to the aggregate product-level results with Bertrand competition. As before, we first present findings for a regression that pools all 2-digit sectors (Table 17) and then run regressions at the HS 2 level product by product, showing descriptive statistics for the resulting coefficient bias (Table 18). As expected from our theoretical finding that the correction term with Bertrand is only a fraction $(1/\sigma)$ of its Cournot equivalent, the changes in coefficient estimates when correcting for oligopoly bias are less pronounced than before. However, there are still 10% of products for which we find a bias larger than 9% for the common language dummy. In Table 19, we present results estimated at the finer 6-digit HS level. Here, we compute HHIs using our firm-level data, as previously explained. Like in the case of Cournot competition, the magnitude of the bias is larger when regressions are estimated at this more disaggregated level.

Table 15: Firm-Level Gravity Estimates by Product with Price Competition, Estimated σ and γ

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	5%	22%	53%	786
Eurozone	4%	17%	61%	784
Common Language	4%	14%	52%	780

Note: Table shows results based on firm-level regressions estimated separately for each product in our data. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

Table 16: Firm-Level Gravity Estimates by Product with Price Competition, $\sigma = 5$ and $\gamma = 0$

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	12%	48%	80%	751
Eurozone	12%	52%	106%	749
Common Language	11%	43%	101%	744

Note: Table shows results based on firm-level regressions estimated separately for each product in our data. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

Table 17: Product-Level Gravity Estimates with Price Competition, $\sigma = 5$ and $\gamma = 0$

Regressor	Without correction	With correction	Bias (%)
ln(distance)	-1.693***	-1.698***	0.3%
	(0.172)	(0.172)	
Euro	0.475**	0.486***	2.2%
	(0.186)	(0.186)	
Common Language	0.589	0.589	0%
	(0.243)	(0.255)	
Observations	35,049	35,049	

Note: Table shows results for PPML estimation of equation (14) in exponential form with exporter-product and importer-product fixed effects, pooling across all 2-digit product in our data. Standard errors in brackets, clustered by country pair. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Bias is defined as $(\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|$.

Table 18: Product-Level Gravity Estimates by 2-digit HS Product with Price Competition, $\sigma = 5$ and $\gamma = 0$

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	0.2%	0.8%	2.6%	72
Euro	1%	4%	11%	72
Common Language	0.5%	2%	9%	72

Note: Table shows results based on product-level regressions estimated separately for each HS 2-digit product in our data. (*), (**), (***) denotes 10, 5 and 1 percent statistical significance, respectively. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

Table 19: Product-Level Gravity Estimates by 6-digit HS Product with Price Competition, $\sigma = 5$ and $\gamma = 0$

	Absolute Percentage Bias (%)			
Regressor	50th percentile	75th percentile	90th percentile	# obs
ln(distance)	4.3%	12.9%	29.9%	799
Euro	4.6%	13.6%	44.1%	799
Common Language	4.5%	12.2%	29.5%	799

Note: Table shows results based on product-level regressions estimated separately for each HS 6-digit product in our data. Absolute percentage bias is defined as the absolute value of the percentage difference in coefficient estimates, $abs((\hat{\beta}_{nocorr} - \hat{\beta}_{corr})/|\hat{\beta}_{corr}|)$.

C Data Appendix

Table 20: List of export destinations included in the firm-level and product-level data

Austria	Latvia
Belgium	Lithuania
Bulgaria	Luxembourg*
Croatia	Malta*
Cyprus*	Netherlands
Czech Rep.	Norway*
Denmark	Poland
Estonia	Portugal
Finland	Romania
France	Serbia*
Germany	Slovakia
Greece	Slovenia
Hungary	Spain
Iceland*	Sweden
Ireland	Turkey*
Italy	UK

Note: * denotes countries only included in the firm-level regressions and in the product-level regressions where the data have been constructed from firm-level export data.

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