# The Design of Trade Agreements Under Monopolistic Competition*

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This draft: January 2022

We study whether trade agreements should also constrain domestic policies from the perspective of monopolistic competition models with heterogeneous firms and multiple sectors. We consider shallow and deep trade agreements, modeled according to GATT-WTO rules. We show that, in contrast to deep agreements, shallow agreements, complying with market access commitments and tariff bindings, are not sufficient to achieve global efficiency because terms-of-trade improvements can be obtained without reducing foreign market access. We also show how the performance of different trade agreements is affected by firm heterogeneity and that the distortions arising from uncoordinated domestic policies increase when trade costs fall.

**Keywords: Heterogeneous Firms, Trade Policy, Domestic Policy, Trade Agreements, Terms of Trade, Efficiency**

**JEL classification codes: F12, F13, F42**

*We thank Costas Arkolakis, Paola Conconi, Jan Haaland, Eckhard Janeba, Ahmad Lashkaripour, Monika Mrazova, Mathieu Parenti, Gonzague Vanoorenberghe, and seminar participants in Glasgow, Mannheim, Norwegian School of Economics, Nottingham, PSE, Sciences Po, Southampton and conference participants at EEA, ETSG, the CRC TR 224 conference, the Midwest International Trade Conference, ESEM, the ITSG, the workshop on Trade Policy and Firm Performance and the 5th CEPR Conference on Global Value Chains, Trade and Development for comments.

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# 1 Introduction

We are witnessing a change in the way countries approach trade policy. In the past, regional and multilateral trade agreements were mostly ”shallow”, i.e. focused on the reduction of import tariffs and export taxes. More recently, there has been a shift to ”deeper” agreements, which, in addition to traditional trade policies, cover various domestic policies, such as production subsidies, product and labor standards, intellectual property rights, competition policy, and many other subjects (e.g., Horn, Mavroidis and Sapir, 2010; Dür, Baccini and Elsig, 2014; Rodrik, 2018).[1] Despite these fundamental changes in countries' actual approach to trade agreements, much of the theoretical literature still focuses on classical trade policies: import and export taxes (see Bagwell and Staiger, 2016, for a survey). An important contribution by Bagwell and Staiger (2001) considers domestic policies in the context of neoclassical trade models and argues that the restrictions imposed by shallow integration under the current GATT-WTO rules (tariff bindings and market access commitments) are sufficient to reach global efficiency without requiring coordination of domestic policies. Whether this result extends to the workhorse trade model featuring monopolistic competition and firm heterogeneity and how to optimally design trade agreements in this framework remains, however, an open question.

To fill this gap, we consider a general trade model with monopolistic competition and free entry (Krugman, 1980), firms that are heterogeneous in terms of productivity (Melitz, 2003) and operate in multiple sectors with CES demand. This model is particularly well suited for studying domestic policies and thus deep trade agreements because it features a clear motive for domestic regulation, even in the absence of international trade: without sector-specific production subsidies, market outcomes are distorted by monopolistic price setting due to multiple sectors with different markups (Ottaviano, Nocco and Salto, 2019). At the same time, our setup allows us to study to what extent policies and the optimal design of trade agreements are af-

[1] To illustrate the increasing depth and complexity of trade agreements, Rodrik (2018) compares the US trade agreements with Israel and Singapore, signed two decades apart. The US-Israel Free Trade Agreement, which went into force in 1985, was the first bilateral trade agreement the US concluded in the postwar period. It contains 22 articles and three annexes, the bulk of which are devoted to free-trade issues such as tariffs, agricultural restrictions, import licensing, and rules of origin. The US-Singapore Free Trade Agreement went into effect in 2004 and contains 20 chapters (each with many articles), more than a dozen annexes, and multiple side letters. Of its 20 chapters, only seven cover conventional trade topics. Other chapters deal with behind-the-border topics such anti-competitive business conduct, electronic commerce, labor, the environment, investment rules, financial services, and intellectual property rights.

fected by the presence of firm heterogeneity. We model domestic policies in terms of production taxes/subsidies because they fit most naturally into the Melitz (2003) framework. However, conceptually one can think more broadly of any policies that aim at correcting a distortion between domestic social marginal costs and domestic social marginal benefits, such as market power, or a consumption or production externality. This covers, e.g., issues such as competition policy, environmental and product standards or subsidies for research and development.

We then study the relative performance of trade agreements with different levels of integration: several forms of *shallow trade agreements* (agreements on trade taxes without coordination of domestic policies) modeled according to GATT-WTO rules; a *deep trade agreement* (cooperation on trade taxes and domestic policies); and a *laissez-faire agreement* (free trade and a commitment to abstain from using domestic policies). We find that – in contrast to the result obtained in the neoclassical trade model – under monopolistic competition shallow agreements in combination with market access commitments and tariff bindings are not sufficient to achieve the full benefits of globalization. Obtaining a globally efficient outcome requires signing a deep trade agreement. Moreover, firm heterogeneity crucially affects the costs and benefits of a shallow free trade agreement relative to a laissez-faire agreement.

In order to interpret our findings in the light of the incentives for trade and domestic policies faced by individual-country policymakers, we make use of a novel welfare decomposition written in terms of the aggregate representation of the model that we derive in a companion paper (Campolmi, Fadinger and Forlati, 2022). In that paper, we show that the general-equilibrium welfare effects induced by trade or domestic policies in monopolistic competition models with a CES demand structure can be exactly decomposed into (i) consumption-efficiency and (ii) production-efficiency effects and (iii) aggregate terms-of-trade effects that operate via changes in international prices.[2] As our welfare decomposition is valid independently of the number of policy instruments, it is particularly useful for studying policy in second-best environments, like those arising under shallow trade agreements, where the available instruments (domestic policies) are not sufficient to separate production-efficiency from terms-of-trade motives and

[2]The terms of trade are defined in terms of ideal price indices of exportables and importables. As a consequence, the terms of trade are affected both by changes in the international prices of individual varieties (intensive margin) and by changes in the set of firms active in foreign markets (extensive margin).

thus policy makers face a trade-off between them.[3]

To establish the trade-off between production-efficiency and aggregate terms-of-trade effects associated with using individual policy instruments, it is useful to recall some results for unilateral deviations from the (inefficient) laissez-faire allocation derived in Campolmi et al. (2022). In particular, consider a small import tariff or a small export or production subsidy. They trigger entry of firms and increase the amount of labor allocated to this sector. This improves production efficiency by reducing wedges due to monopolistic markups, while worsening the aggregate terms of trade via the extensive margin by reducing the ideal price index of the exportable bundle.[4] There exists a sufficient statistic, the variable profit share of the average active firm from sales in its domestic market, that determines which effect dominates. When the profit share from domestic sales is larger than the one from export sales, the terms-of-trade motive is weak relative to the production-efficiency motive: only relatively few firms select into exporting and most sales go to the domestic market. Thus, increasing production efficiency is the dominant motive and policy makers exploit the delocation effect to achieve this outcome. By contrast, when the profit share from domestic sales is smaller than the one from export sales the terms-of-trade motive dominates. Consequently, countries can benefit from a small unilateral import subsidy, a production tax, or an export tax that delocates firms to the foreign market (an anti-delocation effect).

With an understanding of the theoretical mechanisms that govern policy makers' incentives, we then study strategic policy in the absence of a trade agreement and the normative implications of trade agreements with different degrees of integration.

We first consider strategic trade and domestic policies in the absence of any type of trade agreement in order to have a benchmark for the distortions arising without international cooperation. In this case, the targeting principle applies and strategic outcomes are qualitatively independent of firm heterogeneity: in a symmetric Nash equilibrium, production subsidies are

[3]As explained in more detail in Campolmi et al. (2022), our decomposition extends and generalizes the concept of "politically optimal policies" – those policies that policy makers would choose if they did not value terms-of-trade-effects (Bagwell and Staiger, 1999, 2001) – to situations where policy makers do not dispose of a sufficient set of instruments to separately deal with production-efficiency and terms-of-trade effects. By contrast, "politically optimal policies" generally do not allow identifying policy-makers' incentives in such a situation (see, e.g. Bagwell and Staiger (2016), page 26.).

[4]Note that the negative terms-of-trade effect of a tariff stands in contrast with the neoclassical model, where a tariff improves the terms of trade.

set at the globally optimal level and exactly offset monopolistic distortions, while trade policies consist of import subsidies and export taxes. Thus, Nash trade policies aim at delocating firms to the *other* economy in order to improve countries' terms of trade via the extensive margin (anti-delocation effect). This result confirms the insight gained from our welfare decomposition: when policy makers have sufficiently many instruments to deal with production efficiency and terms-of-trade effects separately, the terms-of-trade motive is the only international externality and thus the only reason to enter a trade agreement (Campolmi et al., 2022).

We then study a deep trade agreement that coordinates both trade and domestic policies. Starting from the symmetric Nash equilibrium described above, countries can attain the globally efficient allocation in cooperative negotiations by reducing import subsidies and export taxes reciprocally to zero, while leaving the aggregate terms of trade unaffected and production subsidies unchanged at their globally optimal levels. Thus, a deep trade agreement is sufficient to achieve a globally efficient outcome. We then ask the question if a shallow agreement supplemented with tariff bindings and market access commitments that are implied by GATT-WTO rules achieve the same outcome, as argued by Bagwell and Staiger (2001) for perfectly competitive models. In fact, we show that in the context of our monopolistic competition framework such a shallow agreement is *not* enough to guarantee a globally efficient outcome: without a commitment to coordinate both trade *and* domestic policies, individual-country policy makers have incentives to unilaterally deviate from the previously negotiated globally efficient allocation, e.g., by reducing production subsidies or by subsidizing imports. Such deviations improve domestic aggregate terms of trade by triggering entry of foreign exporters and reducing the price index of importables without reducing foreign market access.

Next, we consider a more stringent scenario modeled along the lines of a shallow free trade agreement according to GATT Article XXIV: we consider strategic domestic policies in a situation where trade taxes are set to zero. In this case domestic policies are governed by the trade-off between improving production efficiency and manipulating the terms of trade, and are thus not set efficiently. When firms are heterogeneous, the relative importance of the two effects depends on whether the profit share from domestic sales is larger than the one from export sales. When it is larger, the production-efficiency effect dominates, and the Nash policy is an (inefficiently low) production subsidy. When it is smaller, the second effect dominates, and the Nash policy is a production tax. Due to selection into exporting (Melitz, 2003), the average variable profit share from domestic sales is endogenous and an increasing function of fixed and variable physical trade costs. When physical trade costs fall, uncoordinated domestic policies become more distortive. Thus, in a highly globalized world with low physical trade costs signing a deep trade agreement becomes more important. However, full coordination of domestic policies may not always be feasible. We thus consider as an alternative a laissez-faire agreement, which forbids both the use of trade and domestic policies. We show that whether or not this welfare dominates a shallow free trade agreement depends on whether the profit share from domestic sales is smaller or larger than the one from export sales.

We contribute to the literature on trade and domestic policies and the design of trade agreements. Much of this literature focuses on the neoclassical model. Bagwell and Staiger (2001) use a perfectly competitive model with a local externality to study the gains from integrating agreements on domestic policies into trade agreements within a two-stage setup. They argue against integrating rules on domestic policies into trade agreements since in their model GATT-WTO rules are sufficient to sustain efficient levels of domestic policies: they prohibit changes in domestic policies that undo the market access commitment of previously granted tariff concessions and thus a shallow trade agreement can achieve the same level of efficiency as a deep agreement. The reason is that under perfect competition any policy that improves the terms of trade simultaneously reduces market access. We show that this is no longer the case under monopolistic competition, where countries can use domestic policies to improve their terms of trade by triggering entry of foreign firms, which increases foreign market access. Lashkaripour and Lugovsky (2019) use a quantitative multi-sector Krugman (1980) model with trade policies and domestic production subsidies to put a number on the welfare gains from deep trade integration relative to unilaterally optimal policies.

A recent related literature focuses on deep trade integration involving the harmonization of product standards. Costinot (2008) considers agreements on vertical and horizontal product standards in a Cournot delocation model with local externalities. He shows that a national treatment clause, which requires the same standards for imported as for domestic goods, tends to induce hidden protection and thus too tight standards. By contrast, standards are chosen too leniently under a mutual recognition clause, which allows exporters to sell to the foreign market using their own standards. Grossman, McCalman and Staiger (2021) investigate various forms of agreements on product standards in a monopolistic competition model with homogeneous firms when domestic and foreign consumers have different preferences over product characteristics and firms can adapt product characteristics to each market at a cost. There are several key difference between their setup and ours: First, in our model there is no disagreement between countries regarding the globally optimal level of regulation. Second, in their model regulations play no role in the absence of international trade: in their main setup inefficiencies in standard setting arise only because policy makers manipulate standards to free ride on the other country, while laissez-faire standards are optimal. By contrast, in our model the laissez-faire allocation is inefficient due to monopolistic distortions, which provides a strong rationale for using and coordinating domestic policies. Parenti and Vanoorenberghe (2021) consider standard harmonization in a differentiated good model with local consumption externalities under preference heterogeneity and show that standard harmonization is welfare improving only when countries' preferences are sufficiently aligned. Also related is Ossa and Maggi (2019) who show that agreements on product standards (which aim at solving a consumption externality) can be welfare-detrimental in the presence of political-economy motives, while agreements on process standards (which aim at solving a production externality) are welfare enhancing. None of these papers addresses the question if shallow integration in combination with other WTO provisions is sufficient to achieve a globally efficient outcome, like we do.

The rest of the paper is structured as follows. In Section 2 we describe a multi-sector Melitz (2003) model expressed in terms of macro bundles. In Section 3 we set up the problems of policy makers who maximizes either global or domestic welfare and we present our welfare decomposition that decomposes welfare effects of policies. Finally, in Section 4 we consider strategic trade and domestic policies under various institutional arrangements. Section 5 presents our conclusions.

# 2 The Model

The setup follows Melitz and Redding (2015). The world economy consists of two countries $i$: Home (H) and Foreign (F). The only factor of production is labor which is supplied inelastically in amount $L$ in each country, perfectly mobile across firms and sectors and immobile across countries. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index corresponds to the location of consumption and the second sub-index to the location of production.

## 2.1 Technology and Market Structure

Each country has two sectors. The first sector produces a continuum of differentiated goods under monopolistic competition with free entry. The other sector is perfectly competitive and produces a homogeneous good.[5] Labor markets are perfectly competitive. Differentiated goods are subject to iceberg transport costs. Firms in the differentiated sector pay a fixed cost in terms of labor, $f_E$, to enter the market and to pick a draw of productivity $\varphi$ from a cumulative distribution $G(\varphi)$.[6] After observing their productivity draw, they decide whether to pay a fixed cost $f$ in terms of domestic labor to become active and produce for the domestic market. Active firms then decide whether to pay an additional market access cost $f_X$ (in terms of domestic labor) to export to the other country, or to produce only for the domestic market. Therefore, labor demand of firm $\varphi$ located in market $i$ for a variety sold in market $j$ is given by:

$$l_{ji}(\varphi) = \frac{q_{ji}(\varphi)}{\varphi} + f_{ji}, \quad i, j = H, F \tag{1}$$

where $f_{ji} = f$ for $j = i$, $f_{ji} = f_X$ for $j \neq i$ and where $q_{ji}(\varphi)$ is the production of a firm with productivity $\varphi$ located in country $i$ for market $j$. Varieties sold in the foreign market are subject to an iceberg transport cost $\tau > 1$. We thus define $\tau_{ji} = 1$ for $j = i$ and $\tau_{ji} = \tau$ for $j \neq i$.

In the homogeneous-good sector countries share the same linear production technology, and labor demand $L_{Zi}$ is given by:

$$L_{Zi} = Q_{Zi}, \tag{2}$$

where $Q_{Zi}$ is the production of the homogeneous good. Since this good is sold in a perfectly competitive market without trade costs, its price is identical in both countries and equals the marginal cost of production $W_i$. We assume that it is always produced in both countries in

[5]The generalization of the model to multiple monopolistically competitive sectors is straightforward.
[6]We assume that $\varphi$ has support $[0, \infty)$ and that $G(\varphi)$ is continuously differentiable with derivative $g(\varphi)$.

equilibrium. This implies equalization of wages $W_i = W_j$ for $i \neq j$. Without loss of generality we normalize $W_j = 1$.

## 2.2 Preferences

Households' utility function is given by:

$$U_i \equiv \alpha \log C_i + (1-\alpha) \log Z_i, \quad i = H, F, \tag{3}$$

where $C_i$ aggregates over the varieties of differentiated goods and $\alpha$ is the expenditure share of the differentiated bundle. $Z_i$ represents consumption of the homogeneous good (Krugman, 1980). The differentiated varieties produced in the two countries are aggregated with a CES function given by:[7]

$$C_i = \left[ \sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i = H, F \tag{4}$$

$$C_{ij} = \left[ N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \tag{5}$$

Here, $C_{ij}$ is the aggregate consumption bundle of country-$i$ consumers of varieties produced in country $j$, $c_{ij}(\varphi)$ is consumption by country-$i$ consumers of a variety $\varphi$ produced in country $j$, $N_j$ is the measure of varieties produced by country $j$. $\varphi_{ij}$ is the cutoff-productivity level, such that a country-$j$ firm with this productivity level makes exactly zero profits when selling to country $i$, while firms with strictly larger productivity levels make positive profits from selling to this market, so that all country-$j$ firms with $\varphi \geq \varphi_{ij}$ export to country $i$. Finally, $\varepsilon > 1$ is the elasticity of substitution between domestic and foreign bundles and between different varieties.

## 2.3 Government

The government of each country disposes of the following fiscal instruments: a sector-specific production tax/subsidy ($\tau_{Li}$) on the fixed and marginal costs of firms in the differentiated

[7]Notice that we can index consumption of differentiated varieties by firms' productivity level $\varphi$ since all firms with a given level of $\varphi$ behave identically. Note also that our definitions of $C_{ij}$ imply $C_i = \left[ N_i \int_{\varphi_{ii}}^{\infty} c_{ii}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) + N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e., the model is the standard one considered in the literature. However, it is convenient to define optimal consumption indices.

sector,[8] a sector-specific tariff/subsidy on imports in the differentiated sector ($\tau_{Ii}$) and a sector-specific tax/subsidy on exports in the differentiated sector ($\tau_{Xi}$).[9] We model domestic policies in terms of sector-specific production taxes/subsidies because they fit most naturally into the Melitz (2003) framework. However, one can interpret them more broadly as any policies that aim at correcting a distortion between domestic social marginal costs and domestic social marginal benefits. Such distortions may arise due to market power, as in our framework, but may also be due, e.g., to local consumption or production externalities. Thus one can think of domestic policies as covering a wide range of issues, including competition policy, environmental and product standards or R&D subsidies. In terms of notation, $\tau_{mi}$ indicates a gross tax for $m \in \{L, I, X\}$, i.e., $\tau_{mi} < 1$ indicates a subsidy and $\tau_{mi} > 1$ indicates a tax. In what follows, we employ the word *tax* whenever we refer to a policy instrument without specifying whether $\tau_{mi}$ is smaller or larger than one and we use the notation $\tau_{Tij} = 1$ for $i = j$ and $\tau_{Tij} = \tau_{Ii}\tau_{Xj}$ for $i \neq j$. Moreover, we assume that taxes are paid directly by the firms[10] and that all government revenues are redistributed to consumers through a lump-sum transfer $T_i$. We use the term *laissez-faire allocation* to refer to the market allocation in which both countries refrain from using any of the policy instruments, i.e., $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$.

## 2.4 Equilibrium

Since the model is standard, we relegate a more detailed description of the setup and the derivation of the market equilibrium to Appendix A. Similarly to Arkolakis, Costinot and Rodríguez-Clare (2012) and Costinot, Rodríguez-Clare and Werning (2020), we write the equilibrium in terms of sectoral aggregates: a good that is domestically produced and consumed (non-tradable good); a domestic exportable good and a domestic importable good. The model additionally features a homogeneous good. This representation in terms of aggregate bundles (i) highlights

[8]Since the only production factor in the model is labor, this is equivalent to a sector-specific labor tax/subsidy. We impose that the same production tax is levied on both fixed and marginal costs (including also the fixed entry cost $f_E$). This assumption is necessary to keep firm size unaffected by production taxes, which is optimal, as shown in Campolmi et al. (2022).

[9]Note that we could easily allow for tax instruments in the perfectly competitive sector but these would be completely redundant. We do not explicitly introduce sector-specific consumption taxes/subsidies but they can be replicated with a combination of production subsidies and import tariffs.

[10]In particular, following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

that models with monopolistic competition and CES preferences have a common macro representation and (ii) makes the connection to standard neoclassical trade models visible. The market equilibrium is described by the following conditions:

$$\widetilde{\varphi}_{ji} = \left[ \int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})} \right]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \tag{6}$$

$$\delta_{ji} = \frac{f_{ji}(1-G(\varphi_{ji})) \left( \frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1}}{\sum\limits_{k=H,F} f_{ki}(1-G(\varphi_{ki})) \left( \frac{\widetilde{\varphi}_{ki}}{\varphi_{ki}} \right)^{\varepsilon-1}}, \quad i,j = H,F \tag{7}$$

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left( \frac{f_{ii}}{f_{ij}} \right)^{\frac{1}{\varepsilon-1}} \left( \frac{\tau_{Li}}{\tau_{Lj}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ij}^{-1} \tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i = H,F, \quad i \neq j \tag{8}$$

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})) \left( \frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})), \quad i = H,F \tag{9}$$

$$C_{ij} = \frac{\varepsilon-1}{\varepsilon} (\varepsilon f_{ij})^{\frac{-1}{\varepsilon-1}} \tau_{ij}^{-1} \varphi_{ij} (\delta_{ij} L_{Cj})^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{10}$$

$$P_{ij} = \frac{\varepsilon}{\varepsilon-1} (\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} \varphi_{ij}^{-1} (\delta_{ij} L_{Cj})^{\frac{-1}{\varepsilon-1}}, \quad i,j = H,F \tag{11}$$

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{12}$$

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij} \tag{13}$$

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij} \quad i = H,F \tag{14}$$

Condition (6) defines $\tilde{\varphi}_{ji}$, the average productivity of country-$i$ firms active in market $j$, which is given by the harmonic mean of productivity of those firms that operate in the respective market. Condition (7) defines $\delta_{ji}$, the variable-profit share of a country-$i$ firm with average productivity $\tilde{\varphi}_{ji}$ arising from sales in market $j$ – henceforth called *domestic profit share.*[11] Equivalently, $\delta_{ji}$ is also the share of total labor used in the differentiated sector in country $i$ that is allocated to production for market $j$. Condition (8) follows from dividing the zero-profit conditions defining the survival-productivity cutoffs – which imply zero profits for a country-$i$ firm with the cutoff-productivity level $\varphi_{ij}$ from selling in market $j$ – for firms in their domestic market by the one for foreign firms that export to the domestic market. Condition (9) is the

[11] It can be shown that $f_{ji}(1-G(\varphi_{ji})) \left( \frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1}$ are variable profits of a the average country-$i$ firm active in market $j$.

free-entry condition combined with the zero-profit conditions. In equilibrium, expected variable profits (left-hand side) have to equal the expected overall fixed cost bill (right-hand side).

Condition (10) can be interpreted as a sectoral aggregate production function $C_{ij} = Q_{Cij}(L_{Cj})$ in terms of aggregate labor allocated to the differentiated sector, $L_{Cj}$, measuring the amount of production of the aggregate bundle produced in country $j$ for consumption in market $i$. Condition (11) defines the equilibrium consumer price index $P_{ij}$ of the aggregate differentiated bundle produced in country $j$ and sold in country $i$.[12]

Importantly, condition (12) defines the trade-balance condition that states that the value of net imports of the homogeneous good plus the value imports of the differentiated bundle (left-hand side) must equal the value of exports of the differentiated bundle (right-hand side). Note that imports and exports of differentiated bundles are evaluated at international prices (before tariffs are applied). The model-consistent definition of the terms of trade then follows directly from this equation.[13] The international price of imports $\tau_{Ii}^{-1} P_{ij}$ defines the inverse of the terms of trade of the differentiated importable bundle (relative to the homogeneous good), while the international price of exports $\tau_{Ij}^{-1} P_{ji}$ defines the terms of trade of the differentiated exportable bundle (relative to the homogeneous good). Given that terms of trade are defined in terms of sectoral ideal price indices of exportables relative to importables, they will be affected not only by changes in the prices of individual varieties but also by changes in the measure of exporters and importers and their average productivity levels. We will discuss this in detail in Section 3.

Finally, (13) is the market-clearing condition for the homogeneous good and condition (14) defines demand for the homogeneous good, presented here for future reference. We thus have a system of 24 equilibrium equations in 24 unknowns, namely $\delta_{ji}$, $\varphi_{ji}$, $\widetilde{\varphi}_{ji}$, $C_{ji}$, $P_{ij}$, $L_{Ci}$, and $Z_i$ for $i, j = H, F$. For a detailed description of the model and the equilibrium see Appendix A.

[12] More precisely, $P_{ij}$ should be interpreted as a relative aggregate price index in terms of the homogeneous good.

[13] This definition is also consistent with Costinot et al. (2020), who also define terms of trade in terms of aggregate international price indices of exportables and importables.

# 3 Policy Incentives and Welfare

We now set up the problems faced by policy makers who are concerned with maximizing either global welfare or the welfare of individual countries. Our ultimate goal is to study the design of various forms of trade agreements with different degrees of integration and thus we first need to understand policy makers' incentives. For this reason, we will consider scenarios where policy makers have access either to all policy instruments (production and trade taxes in the differentiated sector) or to just a subset of them. In this section we will recall a number of results derived in our companion paper, Campolmi et al. (2022),[14] that we will rely on in the following section on the design of trade agreements.

We first consider the problem of the world policy maker who maximizes the sum of individual-country welfare. She sets domestic and foreign policy instruments $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ in order to solve the following problem:[15]

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}, \tau_{Li}, \tau_{Ii}, \tau_{Xi}\}_{i,j=H,F}}} \sum_{i=H,F} U_i \tag{15}$$

$$\textit{subject to conditions } (6)\text{-}(13).$$

We rely on the total differential approach. The main advantage of this choice is that it allows us to derive a welfare decomposition which highlights the motives behind policy choices which is also valid with a limited set of policy instruments. Our approach involves taking the total differential of (15) and the equilibrium conditions to obtain the following welfare decomposition in response to small domestic or foreign policy changes (Proposition 1 in Campolmi et al. (2022)):

[14] More specifically, see Campolmi et al. (2022) for the derivation and a more detailed interpretation of equations (18) and (19) and of the globally efficient outcome.

[15] $U_i$ is defined in (3), (4) and (14).

$$\sum_{i=H,F} dV_i = \underbrace{\sum_{i=H,F} (1-\tau_{Xi}) P_{ii} dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1} P_{ij} dC_{ij}}_{\substack{\textit{global} \\ \textit{consumption-efficiency effect}}} + \underbrace{\sum_{i=H,F} \left( \frac{\varepsilon}{\varepsilon-1} \tau_{Li} \tau_{Xi} - 1 \right) dL_{Ci}}_{\substack{\textit{global} \\ \textit{production-efficiency effect}}} \tag{16}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ and $I_i = W_i L + T_i$ is household income.

Equation (16) identifies the global welfare effects of a small policy change and is valid independently of the number of policy instruments that the global policy maker has at her disposal. To understand the efficiency effects of policy observe that if $\tau_{Ii} = \tau_{Xi} = 1$ (no trade taxes) and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$ (production subsidies equal to inverse markup) the market allocation is globally efficient and reaches the Pareto-efficient outcome. Instead, in the laissez-faire equilibrium, the market allocation of consumption between non-tradables and importables in the differentiated sector is efficient (consumption-efficiency wedges are closed) while the allocation of labor between the differentiated and the homogeneous sector is not because of monopolistic markups. These increase the price of the differentiated bundle inefficiently. As a consequence, too much labor is allocated to the homogeneous sector, which implies a production-efficiency wedge. This distortion can be offset with the use of an appropriate production subsidy in each country that offsets the markup.

The individual-country policy maker instead sets domestic policy instruments $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}\}_{i,j=H,F}, \mathcal{T}_i}} U_i \tag{17}$$

$$\textit{subject to conditions } (6)\text{-}(13),$$

where $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ for $i = H, F$ and taking as given $\mathcal{T}_j \subseteq \{\tau_{Lj}, \tau_{Ij}, \tau_{Xj}\}$, with $j \neq i$.[16]

[16] $U_i$ is defined in (3), (4) and (14).

This problem allows us to obtain the following welfare decomposition for small policy changes:

$$dV_i = \underbrace{(1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij}}_{\substack{\text{domestic} \\ \text{consumption-efficiency effect}}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci}}_{\substack{\text{domestic} \\ \text{production-efficiency effect}}}$$
$$+ \underbrace{C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}_{\substack{\text{domestic} \\ \text{terms-of-trade effect}}}, \tag{18}$$

Equation (18) identifies the domestic welfare effects of a small policy change. It is valid independently of the number of policy instruments that the individual-country policy makers have at their disposal. In this respect it is more general than the concept of "politically optimal" policies, which allow identifying policy makers' incentives only when a complete set of instruments is available (Bagwell and Staiger, 2001, 2016).

Whenever the production subsidy or the export tax do not offset the monopolistic distortion completely, there is a wedge between the marginal value product of labor in the differentiated sector (evaluated at international prices) and the wage. This induces a misallocation of labor across sectors. The *domestic production-efficiency effect* measures the positive (negative) welfare effect of a reallocation of labor towards the differentiated (homogeneous) sector induced by a small policy change. A production subsidy, an import tariff or an export subsidy all trigger entry into the differentiated sector, thereby improving production efficiency. At the same time, whenever individual-country policy makers make use of trade policies, they also introduce a wedge between producer and consumer prices for the importable bundle, which generates a misallocation of consumption across all markets and sectors. For example, in the presence of an import tariff, consumption of the importable bundle is inefficiently low. The *domestic consumption-efficiency effect* then captures the positive (negative) welfare change generated by the reallocation of consumption towards (away from) importables induced by a small policy change. A similar argument applies when export taxes are used and the consumption of non-tradables is inefficiently high. Finally, the *domestic terms-of-trade effect* measures the welfare changes induced by changes in the international prices of exportables and importables. More specifically, an increase in the aggregate price of exportables raises domestic welfare while an increase in the aggregate price of importables has the opposite effect. Observe that the terms-of-trade effect is the only difference between welfare effects from the individual and the global perspective. Differently from the efficiency effects, terms-of-trade effects are *beggar-thy-neighbor* i.e., an increase in domestic welfare due to the terms-of-trade improvement is always compensated by an equal fall in the foreign one.

Note that the impact of an import tariff on the terms of trade is different from its effect in the neoclassical model. Indeed, in the multi-sector model with monopolistic competition, an import tariff worsens the terms of trade. To better understand why this is the case, we decompose the domestic terms-of-trade effect in (18) when starting from a symmetric allocation as follows:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = \tag{19}$$
$$\tau_{Ii}^{-1}P_{ij}C_{ij}\left[\frac{d\tau_{Li}}{\tau_{Li}} + \frac{d\tau_{Xi}}{\tau_{Xi}} + (\varepsilon-1)^{-1}\underbrace{\left(\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}}\right)}_{(i)} + (\varepsilon-1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(ii)} + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iii)}\right]$$

A small import tariff ($d\tau_{Ii} > 0$ and $d\tau_{Ii} = d\tau_{Xi} = 0$) raises home demand for domestically produced varieties, thus shifting labor towards the differentiated sector ($dL_{Ci} > 0$), while having the opposite effect on the foreign country ($dL_{Cj} < 0$). This worsens the terms of trade by reducing the price index of exportables and increasing the one of importables. Moreover, when firms make the larger share of profits in their domestic market ($\delta_{ii>}\frac{1}{2}$) the tariff increases the export profit share and thus triggers entry into exporting ($d\delta_{ji} > 0$), while leading to less stringent selection into exporting ($d\varphi_{ji} < 0$). The opposite occurs in the foreign market so that $(ii) < 0$ and $(iii) > 0$. When $\delta_{ii} < \frac{1}{2}$ the signs of $(ii)$ and $(iii)$ switch. Nonetheless, independently of the size of $\delta_{ii}$, the overall effect of a small tariff is always a terms-of-trade worsening. Similarly, negative terms-of trade effects are induced by a positive export or production subsidy.

In conclusion, (18) clarifies that individual-country policy makers use trade and/or domestic policies either to improve efficiency or to manipulate the terms of trade in their favor. When only a limited set of instruments is available, individual-country policymakers always face a trade-off between these two objectives. For instance, starting from the *laissez-faire* equilibrium, a small production subsidy, a small export subsidy or a small import tariff all increase efficiency at the expense of worsening the terms of trade. Which effect on welfare prevails depends on the share of profits firms make in their domestic market. Indeed, when $\delta_{ii} > \frac{1}{2}$ the production-efficiency effect dominates the terms-of-trade effect and a small production subsidy, a tariff, or an export subsidy increase domestic welfare. In the next section we will see that this trade-off disappears when a full set of instruments is at the disposal of individual-country policymakers while it is key to understand the welfare effects of shallow trade agreements.

# 4 The Design of Trade Agreements in the Presence of Domestic Policies

After having laid out the global and the individual-country policy maker problem and the incentives to set taxes, we now move to strategic policies in order to study how trade agreements should be designed. Recall that in our model production in the laissez-faire equilibrium is inefficient, so that there exists a motive for domestic policy intervention even in the absence of international trade.

We first consider strategic trade and domestic policies in the absence of any type of trade agreement in order to have a benchmark for the distortions arising without international cooperation. Next, we show that cooperative negotiations on trade and domestic policies under a deep trade agreement are sufficient to achieve global efficiency. In the remainder of the section, we consider various trade agreements with different levels of integration. First, we consider a shallow trade agreement modeled along the lines of GATT-WTO membership: in the first stage, countries negotiate reciprocal reductions in trade taxes, taking as given domestic policies. In the second stage, they can deviate unilaterally from the negotiation outcome subject to market access constraints and tariff bindings imposed by WTO rules. Second, we consider a more stringent scenario modelled along the lines of a shallow free trade agreement according to GATT Article XXIV: we consider strategic domestic policies in a situation where trade taxes are set to zero. Finally, we compare welfare under the previous scenario with a laissez-faire agreement, where countries commit to abstain from using both trade and domestic policies.

## 4.1 Trade and Domestic Policies in the Absence of a Trade Agreement

We first consider a situation without any type of agreement, so that individual-country policy makers can set both trade and domestic policies non-cooperatively. We thus allow domestic policies $\tau_{Li}$ and trade policies $\tau_{Ii}$, $\tau_{Xi}$, for $i = H, F$ to be set strategically and simultaneously by the policy makers of both countries. Individual-country policy makers solve the problem described in (17). The welfare decomposition in (18) holds independently of the number of instruments at the disposal of the individual-country policy maker and corresponds to the policy maker's objective. After substituting additional equilibrium conditions, this objective can be rewritten in terms of three wedges that are all individually equal to zero at the optimum. Proposition 1 states this more formally and characterizes the symmetric Nash equilibrium of this policy game.

**Proposition 1** ***Strategic trade and domestic policies***

*When production, import and export taxes are available in the differentiated sector,*

*(a) it is possible to rewrite* (18) *as follows:*

$$dV_i = [\Omega_{Cii} dC_{ii} + \Omega_{Cij} dC_{ij} + \Omega_{LCi} dL_{Ci}] \tag{20}$$

*where* $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and the wedges* $\Omega_{Cii}$, $\Omega_{Cij}$ *and* $\Omega_{LCi}$ *are defined in Appendix C.1.*

*(b) Solving the individual-country policy maker problem stated in* (17) *by using the total-differential approach requires setting* $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$.

*(c) As a result, any symmetric Nash equilibrium in the two-sector model with heterogeneous firms when both countries can simultaneously set all policy instruments entails the first-best level of production subsidies, and inefficient import subsidies and export taxes in the differentiated sector. Formally,*

$$\tau_L^N = \frac{\varepsilon - 1}{\varepsilon},\ \tau_I^N < 1 \text{ and } \tau_X^N > 1.$$

**Proof** See Appendix C.1 ■

Our welfare decomposition allows us to interpret the Nash policy outcome stated in Proposition 1. Domestic policies are set fully efficiently even under strategic interaction and do not cause any beggar-thy-neighbor effects. By contrast, trade policy instruments are set with the intention to manipulate the terms of trade. As made clear in Section 3, an import subsidy or an export tax both aim at improving the terms of trade by delocating firms to the other economy (anti-delocation effect). Because there are two international relative prices (the one of the differentiated exportable bundle and the one of the differentiated importable bundle relative to the homogeneous good) two trade-policy instruments are necessary to target both. In the symmetric Nash equilibrium, policy makers do not achieve this objective and the trade taxes just create consumption and production-efficiency wedges.

The result that production subsidies are set so as to completely offset monopolistic distortions is an application of the Bhagwati-Johnson targeting principle in public economics (Dixit, 1985). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. If the policy maker disposes of sufficiently many instruments to deal with each incentive separately, she uses the production subsidy to address production efficiency. The trade policy instruments are instead used to exploit the terms-of-trade effect, which is the only remaining incentive.[17]

## 4.2 A Deep Trade Agreement – Globally Efficient Trade and Domestic Policies

Proposition 1 implies that some type of trade agreement is necessary to prevent countries from trying to exploit the terms-of-trade effects of their policies. Thus, the question arises how to design such an agreement and how much cooperation is necessary to achieve a globally efficient outcome.

Let us first address the question if countries can move from a situation of no cooperation, i.e. the situation described in Proposition 1, to a fully efficient outcome by negotiating cooperatively over trade taxes and production taxes and then to commit to the negotiation outcome. We

[17] Proposition 1 extends the result of Campolmi, Fadinger and Forlati (2014) – who find that in the two-sector model with homogeneous firms strategic trade policy consists of globally efficient wage subsidies and inefficient import subsidies and export taxes – to the case of heterogeneous firms. This implies that firm heterogeneity neither adds further motives for signing a trade agreement beyond the classical terms-of-trade effect nor changes the qualitative results (import subsidies and export taxes in the differentiated sector) of the equilibrium outcome compared to the case with homogeneous firms.

call such a setup a *deep trade agreement*. Indeed, it is easy to show that this is possible: because countries are fully symmetric, the symmetric point on the Pareto-efficiency frontier (the globally efficient allocation) makes both countries better off than the Nash equilibrium described in Proposition 1. Moreover, moving from the Nash equilibrium to this point can be achieved without changes in the terms of trade (which would require compensating international transfers): a reciprocal reduction in import subsidies $\tau_{Ii}$ and export taxes $\tau_{Xi}$ for $i = H, F$ all the way to zero does not change the terms of trade and leads to full consumption and production efficiency as discussed in the previous section. Observe that domestic policies are left unchanged during this process, since the Nash production subsidies already correspond to the optimal ones. We have thus established the following result:

**Corollary 1** ***Global efficiency of a deep trade agreement***

*In our set up, countries can negotiate a mutually beneficial deep trade agreement with cooperation on trade and domestic policies. This agreement implements the globally efficient outcome by forbidding the use of trade policy instruments ($\tau_{Ii} = 1$ and $\tau_{Xi} = 1$ for $i = H, F$) and setting production subsidies in both countries equal to the inverse of the monopolistic markup ($\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$).*[18]

Thus, a deep trade agreement is sufficient to achieve global efficiency. But is it also necessary to achieve it, or would a shallow trade agreement, which does not comprise coordination of domestic policies, achieve a similarly efficient outcome?

## 4.3 Shallow Trade Agreements

We now consider various forms of *shallow trade agreements*, which focus purely on coordination of trade taxes. We first consider a situation of trade negotiations under the current GATT-WTO rules. We follow Bagwell and Staiger (2001) in modeling the negotiation process of a shallow trade agreement in the presence of domestic policies as a two-stage process. In the first stage, countries negotiate cooperatively over trade taxes while keeping domestic policies constant at the Nash levels (which, in our setup, correspond to the first-best production subsidies).

[18] In principle, countries could alternatively continue to use tariffs and export taxes as long as they agree to set $\tau_{Tij} = 1$ and $\tau_{Ii} = \tau_{Ij}$ and $\tau_{Xi} = \tau_{Xj}$ for $i, j = H, F$. Since this is not very practical, we focus on zero trade taxes.

In the second stage, countries can deviate non-cooperatively from the stage-one outcome by setting trade taxes and/or production taxes non-cooperatively but subject to two additional constraints imposed by WTO rules: first, import taxes are subject to tariff bindings (they cannot be increased relative to the outcome negotiated in stage one); second, market access cannot be reduced: countries are not allowed to offset market access commitments (i.e., they cannot reduce imports) from the first stage by an unanticipated change in their policies.[19]

We have already shown above that the stage-one negotiation outcome leads to the globally optimal allocation. We now show that in stage two there exist unilateral deviations from the stage-one outcome that make the deviating country better off without violating the tariff-binding or market access constraints. As a consequence, a shallow trade agreement under the current WTO rules is not sufficient to obtain the globally optimal outcome.

To see this, note first that any deviations from the globally efficient outcome are due to terms-of-trade effects because production efficiency is already guaranteed (this is established in Corrolary 1). Second, we now show that there exist unilateral deviations from the globally efficient outcome that improve domestic terms of trade and welfare and do not reduce market access. In particular, a reduction in the production subsidy below the globally optimal level, a small import subsidy or an export tax all improve the terms of trade, while also increasing imports in the differentiated sector.[20]

**Lemma 1** ***Unilateral deviations from the globally efficient allocation***

*Consider a marginal unilateral increase in each policy instrument at a time starting from the globally efficient allocation, i.e., a situation with* $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$ *and let* $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\varepsilon-1)}$ *or* $\delta_{ii} > \frac{1}{2}$. *Then:*

*(a) the domestic welfare effect is negative for* $\tau_{Ii}$ *and positive for* $\tau_{Xi}$ *and* $\tau_{Li}$.

*(b) the volume of imports in the differentiated sector decreases in* $\tau_{Ii}$ *and increases in* $\tau_{Xi}$ *and* $\tau_{Li}$.

[19] As Bagwell and Staiger (2001) argue, the legal basis for such "nonviolation" complaints is provided in GATT Article XXIII: countries are not allowed to reduce foreign countries' access to their markets with policy changes, even if these policy changes broke no explicit WTO rules.

[20] Note that imports in the homogeneous sector are zero in the globally optimal allocation, so that any changes in imports in this sector due to policy changes do not affect market access commitments. Lemma 1 also holds with homogeneous firms. The proof is available upon request.

**Proof** See Appendix C.2. ■

Observe the important difference of this result to the one of Bagwell and Staiger (2001). They have shown that under perfect competition a shallow agreement in combination with tariff bindings and market access commitments guarantees a globally efficient outcome. Intuitively, under perfect competition the terms-of-trade motive is the only reason for a trade agreement and any policy that improves the terms of trade simultaneously reduces foreign market access. Thus, by committing not to reduce imports, policy makers are simultaneously prevented from using trade or domestic policies to improve their terms of trade. By contrast, under monopolistic competition, a terms-of-trade improvement may be achieved by increasing the number or productivity of foreign firms selling to the domestic market. Hence, a terms-of-trade improvement is perfectly compatible with an increase in foreign market access. We summarize this important result in the following Corollary.

**Corollary 2** ***Insufficiency of a shallow trade agreement for global efficiency***

*In our setup, a shallow trade agreement in combination with tariff bindings and market access commitments is not sufficient to achieve the globally efficient outcome.*

Having shown that a shallow agreement in combination with WTO rules on tariff bindings and market access is not sufficient to replicate the outcome of a deep trade agreement, we now analyze a situation that mimics the more stringent setup of a shallow free trade agreement under Article XXIV of GATT-WTO. Such an agreement requires full trade liberalization among its members (zero trade taxes), while leaving domestic policies uncoordinated. We characterize in detail the Nash equilibrium arising from strategic domestic policies (production taxes) under such an agreement. In this case, individual-country policy makers face a missing-instrument problem and consequently a trade-off between changing production efficiency (calling for a production subsidy) and the terms of trade (calling for a production tax). We have already discussed that in the presence of firm heterogeneity the relative weight of these motives depends on the profit share from sales in the domestic market. We now show that this intuition carries over to the Nash policies.

**Proposition 2** ***Strategic domestic policies in the presence of a shallow trade agreement*** *When only production taxes in the differentiated sector are available,* *(a) it is possible to rewrite* (18) *as follows:*

$$dV_i = \Omega_i dL_{Ci} \tag{21}$$

*where* $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and where the wedge* $\Omega_i$ *is defined in Appendix C.3.*

*(b) Solving the individual-country policy maker problem in* (17) *by using the total-differential approach when* $\tau_{Ii} = \tau_{Xi} = 1$, $i = H, F$ *requires setting* $\Omega_i = 0$.

*(c) As a result, the symmetric Nash equilibrium when trade taxes are not available and both countries can simultaneously set production taxes in the differentiated sector is characterized as follows: it exists, is unique and entails positive, but inefficiently low, production subsidies when the domestic profit share,* $\delta_{ii}$*, is larger or equal than 1/2. Otherwise, the Nash equilibrium entails positive production taxes. Formally:*

*(i) If* $\delta_{ii} \geq \frac{1}{2}$*, then there exists a unique symmetric Nash equilibrium with* $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$*;*

*(ii) If either* $0 < \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon \geq \frac{3-\alpha}{2}$ *or* $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon < \frac{3-\alpha}{2}$*, there exists a unique symmetric Nash equilibrium with* $\tau_L^N > 1$*;*

**Proof** See Appendix C.3. ■

The domestic profit share $\delta_{ii}$, is a sufficient statistic for the impact of firm heterogeneity and selection. Proposition 2 states that if it is larger than the export profit share, strategic domestic policies feature positive production subsidies. As discussed in the previous section, we know that this outcome reflects that the (positive) production-efficiency effect dominates the (negative) terms-of-trade effect. However, these subsidies are inefficiently low due the trade-off between these motives. By contrast, when the domestic profit share is smaller than the export profit share, strategic domestic policies feature production taxes, which worsen the allocation compared to the laissez-faire allocation.[21][22] In this case, the terms-of-trade effect dominates

[21]Proposition 2 stands in contrast to the result of Campolmi et al. (2014). While they find that in the two-sector model with homogeneous firms strategic domestic policies always feature positive but inefficiently low production subsidies, this is no longer the case under firm heterogeneity and endogenous selection into exporting.

[22]Observe that if we impose the assumption that the export cutoff $\varphi_{ji}$ for $j \neq i$ must be larger than the domestic survival cutoff $\varphi_{ii}$ at the symmetric Nash equilibrium, i.e. $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} > 1$, then $\delta_{ii}$ is always strictly greater than 1/2.

the production-efficiency effect because firms make the bulk of their profits from exporting, so that manipulating international prices is key. In the presence of firm heterogeneity, the relative importance of the two effects thus depends on the magnitude of the domestic profit share. Therefore, when the set of policy instruments is limited, firm heterogeneity plays a crucial role in shaping the equilibrium policies, and thus the desirability of specific institutional arrangements, as we show next.

## 4.4 A Laissez-faire Agreement

As shown above, a sufficient condition for reaping the full benefits of integration is to sign a deep trade agreement with cooperation on trade and domestic policies. However, full cooperation on domestic policies may not be feasible in practice. Alternatively, countries may be able to commit to free trade and not to use domestic policies at all. We thus consider as an alternative scenario a *laissez-faire agreement*, which forbids both the use of trade and domestic policies and we compare its performance with the one of a shallow free trade agreement. Whether or not such an arrangement dominates a shallow free trade agreement when firms are heterogeneous depends on whether the profit share from domestic sales is smaller or larger than the one from export sales. This is straightforward: a Nash production subsidy improves equilibrium production efficiency, and thus welfare, compared to the laissez-faire allocation, while a Nash production tax worsens it. (Terms-of-trade effects of domestic policies offset each other in the symmetric Nash equilibrium.)

Finally, note that in the presence of firm heterogeneity and selection effects, the domestic profit share is endogenous to physical trade costs: one can show that $\delta_{ii}$ is increasing in $\tau_{ij}$ and $f_{ij}$ for $j \neq i$. Thus, as physical trade barriers fall, the domestic profit share falls and may even become smaller than one half. Therefore, with sufficiently low physical trade barriers a laissez-faire agreement can be better than a shallow free trade agreement. These insights on the welfare effects of shallow vs. laissez-faire agreements are summarized by the following Proposition.[23]

[23] In numerical simulations with Pareto-distributed productivity we have obtained the robust result that when physical trade barriers fall Nash-equilibrium production subsidies decrease smoothly until they turn into positive taxes at a level of trade barriers that implies $\delta_{ii} = 1/2$. From that point on, production taxes strictly increase as trade barriers fall further. These results imply that the proportional welfare gains from moving from a shallow to a deep trade agreement rise as physical trade barriers fall.

**Lemma 2** ***Welfare effects of strategic domestic policies in the presence of a shallow free trade agreement*** *Assume that $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and let firms' average variable-profit share from sales in their domestic market be given by $\delta_{ii}$.*

*(a) When $\delta_{ii} \geq \frac{1}{2}$ the symmetric Nash equilibrium when countries can only set domestic policies strategically welfare-dominates the laissez-faire allocation with $\tau_{Li} = 1$, $i = H, F$.*

*(b) $\delta_{ii}$ is increasing in $\tau_{ij}$ and $f_{ij}$, $j \neq i$.*

**Proof** See Appendix C.4. ■

To summarize, when $\delta_{ii} \geq \frac{1}{2}$, a shallow free trade agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely welfare-dominates a laissez-faire agreement that forbids countries to use domestic and trade policies. When instead $\delta_{ii} < \frac{1}{2}$ and as long as there exists a unique symmetric Nash equilibrium, a laissez-faire agreement welfare-dominates a shallow free trade agreement. Thus, a laissez-faire agreement is less distortive than a shallow free trade agreement when physical trade costs are sufficiently low.

# 5 Conclusion

In this paper we have studied the design of shallow and deep trade agreements in a multi-sector model with monopolistic competition and firm heterogeneity. Starting from the observation that trade models with CES preferences and monopolistic competition have a common macro representation, we have used a novel welfare decomposition (Campolmi et al., 2022) to study the motives for trade and domestic policies. From the global perspective, incentives are governed by production and consumption efficiency considerations. Production subsidies equal to the inverse of monopolistic markups are necessary and sufficient to correct monopolistic distortions and to implement the globally optimal allocation. From the individual-country perspective, welfare incentives are additionally governed by terms-of-trade motives.

Then we have discussed that using individual policy instruments always leads to a trade-off between production-efficiency and terms-of-trade effects. Firm heterogeneity in combination with physical trade costs matter for unilateral policies because they determine the profit share from sales in each market. This variable governs how the trade-off between these motives plays out: when physical trade barriers are high, firms make most of their profits domestically, and thus the desire to increase production efficiency dominates terms-of-trade manipulation.

Finally, we have used these insights to study the design of trade agreements from the perspective of the multi-sector heterogeneous-firm model. We have shown that in the absence of any trade agreement, the Nash equilibrium entails the globally-optimal production subsidies and inefficient import subsidies and export taxes that aim at improving the terms of trade. Thus, even in the presence of firm heterogeneity and domestic policies terms-of-trade motives remain the only reason for signing a trade agreement. We have shown that a deep trade agreement with coordination of trade and domestic policies can implement the globally optimal allocation. We have then considered trade negotiations under current WTO rules: countries first negotiate reciprocal reductions in trade taxes and can then adjust their policies unilaterally subject to tariff bindings and market access commitments. We have shown that such an institutional setup is not sufficient to guarantee an efficient outcome. Moreover, when a shallow free trade agreement prevents countries from using trade policy strategically, strategic domestic policies are set to balance a trade-off between improving the terms of trade and increasing production efficiency. In this case, Nash-equilibrium domestic policies depend on firm heterogeneity via the profit share from domestic sales: when it is larger than the one from export sales, the Nash equilibrium features positive (albeit inefficiently low) production subsidies. By contrast, when it is smaller, the Nash equilibrium is characterized by positive production taxes. This result implies that achieving the full benefits of globalization requires a deep trade agreement that allows countries to coordinate both trade and domestic policies. Moreover, it means that shallow free trade agreements are more distortive when physical trade costs are lower and thus signing deep trade agreements becomes more desirable.

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## APPENDIX - FOR ONLINE PUBLICATION

# A The Model

In this section lay out the model set-up and derive the equilibrium conditions (6)-(14). Finally, we derive the laissez-faire allocation.

## A.1 Households

Given the Dixit-Stiglitz structure of preferences in (4), the households' maximization problem can be solved in three stages. At the first two stages, households choose how much to consume of each domestically produced and foreign produced variety, and how to allocate consumption between the domestic and the foreign bundles. The optimality conditions imply the following demand functions and price indices:

$$c_{ij}(\varphi) = \left[\frac{p_{ij}(\varphi)}{P_{ij}}\right]^{-\varepsilon} C_{ij}, \qquad C_{ij} = \left[\frac{P_{ij}}{P_i}\right]^{-\varepsilon} C_i, \quad i,j = H,F \tag{A-1}$$

$$P_i = \left[\sum_{j\in H,F} P_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \qquad P_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} p_{ij}(\varphi)^{1-\varepsilon} dG(\varphi)\right]^{\frac{1}{1-\varepsilon}}, \quad i,j = H,F \tag{A-2}$$

Here $P_i$ is the price index of the differentiated bundle in country $i$, $P_{ij}$ is the country-$i$ price index of the bundle of differentiated varieties produced in country $j$, and $p_{ij}(\varphi)$ is the country-$i$ consumer price of variety $\varphi$ produced by country $j$.

In the last stage, households choose how to allocate consumption between the homogeneous good and the differentiated bundle. Thus, they maximize (3) subject to the following budget constraint:

$$P_i C_i + p_{Zi} Z_i = I_i, \quad i = H,F$$

where $I_i = W_i L + T_i$ is total income and $T_i$ is a lump sum transfer which depends on the tax scheme adopted by the country-$i$ government. The solution to the consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_i}{C_i} = \frac{P_i}{p_{Zi}}, \quad i = H, F \tag{A-3}$$

Then following Melitz and Redding (2015), we can rewrite the demand functions as

$$c_{ij}(\varphi) = p_{ij}(\varphi)^{-\varepsilon} A_i, \quad C_{ij} = P_{ij}^{-\varepsilon} A_i, \quad C_i = P_i^{-\varepsilon} A_i, \quad i, j = H, F, \tag{A-4}$$

where $A_i \equiv P_i^{\varepsilon-1} \alpha I_i$. $A_i$ can be interpreted as an index of market (aggregate) demand.

## A.2 Firms

### A.2.1 Firms' behavior in the differentiated sector

Given the constant price elasticity of demand, optimal prices charged by country-$i$ firms in their domestic market are a fixed markup over their perceived marginal cost $\left(\tau_{Li}\frac{W_i}{\varphi}\right)$, and optimal prices charged to country-$j$ consumers for varieties produced in country $i$ equal country-$i$ prices augmented by transport costs and trade taxes

$$p_{ji}(\varphi) = \tau_{ji}\tau_{Tji}\tau_{Li}\frac{\varepsilon}{\varepsilon-1}\frac{W_i}{\varphi}, \quad i, j = H, F \tag{A-5}$$

The optimal pricing rule implies the following firm revenues:

$$r_{ji}(\varphi) \equiv \tau_{Tji}^{-1} p_{ji}(\varphi) c_{ji}(\varphi) = \tau_{Tji}^{-1} p_{ji}(\varphi)^{1-\varepsilon} A_j = \varepsilon \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} \tau_{Li}^{1-\varepsilon} W_i^{1-\varepsilon} \varphi^{\varepsilon-1} B_j, \quad i, j = H, F, \tag{A-6}$$

where $B_i \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \frac{1}{\varepsilon} A_i$. Profits are given by:

$$\pi_{ji}(\varphi) \equiv B_j \left(\frac{\tau_{Li} W_i}{\varphi}\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} = \frac{r_{ji}(\varphi)}{\varepsilon} - \tau_{Li} W_i f_{ji}, \quad i, j = H, F \tag{A-7}$$

### A.2.2 Zero-profit conditions

Firms choose to produce for the domestic (export) market only when this is profitable. Since profits are monotonically increasing in $\varphi$, we can determine the equilibrium productivity cutoffs for firms active in the domestic market and export market, $\varphi_{ji}$, by setting $\pi_{ji}(\varphi_{ji}) = 0$, namely:

$$\pi_{ji}(\varphi_{ji}) = 0 \Rightarrow \frac{r_{ji}(\varphi_{ji})}{\varepsilon} = \tau_{Li} W_i f_{ji}, \quad i, j = H, F \tag{A-8}$$

As in Melitz (2003), we call these conditions the *zero profit* (**ZCP**) conditions. Using (A-7) we rewrite (A-8) as follows:

$$B_j = \tau_{ji}^{\varepsilon-1} \tau_{Li}^{\varepsilon} \tau_{Tji}^{\varepsilon} W_i^{\varepsilon} \varphi_{ji}^{1-\varepsilon} \quad j = H, F, \quad i \neq j \tag{A-9}$$

### A.2.3 Free-entry conditions (FE)

The *free entry* (**FE**) conditions require expected profits (before firms know the realization of their productivity) in each country to be zero in equilibrium:

$$\sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} \pi_{ji}(\varphi) dG(\varphi) = \tau_{Li} W_i f_E, \quad i = H, F$$

Substituting optimal profits (A-7), we obtain

$$\sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} \left[ B_j \left( \frac{\tau_{Li} W_i}{\varphi} \right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} \right] dG(\varphi) = \tau_{Li} W_i f_E, \quad i = H, F \tag{A-10}$$

### A.2.4 Firms' behavior in the homogeneous sector

Since the homogeneous good is sold in a perfectly competitive market without trade costs, its price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization:

$$p_{Zi} = p_{Zj} = W_i = W_j = 1, \quad i = H, j = F \tag{A-11}$$

## A.3 Government

The government is assumed to run a balanced budget. Hence, country-i government's budget constraint is given by:

$$\begin{aligned} T_i = (\tau_{Ii} - 1)\tau_{Ii}^{-1} P_{ij} C_{ij} + (\tau_{Xi} - 1)\tau_{Tji}^{-1} P_{ji} C_{ji} + \\ + (\tau_{Li} - 1) N_i W_i \left[ \sum_{k=H,F} \int_{\varphi_{ki}}^{\infty} \left( \frac{q_{ki}(\varphi)}{\varphi} + f_{ki} \right) dG(\varphi) + f_E \right], \quad i = H, F, \quad j \neq i \end{aligned} \tag{A-12}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and foreign export taxes (thus, tariffs are charged on CIF values of foreign exports), export tax revenues charged on exports gross of transport costs, and production tax revenues.

## A.4 Equilibrium

From now on we use (A-11) to eliminate wages from all equilibrium conditions. Substituting **ZCP** (A-9) into **FE** (A-10), we obtain:

$$\sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) \left( \frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})), \quad i = H, F, \tag{A-13}$$

where

$$\widetilde{\varphi}_{ji} = \left[ \int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1 - G(\varphi_{ji})} \right]^{\frac{1}{\varepsilon-1}}, \quad i, j = H, F, \tag{A-14}$$

which correspond to (9) and (6) in the main text. Moreover, dividing the **ZCP** conditions (A-9), we obtain condition (8) in the main text:

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left( \frac{f_{ii}}{f_{ij}} \right)^{\frac{1}{\varepsilon-1}} \left( \frac{\tau_{Li}}{\tau_{Lj}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ij}^{-1} \tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \tag{A-15}$$

The remaining equilibrium equations are then given as follows:

**Consumption sub-indices**, which can be determined using (A-4) jointly with (A-9):

$$C_{ij} = P_{ij}^{-\varepsilon} \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{\varepsilon - 1} \varepsilon \tau_{Lj}^{\varepsilon} \tau_{ij}^{\varepsilon - 1} \tau_{Tij}^{\varepsilon} \varphi_{ij}^{1-\varepsilon} f_{ij}, \quad i, j = H, F \tag{A-16}$$

**Price sub-indices**, which emerge from substituting (A-5) into (A-2):

$$P_{ij}^{1-\varepsilon} = \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{1-\varepsilon} N_j (1 - G(\varphi_{ij}))(\tau_{ij} \tau_{Tij} \tau_{Lj})^{1-\varepsilon} \widetilde{\varphi}_{ij}^{\varepsilon - 1}, \quad i, j = H, F \tag{A-17}$$

Aggregate profits $\Pi_i$ are given by $\Pi_i = R_i - \tau_{Li} L_{Ci} + \tau_{Li} N_i f_E$, where $R_i$ is aggregate revenue, $R_i \equiv N_i \sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} r_{ji}(\varphi) dG(\varphi)$. From the **FE** condition (A-10) it then follows that $\Pi_i = \tau_{Li} N_i f_E$ and thus $R_i = \tau_{Li} L_{Ci}$. Substituting the definition of optimal revenues (A-6) into the previous condition, we get

$$\tau_{Li} L_{Ci} = \varepsilon N_i \sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} B_j \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} \tau_{Li}^{1-\varepsilon} \varphi^{\varepsilon - 1} dG(\varphi), \quad i = H, F$$

Combining this condition with (9) and (A-9), we obtain:

**Labor market clearing in the differentiated sector**

$$L_{Ci} = \varepsilon N_i \sum_{j=H,F} f_{ji} (1 - G(\varphi_{ji})) + \varepsilon f_E N_i, \quad i = H, F \tag{A-18}$$

This can be solved for the equilibrium level of $N_i$:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F} f_{ji} (1 - G(\varphi_{ji})) + \varepsilon f_E}, \quad i = H, F \tag{A-19}$$

Combining this last condition with (9), plugging into (A-16) and (A-17) and taking into account the definition (6), allows us to recover (10) and (11) in the main text.

The trade-balance condition is given by:[24]

$$Q_{Zi} - Z_i + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F \tag{A-20}$$

[24] Import taxes are collected directly by the governments at the border so they do not enter into this condition.

We can use the fact that $\sum_{j=H,F} P_{ij}C_{ij} = P_iC_i$ to rewrite (A-3) as:

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij}C_{ij}, \quad i = H, F$$

We can combine this equation with the trade-balance condition above and the aggregate labor market clearing $L = L_{Ci} + L_{Zi}$ to obtain:

**Trade-balance condition**

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik}C_{ik} + \tau_{Ij}^{-1} P_{ji}C_{ji} = \tau_{Ii}^{-1} P_{ij}C_{ij}, \quad i = H, j = F,$$

which corresponds to condition (12).

Finally, we also require equilibrium in the market for the homogenous good, i.e. $\sum_{i=H,F} Q_{Zi} = \sum_{i=H,F} Z_i$. Combining this condition with aggregate labor market clearing and demand for the homogeneous good (A-3) we obtain:

**Homogeneous-good market clearing condition**

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij}C_{ij},$$

which coincides with condition (13).

Conditions (6)-(14) identify a system of 24 equilibrium equations in 24 unknowns, namely $\delta_{ji}$, $\varphi_{ji}$, $\widetilde{\varphi}_{ji}$, $C_{ji}$, $P_{ij}$, $L_{Ci}$, $Z_i$ for $i, j = H, F$.

### A.4.1 The allocation under the laissez-faire agreement

Using equations (10) and (11), we find that

$$P_{ij}C_{ij} = \delta_{ij} L_{Cj} \tau_{Tij} \tau_{Lj}, \quad i, j = H, F$$

Substituting into the trade-balance condition (12), we obtain:

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} \delta_{ik} L_{Ck} \tau_{Tik} \tau_{Lk} + \delta_{ji} L_{Ci} \tau_{Xi} \tau_{Li} = \delta_{ij} L_{Cj} \tau_{Xj} \tau_{Lj}, \quad i = H, j = F$$

Under the laissez-faire agreement, $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Since the countries are symmetric, the equilibrium is also symmetric and thus $L_{Ci} = L_{Cj}$ and $\delta_{ij} = \delta_{ji}$ for $i = H, F$ and $j \neq i$. Substituting these conditions, we find that

$$L_{Ci}^{LF} = \alpha L, \quad i = H, F$$

Using this result together with (A-18) and (A-13), we obtain

$$N_i^{LF} = \frac{\alpha L}{\varepsilon \sum_{j=H,F} \left[ f_{ji}(1 - G(\varphi_{ji})) \left( \frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon - 1} \right]}, \quad i = H, F$$

# B The Total-Differential Approach

We use the total-differential approach to optimization to solve the optimal-policy problem.[25] One advantage of this approach is that the same methodology can be used to derive the welfare decomposition (see Campolmi et al. (2022)), the unilateral and the strategic policies.

We first discuss how we apply this approach to find the optimal deviations of domestic and trade policies. Then, we explain how to employ it to solve constrained optimization problems. Finally, we derive a number of preliminary results that we will use in Section 4.

## B.1 How to apply the total-differential approach

### B.1.1 Unilateral policy deviations

The unilateral deviations of each policy instrument can be determined following these steps:

**(1)** Take the total differential of the objective function and the equilibrium conditions.

**(2)** Use the total differential of the equilibrium conditions to solve for the total differentials of the endogenous variables as linear functions of the total differentials of the policy instruments. Since we consider each policy instrument at a time, set the total differentials of the policy instruments that are not of interest to zero.

[25] Observe that using this approach implies restricting our analysis to interior solutions.

**(3)** Substitute the solution of the total differentials of the endogenous variables into the total differential of the objective function. Collect all the terms and sign the coefficient multiplying the total differential of the policy instrument to determine the direction of the optimal deviations.

#### B.1.2 Constrained optimization problems

A constrained optimization problem in $n$ variables given $m$ constraints with $n > m$ can be solved using the total-differential approach according to the following steps:

**(1)** Take the total differential of the objective function and the constraints.

**(2)** Use the total differential of the constraints to solve for $m$ total differentials as a function of the $n - m$ other total differentials.

**(3)** Substitute the solution of the $m$ total differentials into the total differential of the objective function. Then the total differential of the objective function must be zero for *any* of the $n-m$ total differentials (i.e., for any *arbitrary* perturbation of the $n - m$ relevant variables). Collect the terms multiplied by the $n - m$ differentials to find the $n - m$ conditions that need to be zero at the optimum.

**(4)** The $n - m$ conditions found in (3) jointly with the $m$ constraints determine the solution of the $n$ variables.

### B.2 Preliminary steps for applying the total-differential approach

In this section, we derive some preliminary results that will be useful to derive the results of Section 4.

As explained above, the first steps to apply the total-differential approach – independently of whether the optimal policy problem or unilateral deviations are considered – is to take the total differential of the equilibrium equations (6)-(13), which we do in Section B.2.1 below. Then, we evaluate the total differentials at a symmetric allocation. Finally, we set $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$, and combine the equations so as to be left with 3 equations, which are linear functions of 6 differentials: $dL_{Ci}$, $dC_{ii}$, $dC_{ij}$, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. We can then use these 3 equations to express 3 differentials as functions of the remaining 3. For the unilateral deviations, we solve for $dL_{Ci}$, $dC_{ii}$ and $dC_{ij}$ as linear functions of the deviations of the policy instruments $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. Then, we allow only a single policy instrument to vary at a time, while setting the deviations for the other two to zero. Differently, for the cases of strategic interaction we use the 3 equations to write the differentials of the tax instruments, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ as linear functions of the other 3 differentials, $dL_{Ci}$, $dC_{ii}$ and $dC_{ij}$. Finally, for the case of strategic interaction when only production taxes are available (shallow trade agreement) we set the deviations for $d\tau_{Ii}$ and $d\tau_{Xi}$ to zero. This allow us to express $d\tau_{Li}$ as a function of $dL_{Ci}$ only.

### B.2.1 Total differentials of some equilibrium conditions

Since the total differentials of the equilibrium equations (6)-(10) are extensively used in the proofs of Section 4 we present them here for future reference. The total differential of (6) gives:

$$d\widetilde{\varphi}_{ji} = \frac{1}{\varepsilon-1}\frac{g(\varphi_{ji})}{[1-G(\varphi_{ji})]}\widetilde{\varphi}_{ji}\left[1-\left(\frac{\varphi_{ji}}{\widetilde{\varphi}_{ji}}\right)^{\varepsilon-1}\right]d\varphi_{ji}, \quad i,j=H,F \tag{B-1}$$

Substituting this condition into the total differential of (9), we get:

$$d\varphi_{ji} = -\frac{f_{ii}[1-G(\varphi_{ii})]\varphi_{ii}^{-\varepsilon}\widetilde{\varphi}_{ii}^{\varepsilon-1}}{f_{ji}[1-G(\varphi_{ji})]\varphi_{ji}^{-\varepsilon}\widetilde{\varphi}_{ji}^{\varepsilon-1}}d\varphi_{ii}, \quad i=H,F, \quad i\neq j \tag{B-2}$$

Using (7) and (9), this condition can be rewritten as

$$d\varphi_{ji} = -\frac{\delta_{ii}}{1-\delta_{ii}}\frac{\varphi_{ji}}{\varphi_{ii}}d\varphi_{ii}, \quad i=H,F, \quad i\neq j, \tag{B-3}$$

which expresses the total differential of the productivity cut-offs for the domestically produced goods in the export markets as a function of the cut-offs in the domestic markets. Taking the total differential of (7) combined with (9) and substituting (B-1) and (B-2) into the resulting condition, we get:

$$d\delta_{ji} = -\frac{\delta_{ji}}{\varphi_{ji}}\left(\Phi_i+(\varepsilon-1)\right)d\varphi_{ji}, \quad i,j=H,F \tag{B-4}$$

where $\Phi_i \equiv \delta_{ii}\frac{g(\varphi_{ji})\varphi_{ji}^{\varepsilon}\widetilde{\varphi}_{ji}^{1-\varepsilon}}{1-G(\varphi_{ji})} + \delta_{ji}\frac{g(\varphi_{ii})\varphi_{ii}^{\varepsilon}\widetilde{\varphi}_{ii}^{1-\varepsilon}}{1-G(\varphi_{ii})} > 0$, $i=H,F$ and $j\neq i$. Condition (B-4) states that as the productivity cut-off rises, the corresponding variable-profit share shrinks. Moreover, by totally differentiating (10), we obtain:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{C_{ij}}dC_{ij} - \frac{\varepsilon}{\varepsilon-1}\frac{\varphi_{ij}}{\delta_{ij}}d\delta_{ij} - \frac{\varepsilon}{\varepsilon-1}\frac{\varphi_{ij}}{L_{Cj}}dL_{Cj}, \quad i,j=H,F, \tag{B-5}$$

which, using the symmetric condition of (B-4) to substitute out $d\delta_{ij}$, becomes:

$$d\varphi_{ij} = \frac{\varepsilon\varphi_{ij}}{L_{Cj}(\varepsilon-1)\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_j\right)}dL_{Cj} - \frac{\varphi_{ij}}{C_{ij}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_j\right)}dC_{ij}, \quad i,j=H,F \tag{B-6}$$

For future use, we substitute the symmetric condition of (B-6) into (B-4):

$$d\delta_{ji} = \frac{\delta_{ji}(\varepsilon-1+\Phi_i)}{C_{ji}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}dC_{ji} - \frac{\delta_{ji}\varepsilon(\varepsilon-1+\Phi_i)}{L_{Ci}(\varepsilon-1)\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}dL_{Ci}, \quad i,j=H,F \tag{B-7}$$

Finally, taking the total differential of (8), we have:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}}d\varphi_{ii} + \frac{\varepsilon}{\varepsilon-1}\varphi_{ij}\left[\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} + \frac{d\tau_{Tij}}{\tau_{Tij}}\right], \quad i,j=H,F, \quad i\neq j \tag{B-8}$$

where $d\tau_{Tji}=0$ if $i=j$ while $d\tau_{Tji}=\tau_{Xi}d\tau_{Ij}+\tau_{Ij}d\tau_{Xi}$ if $i\neq j$.

### B.2.2 System of 3 equations in 6 differentials

In this section we combine the total differentials of the equilibrium equations to find 3 conditions that can be expressed as functions of $dL_{Ci}$, $dC_{ii}$, $dC_{ij}$, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ only.[26]

**(1)** The first condition can be derived in the following way. Taking the symmetric condition of (B-8), using (B-3) to substitute out $d\varphi_{ji}$, solving for $d\varphi_{jj}$ and finally using (B-6) to substitute out $d\varphi_{ii}$, we obtain:

$$d\varphi_{jj} = -\frac{\varphi_{jj}}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right) - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) \tag{B-9}$$

Using (B-3) to substitute out $d\varphi_{jj}$ from (B-9) we find the following expression for $d\varphi_{ij}$:

$$d\varphi_{ij} = -\frac{\delta_{jj}\varphi_{ij}}{1-\delta_{jj}}\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Tji}}{\tau_{Tji}}\right) - \frac{\delta_{ii}}{1-\delta_{ii}}\frac{1}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right)\right] \tag{B-10}$$

Moreover, we combine (B-6), (B-8) and (B-10) to obtain:

$$-\frac{d\tau_{Tij}}{\tau_{Tij}}(1-\delta_{jj}) + \frac{d\tau_{Tji}}{\tau_{Tji}}\delta_{jj} + \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{1-\delta_{ii}-\delta_{jj}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0$$

Finally we impose symmetry as well as $d\tau_{Lj}=d\tau_{Xj}=d\tau_{Ij}=0$. This means that $d\tau_{Tji}=\tau_{Ij}d\tau_{Xi}$ and $d\tau_{Tij}=\tau_{Xj}d\tau_{Ii}$. Under these restrictions, we can rewrite the last equation as:

$$\frac{d\tau_{Li}}{\tau_{Li}} - (1-\delta_{ii})\frac{d\tau_{Ii}}{\tau_{Ii}} + \delta_{ii}\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{1-2\delta_{ii}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0 \tag{B-11}$$

[26] For the sake of brevity we omit to specify for which countries the equations hold.

**(2)** The second condition can be found as follows. First, we combine (10) and (11) $P_{ij}C_{ij} = L_{Cj}\delta_{ij}\tau_{Tij}\tau_{Lj}$ with $i, j = H, F$. Second, we use this condition to rewrite (12) as follows:

$$L_{Cj} = \frac{\alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{(1-\delta_{jj})\tau_{Lj}\tau_{Xj}(\alpha + (1-\alpha)\tau_{Ii})} \tag{B-12}$$

Third, using (B-4) to find an expression for $d\delta_{jj}$ and combining it with (B-9) we get:

$$\begin{aligned} d\delta_{jj} =& \delta_{jj}(\varepsilon - 1 + \Phi_j)\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right)\right. \\ & \left. - \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right)\right] \end{aligned} \tag{B-13}$$

Taking the total differential of (B-12) and using (B-7) and (B-13) to substitute out $d\delta_{ii}$ and $d\delta_{jj}$, imposing symmetry and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, we obtain:

$$\begin{aligned} &\frac{dL_{Cj}}{L_{Cj}} = \\ &\left(\frac{\alpha}{(1-\alpha)\tau_{Ii}+\alpha} + \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})\left(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}\right)}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dC_{ii}}{C_{ii}} \\ &- \frac{\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})\tau_{Li}\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} - \left(\frac{(1-\alpha)\delta_{ii} - \alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Li}}{\tau_{Li}} \\ &+ \frac{\varepsilon\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii}(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dL_{Ci}}{L_{Ci}} - \frac{1-\alpha}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} \end{aligned} \tag{B-14}$$

In addition, we combine the condition $P_{ij}C_{ij} = L_{Cj}\delta_{ij}\tau_{Tij}\tau_{Lj}$ for $i, j = H, F$ with (13), we take its total differential, and then we substitute out $d\delta_{ii}$ and $d\delta_{jj}$ using (B-7) and (B-13), respectively. We then impose symmetry and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$ to get:

$$\begin{aligned} &-(1-\alpha)L_{Ci}\left(\delta_{ii} + (1-\delta_{ii})\tau_{II}\tau_{Xi} + \frac{\delta_{ii}\varepsilon(1-\tau_{Ii}\tau_{XI})(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right)d\tau_{Li} \\ &-(1-\alpha)L_{Ci}\left((1-\delta_{ii})\tau_{Li} + \frac{\delta_{ii}\varepsilon\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{(\varepsilon-1)\tau_{Ii}\tau_{Xi}}\right)\tau_{Ii}d\tau_{Xi} - (1-\alpha)L_{Ci}(1-\delta_{ii})\tau_{Li}\tau_{Xi}d\tau_{Ii} \\ &+ \frac{(1-\alpha)L_{Ci}\delta_{ii}}{\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}}\left(\frac{\delta_{ii}\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{1-\delta_{ii}} - \tau_{Li}(1-\tau_{Ij}\tau_{Xi})(\varepsilon-1+\Phi_i)\right)\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right) \\ &-(\alpha + (1-\alpha)\tau_{Li}(\delta_{ii} + (1-\delta_{ii})\tau_{Ii}\tau_{Xi}))dL_{Cj} - (\alpha + (1-\alpha)\tau_{Li}(\delta_{ii} + (1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\,dL_{Ci} = 0 \end{aligned} \tag{B-15}$$

We can then use condition (B-14) to substitute out $dL_{Cj}$ from (B-15) and to rewrite (B-15) as follows:

$$\begin{aligned}
&-\frac{(1-\alpha)(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii}\\
&-\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{d\tau_{Li}}{\tau_{Li}}\\
&+\left((1-\alpha)(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})+\frac{\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Li}}\right)d\tau_{Li}\\
&+\frac{\alpha(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}d\tau_{Xi}\\
&+\left((1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}+\frac{\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Xi}}\right)d\tau_{Xi}\\
&+\frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(-\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}}+(1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(1-\delta_{ii})\right.\\
&\left.-\frac{(1-\alpha+\alpha\tau_{Xi})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)\frac{dC_{ii}}{C_{ii}}\\
&-\left[\left(\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}}-1\right)(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\right.\\
&-\frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left((\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{1-\alpha+\alpha\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right.\\
&\left.\left.+\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}}-(1-\delta_{ii})(1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})\right)\right]\frac{dL_{Ci}}{L_{Ci}}=0 \qquad \text{(B-16)}
\end{aligned}$$

**(3)** The third condition can be retrieved as follows. First, we use (10) to solve for $\varphi_{ii}$. Second, we substitute the expression for $\varphi_{ii}$ into (8) and solve for $\varphi_{ij}$. Finally, we employ this expression for $\varphi_{ij}$ together with $\delta_{ij}=1-\delta_{ii}$, and (B-12) to rewrite (10) as follows:

$$C_{ij}=C_{ii}\left(\frac{L_{Ci}\delta_{ii}\tau_{Ii}(L\alpha-L_{Ci}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}))}{\tau_{Li}(\alpha+(1-\alpha)\tau_{Ii})}\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{(B-17)}$$

Taking the total differential of (B-17), using (B-7) to substitute out $d\delta_{ii}$ and (B-12) and (B-17) to define, respectively, $L_{Cj}$ and $C_{ij}$, we have:

$$\begin{aligned}
0=&\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}}-\left(\frac{dC_{ii}}{C_{ii}}\frac{\varepsilon-1}{\varepsilon}-\frac{dL_{Ci}}{L_{Ci}}\right)\left(1-\frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1+\frac{L_{Ci}\delta_{ii}\tau_{Li}}{\Lambda_i}(1-\alpha+\alpha\tau_{Xi})\right)\right)\\
&+\frac{dL_{Ci}}{\Lambda_i}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})-\frac{d\tau_{Ii}}{\tau_{Ii}}\frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}}-d\tau_{Xi}\alpha\frac{L_{Ci}(1-\delta_{ii})\tau_{Li}}{\Lambda_i}\\
&+d\tau_{Li}\left(\frac{L_{Ci}}{\Lambda_i}((1-\alpha)\delta_{ii}-(1-\delta_{ii})\alpha\tau_{Xi})+\frac{1}{\tau_{Li}}\right)
\end{aligned}$$

where $\Lambda_i \equiv \alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})$. Using (B-12), under symmetry we can rewrite the previous expression as follows:

$$\begin{aligned}0 = &\frac{\varepsilon - 1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \frac{\alpha}{\alpha + (1-\alpha)\tau_{Ii}}\left(\frac{d\tau_{Ii}}{\tau_{Ii}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) + \left(1 + \frac{(1-\alpha)\delta_{ii} - \alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha + (1-\alpha)\tau_{Ii})}\right)\frac{d\tau_{Li}}{\tau_{Li}} \\ &- \left(1 - \frac{\varepsilon(\varepsilon - 1 + \Phi_i)}{(\varepsilon - 1)(\varepsilon - 1 + \frac{\varepsilon\Phi_i}{\varepsilon - 1})}\left(1 + \frac{\delta_{ii}(1 - \alpha + \alpha\tau_{Xi})}{(1-\delta_{ii})\tau_{Xi}((1-\alpha)\tau_{Ii} + \alpha)}\right)\right)\left(\frac{\varepsilon - 1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) \\ &+ \frac{\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Xi}\tau_{Li}}{(1-\delta_{ii})\tau_{Xi}\tau_{Li}((1-\alpha)\tau_{Ii} + \alpha)}\frac{dL_{Ci}}{L_{Ci}} \qquad \text{(B-18)}\end{aligned}$$

Conditions (B-11), (B-16), and (B-18) can be used to find an explicit solution for either $dL_{Ci}$, $dC_{ii}$ and $dC_{ij}$ as linear functions of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ (i.e., the solution for the unilateral deviations) or for $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ as linear functions of $dL_{Ci}$, $dC_{ii}$ and $dC_{ij}$(i.e., the solution for the Nash problem with all policy instruments). Conditions (B-11), (B-16), and (B-18) also allow us to retrieve the solution for the Nash problem with only the production tax. All these expressions are available upon request.

# C The Design of Trade Agreements in the Presence of Domestic Policies

In this section we prove Propositions 1 and 2 and Lemma 2, which state the main results on strategic policies when all policy instruments (Proposition 1) or only production taxes (Propositions 2 and Lemma 2) are available. In both cases, we solve the Nash problems using the total-differential approach described in Appendix B. We focus on symmetric Nash equilibria. We also prove Lemma 1, which concerns unilateral deviations from the first-best allocation.

## C.1 Proof of Proposition 1

**Proof** We prove Proposition 1 point by point.

**(a)** First, we write the differential of the terms-of trade effect in (18) in terms of $dL_{Ci}$, $dC_{ii}$, $dC_{ij}$. For this purpose, we use the differentials of the equilibrium conditions derived in Appendix B.2.2 – imposing symmetry and the restrictions $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ – to evaluate each component of the terms-of-trade effects as decomposed in (19). In particular, we use: conditions

(B-14) and (B-15) for term (i) (differential of the amount of labor in both countries allocated to the differentiated sectors); conditions (B-7) and (B-13) jointly with the fact that $d\delta_{ji} = -d\delta_{ii}$ for term (ii) (differential of the average-profit shares in the export markets) and conditions (B-3), (B-6) and (B-10) for term (iii) (differentials of the export productivity cut-offs). Finally, we employ (B-11), (B-16) and (B-18) to substitute out $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ to obtain:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \tag{F-1}$$

where:

$$\Sigma_{Cii} = -\frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\tau_{Li}\tau_{Xi}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}}\delta_{ii}(\varepsilon-1)^2} \frac{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})+\alpha\delta_{ii}(\varepsilon-1)+\alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}]+\delta_{ii}\varepsilon[\alpha+(1-\alpha)\tau_{Li}]\Phi_i}{\delta_{ii}[\alpha+(1-\alpha)\delta_{ii}\tau_{Li}]-(1-\delta_{ii})\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}]-\frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}\delta_{ii}[\alpha+(1-\alpha)\tau_{Li}]}$$

$$\Sigma_{Cij} = \frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\varphi_{ij}} \frac{[(\varepsilon-1+\delta_{ii})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})(\alpha\varepsilon+(1-\alpha)(1-\delta_{ii})\tau_{Ii})\tau_{Li}\tau_{Xi}-\frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\varepsilon-1}((1-\alpha)\tau_{Li}+\alpha)]}{(\delta_{ii}H-\Pi)(\varepsilon-1)-\delta_{ii}\varepsilon[(1-\alpha)\tau_{Li}+\alpha](\varepsilon-1+\Phi_i)}$$

$$\Sigma_{LCi} = \frac{\tau_{Li}\tau_{Xi}\left[(\varepsilon-\delta_{ii})\frac{1-\alpha}{\varepsilon-1}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})+\alpha\delta_{ii}+\alpha\frac{\varepsilon}{\varepsilon-1}(1-\delta_{ii})\tau_{Li}\tau_{Xi}+\delta_{ii}\frac{\varepsilon}{(\varepsilon-1)^2}(\alpha+(1-\alpha)\tau_{Li})\Phi_i\right]}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where $\Sigma_{Cii}$, $\Sigma_{Cij}$, and $\Sigma_{LCi}$ have been simplified using equations (7)-(13). Moreover, $\Pi = (1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}$ and $H = \alpha+(1-\alpha)\tau_{Li}[\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}]$. Condition (F-1) allows us to write (18) as follows:

$$\begin{aligned} dV_i =& (1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) \\ =& E_{Cii}dC_{ii} + E_{Cij}dC_{ij} + E_{LCi}dLC_i + \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \\ =& \Omega_{Cii}dC_{ii} + \Omega_{Cij}dC_{ij} + \Omega_{LCi}dL_{Ci} \end{aligned} \tag{F-2}$$

where $E_{Cii} \equiv (1-\tau_{Xi})P_{ii}$, $E_{Cij} \equiv (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}$, $E_{LCi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1$, $\Omega_{Cii} \equiv E_{Cii}+\Sigma_{Cii}$, $\Omega_{Cij} \equiv E_{Cij}+\Sigma_{Cij}$, and $\Omega_{LCi} \equiv E_{LCi}+\Sigma_{LCi}$. Condition (F-2) corresponds to condition (20) in the main text.

**(b)** In appendix B.1.2 we explained how to apply the total differential approach to solve a constrained optimization problem in $n$ variables with $m$ constraints. In this case our equilibrium system of equations (6)-(14) is characterized by 27 variables (24 endogenous variables plus 3 policy instruments) and 24 constraints i.e., exactly 3 degrees of freedom to choose the policy instruments so as to maximize individual-country welfare. In point (a) we showed how to rewrite the total differential of (20) as function of 3 total differentials ($dC_{ii}, dC_{ij}, dL_{Ci}$ with $i = H, F$ and $i \neq j$). As explained in B.1.2, at the optimum the wedges multiplying each differential need to be individually equal to zero, i.e., $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$. This gives a set of 3 additional equations which can be used to solve for the optimal policy instruments. Once we have the solution for the instruments we can use the 24 constraints to determine the solution of the remaining 24 variables.

Before moving to point (c) we simplify each of these wedges to make them tractable.

First, consider $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$. Using (11) and imposing symmetry, the consumption-efficiency wedge $E_{Cij}$ in (F-2) can be written as $E_{Cij} = \frac{(\tau_{Ii}-1)(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ij}}$ Then, recalling condition (F-1) we obtain

$$\Omega_{Cij} = \frac{\overline{\Omega}_{Cij}\tau_{ij}\tau_{Li}\tau_{Xi}(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}}{\varphi_{ij}(\varepsilon-1)(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\left[(\delta_{ii}H - \Pi)(\varepsilon-1) - \delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)\right]},$$

where

$$\overline{\Omega}_{Cij} = (\varepsilon-1)((\varepsilon-1)(1-\delta_{ii})H + \varepsilon\tau_{Ii}(\delta_{ii}H-\Pi)) - \delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)((1-\alpha)\tau_{Li}+\alpha)(\varepsilon\tau_{Ii}-\varepsilon+1). \quad \text{(F-3)}$$

Second, consider $\Omega_{Cii} \equiv E_{Cii} + \Sigma_{Cii}$. Again using (11), the consumption-efficiency wedge $E_{Cii}$ in (F-2) can be simplified as $E_{Cii} = \frac{(\tau_{Xi}-1)(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{Li}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ii}}$. Therefore, by (F-1)

$$\Omega_{Cii} = \frac{\overline{\Omega}_{Cii}(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\tau_{Li}(L_{Ci}\delta_{ii})^{-\frac{1}{\varepsilon-1}}(\varepsilon-1)^{-2}\varphi_{ii}^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})},$$

where

$$\begin{aligned}
\overline{\Omega}_{Cii} \equiv\; & (1-\tau_{Xi})[\varepsilon(\varepsilon-1)(\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})) \\
& - (\varepsilon-1+\Phi_i)\varepsilon^2\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})] \\
& - \tau_{Xi}[(\varepsilon-1)(\varepsilon(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) - (1-\alpha)\delta_{ii}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) \\
& + \alpha\delta_{ii}(\varepsilon-1) + \alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}) + \delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i] \qquad \text{(F-4)}
\end{aligned}$$

Finally, consider $\Omega_{LCi} \equiv E_{LCi} + \Sigma_{LCi}$. Combining the production-efficiency wedge in (F-2) and condition (F-1) we obtain:

$$\Omega_{LCi} = \frac{\overline{\Omega}_{LCi}(\varepsilon-1)^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where

$$\overline{\Omega}_{LCi} \equiv \delta_{ii}(\varepsilon-1)\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})-\varepsilon(\alpha+(1-\alpha)\tau_{Li})]$$
$$-(\varepsilon-1)[\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})]$$
$$-(\tau_{Li}\tau_{Xi}-1)\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i \tag{F-5}$$

Notice that from (F-3), (F-4) and (F-5) we can conclude that $\Omega_{LCi}=\Omega_{Cii}=\Omega_{Cij}=0$ iff $\overline{\Omega}_{LCi}=\overline{\Omega}_{Cii}=\overline{\Omega}_{Cij}=0$.

**(c)** First recall that from point (b) in the Nash equilibrium

$$\overline{\Omega}_{LCi}=\overline{\Omega}_{Cii}=\overline{\Omega}_{Cij}=0, \tag{F-6}$$

where $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ are defined in (F-3), (F-4), and (F-5). These wedges are functions of 8 variables only: $\tau_{Li}$, $\tau_{Ii}$, $\tau_{Xi}$, $\varphi_{ii}$, $\varphi_{ij}$, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and $\delta_{ii}$. Observe that once we impose symmetry and we take into account that $\delta_{ji}=1-\delta_{ii}$ also conditions (6)-(9) are a system of 5 equations functions of these 5 variables only. Therefore, we can fully characterize the symmetric Nash equilibrium using the 3 conditions in (F-6) jointly with the 5 equilibrium equations (6)-(9). In what follows we use the superscript $N$ to indicate that a variable is evaluated at the Nash equilibrium.

To prove point (c), we proceed in 3 steps. First, we show that in the Nash equilibrium it must be the case that $\tau_L^N=\frac{\varepsilon-1}{\varepsilon}$. Second, we show that $\overline{\Omega}_{LCi}>0$ always when $\tau_X<1$ and $\tau_L=\tau_L^N$. Therefore, when a Nash equilibrium exists it must be such that $\tau_X^N>1$. Finally, we show that $\overline{\Omega}_{Cij}<0$ always when $\tau_I>1$, $\tau_X>1$ and $\tau_L=\tau_L^N$. Hence, when a Nash equilibrium exists it must be such that $\tau_I^N<1$.

**(1)** We use $\overline{\Omega}_{LCi}=\overline{\Omega}_{Cii}=0$ to solve for $\tau_L$ and $\tau_I$ and we obtain two sets of solutions, $(\tau_L^1,\tau_I^1)$ and $(\tau_L^2,\tau_I^2)$:

$$\tau_I^1=\frac{(1-\alpha)\delta_{ii}^2(\varepsilon(1-\tau_X)+\tau_X)-\alpha\varepsilon\tau_X+\delta_{ii}\varepsilon((\varepsilon-1+\alpha)\tau_X-\varepsilon)}{(1-\alpha)(1-\delta_{ii})\tau_X[\varepsilon(1-\delta_{ii})+\delta_{ii}\tau_X(\varepsilon-1)]}+\frac{\delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon(\tau_X-1)-\tau_X)\Phi_i}{(1-\alpha)(1-\delta_{ii})(\varepsilon-1)^2\tau_X[\varepsilon(1-\delta_{ii})+\delta_{ii}\tau_X(\varepsilon-1)]}$$
$$\tau_L^1=\frac{\varepsilon-1}{\varepsilon},\quad \tau_L^2=-\alpha\frac{1+\varepsilon(\varepsilon-2+\Phi_i)}{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})+\alpha(1-\delta_{ii})\tau_X]+(1-\alpha)\varepsilon\Phi_i},\quad \tau_I^2=-\frac{\alpha}{1-\alpha}$$

Note that $\tau_I^2<0$, which is outside the admissible range for $\tau_I$. Thus, the only possible solution is $(\tau_L^1,\tau_I^1)$, implying that when a Nash equilibrium exists, it must be that $\tau_L^N=\frac{\varepsilon-1}{\varepsilon}$. We can thus substitute $\tau_L^N$ into $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ (labeling these expressions $\overline{\Omega}_{LCi}^N$, $\overline{\Omega}_{Cii}^N$, and $\overline{\Omega}_{Cij}^N$

respectively) to obtain $\overline{\Omega}^{N}_{LCi} = \overline{\Omega}^{\mathcal{N}}_{LCi} + \overline{\Omega}^{\Phi}_{LCi}$ $\overline{\Omega}^{N}_{Cii} = -\frac{\overline{\Omega}^{N}_{LCi}}{\varepsilon}$ and $\overline{\Omega}^{N}_{Cij} = \overline{\Omega}^{\mathcal{N}}_{Cij} + \overline{\Omega}^{\Phi}_{Cij}$ where $\overline{\Omega}^{\mathcal{N}}_{LCi} \equiv (\varepsilon-1)^2[\delta_{ii}(\varepsilon-(\varepsilon-1)\tau_X)(\varepsilon-(1-\alpha)\delta_{ii}) + \delta_{ii}(\varepsilon-1)\tau_X((1-\alpha)(1-\delta_{ii})\tau_I\tau_X) + \varepsilon((1-\delta_{ii})(\alpha+(1-\alpha)(1-\delta_{ii})\tau_I)\tau_X)]$, $\overline{\Omega}^{\Phi}_{LCi} \equiv \delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon-(\varepsilon-1)\tau_X)\Phi_i$, $\overline{\Omega}^{\mathcal{N}}_{Cij} \equiv (\varepsilon-1)[\delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1) + \delta_{ii}\tau_I(\alpha\varepsilon+\delta_{ii}(\varepsilon-1)(1-\alpha)) + (1-\delta_{ii})(\varepsilon-1)\left(\alpha+\varepsilon^{-1}\delta_{ii}(1-\alpha)(\varepsilon-1)\right) + (1-\delta_{ii})(\varepsilon-1)\tau_I\tau_X\left(\varepsilon^{-1}(1-\alpha)(\varepsilon-1)(1-\delta_{ii})-\alpha-(1-\alpha)(1-\delta_{ii})\tau_I\right)]$ and $\overline{\Omega}^{\Phi}_{Cij} \equiv \delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1)\Phi_i$ Note that $\overline{\Omega}^{N}_{Cii}$ and $\overline{\Omega}^{N}_{LCi}$ are collinear. In the next steps we thus use only $\overline{\Omega}^{N}_{LCi}$ and $\overline{\Omega}^{N}_{Cij}$ to characterize the Nash equilibrium for the remaining two instruments, $\tau^N_X$ and $\tau^N_I$.

**(2)** First, observe that $\varepsilon-(\varepsilon-1)\tau_X > 0$ iff $\tau_X < \frac{\varepsilon}{\varepsilon-1}$. This implies that when $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then both $\overline{\Omega}^{\mathcal{N}}_{LCi} > 0$ and $\overline{\Omega}^{\Phi}_{LCi} > 0$. Therefore, $\overline{\Omega}^{N}_{LCi} > 0$ for all $\tau_X < \frac{\varepsilon}{\varepsilon-1}$, implying that there cannot be a Nash equilibrium in this region as it will never be the case that $\overline{\Omega}^{N}_{LCi} = 0$. Thus, in the Nash equilibrium it must be the case that $\tau^N_X > \frac{\varepsilon}{\varepsilon-1} > 1$.

**(3)** What remains to show is that $\tau^N_I < 1$. We prove this by contradiction. Assume $\tau^N_I > 1$. In the previous point, we already showed that $\tau^N_X > 1$, thus if $\tau^N_I > 1$ also $\tau^N_I\tau^N_X > 1$. First, consider that $\overline{\Omega}^{\Phi}_{Cij} < 0$ when $\tau^N_I > 1$. As a consequence, a necessary condition for the Nash equilibrium to exist in the region $\tau_I > 1$ is that there exist a $\tau_I > 1$ such that $\overline{\Omega}^{\mathcal{N}}_{Cij} > 0$. To see whether this is the case, observe that $\overline{\Omega}^{\mathcal{N}}_{Cij}$ is linear in $\alpha$ since $\delta_{ii}$ (as implicitly determined by conditions (6)-(9)) is independent of $\alpha$. Moreover, when $\alpha = 0$ $\overline{\Omega}^{\mathcal{N}}_{Cij} = (\varepsilon-1)^2\left[-\delta_{ii}(1-\delta_{ii}+\varepsilon(\tau_I-1)(\varepsilon-\delta_{ii})) - (1-\delta_{ii})^2(1+\varepsilon(\tau_I-1))\tau_I\tau_X\right] < 0$ while when $\alpha = 1$, $\overline{\Omega}^{\mathcal{N}}_{Cij} = -(\varepsilon-1)^2[(\tau_I\tau_X-1)(1-\delta_{ii}) + \delta_{ii}\varepsilon(\tau_I-1)] < 0$. This implies that $\overline{\Omega}^{\mathcal{N}}_{Cij} < 0$ for all $\tau_I > 1$. Therefore, $\overline{\Omega}^{N}_{Cij} < 0$ for all $\tau_I > 1$ which contradicts our original hypothesis of a Nash equilibrium with $\tau^N_I > 1$. Thus, if a Nash equilibrium exists it must be such that $\tau^N_I < 1$. ■

## C.2 Proof of Lemma 1

**Proof** We prove Lemma 1 point by point.

**(a)** First note that, when $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$, both production efficiency and consumption efficiency effects are zero so that condition (18) simplifies to:

$$dV_i = \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \tag{F-7}$$

where we made use of (F-1) to write the terms-of-trade effect as function of $dL_{Ci}$, $dC_{ii}$, and $dC_{ij}$. As explained in section B.2.2, conditions (B-11), (B-16), and (B-18) can be used to find an explicit solution for $dL_{Ci}$, $dC_{ii}$ and $dC_{ij}$ as linear functions of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ for $i = H, F$. Imposing symmetry of the initial conditions, $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$, as well as $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$, we can rewrite (F-7) as function only of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$, and evaluate the welfare effects of a unilateral marginal change in each of the policy instruments.

When $d\tau_{Li} = d\tau_{Xi} = 0$ then $dV_i = \frac{L_{Ci}(\delta_{ii}-1)((1-\delta_{ii})(\varepsilon-1)^2(\varepsilon 2\delta_{ii}-(1-\alpha)(2\delta_{ii}-1))+\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)\varepsilon} d\tau_{Ii}$. Note that $\varepsilon > 1-\alpha$ and $2\delta_{ii} > 2\delta_{ii}-1$. Therefore, $\varepsilon 2\delta_{ii} - (1-\alpha)(2\delta_{ii}-1) > 0$ implying that the numerator is always negative. The sign of the denominator depends on $AB$ where $A \equiv 2\delta_{ii} - 1$ and $B \equiv \alpha + (2\delta_{ii}-1)(\varepsilon-1)$. Note that $A > 0$ if and only if $\delta_{ii} > \frac{1}{2}$ and $B > 0$ if and only if $\delta_{ii} > \frac{1}{2} - \frac{\alpha}{2(\varepsilon-1)}$. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Ii} < 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

When $d\tau_{Li} = d\tau_{Ii} = 0$ then $dV_i = \frac{L_{Ci}(1-\delta_{ii})((\varepsilon-1)^2(\varepsilon(1-\delta_{ii})-\delta_{ii}(\varepsilon-1+\alpha)(1-2\delta_{ii}))+\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)\varepsilon} d\tau_{Xi}$. Note that $\varepsilon > \varepsilon-1+\alpha$ and $1-\delta_{ii} > 1-2\delta_{ii}$ thus, $\varepsilon(1-\delta_{ii}) > \delta_{ii}(\varepsilon-1+\alpha)(1-2\delta_{ii})$ and the numerator is always positive. The denominator is the same as in the previous point. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Xi} > 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $dV_i = \frac{L_{Ci}(1-\delta_{ii})\varepsilon((\varepsilon-1)^2+2\delta_{ii}(\varepsilon-1+\alpha)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)^2} d\tau_{Li}$. Note that the numerator is always positive. The sign of the denominator depends on $AB$ where $A$ and $B$ have been defined above. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Li} > 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

**(b)** We now compute the imports from the differentiated sector in the 3 scenarios. When $d\tau_{Li} = d\tau_{Xi} = 0$ then $dC_{ij} = -C_{ij}\varepsilon\frac{A_{\tau_{Ii}}+\Phi_i B_{\tau_{Ii}}}{C_{\tau_{Ii}}} d\tau_{Ii}$ where $A_{\tau_{Ii}} \equiv (\varepsilon-1)^2((\varepsilon-1)(2\alpha\delta_{ii}(\delta_{ii}^2+1-\delta_{ii})+((1-\delta_{ii})^2+\delta_{ii}^2)((1-\alpha)(1-\delta_{ii})+\delta_{ii}(\varepsilon-1)))+\alpha\delta_{ii}(1-\delta_{ii})+\alpha^2\delta_{ii}^2)$, $B_{\tau_{Ii}} \equiv \alpha(\varepsilon-1)(2\delta_{ii}^2+1-\delta_{ii})+\alpha^2\delta_{ii}+((1-\delta_{ii})^2+\delta_{ii}^2)(\varepsilon-1)^2$, and $C_{\tau_{Ii}} \equiv -(1-2\delta_{ii})2(\varepsilon-1)\left(\delta_{ii}-\frac{1}{2}\left(1-\frac{\alpha}{\varepsilon-1}\right)\right)(\varepsilon-1)^2(\varepsilon-1+\alpha)$

When $d\tau_{Li} = d\tau_{Ii} = 0$ then $dC_{ij} = C_{ij}(1-\delta_{ii})\varepsilon\frac{A_{\tau_{Xi}}+\Phi_i B_{\tau_{Xi}}}{C_{\tau_{Xi}}} d\tau_{Xi}$ where $A_{\tau_{Xi}} \equiv 2(\varepsilon-1)(1-\alpha)\delta_{ii}(1-\delta_{ii})+\alpha(\varepsilon-(1-\alpha)\delta_{ii})+2\delta_{ii}^2(\varepsilon-1)(\varepsilon-1+\alpha)$, $B_{\tau_{Xi}} \equiv \delta_{ii}(\alpha+2\delta_{ii}(\varepsilon-1))\varepsilon(\alpha+\varepsilon-1)$, and $C_{\tau_{Xi}} = C_{\tau_{Ii}}$

When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $dC_{ij} = C_{ij}\varepsilon^2\frac{A_{\tau_{Li}}+\Phi_i B_{\tau_{Li}}}{C_{\tau_{Li}}} d\tau_{Li}$ where $A_{\tau_{Li}} \equiv 2\delta_{ii}(\varepsilon-1)(1-\delta_{ii})+\delta_{ii}(\varepsilon-1)^2+\alpha(\varepsilon-1)(1-\delta_{ii}+2\delta_{ii}^2)+\alpha(1-\delta_{ii})+\delta_{ii}\alpha^2$, $B_{\tau_{Li}} \equiv \delta_{ii}\varepsilon(\alpha+\varepsilon-1)^2$, and $C_{\tau_{Li}} = C_{\tau_{Ii}(\varepsilon-1)}$

First note that $A_{\tau_{Ii}}$, $A_{\tau_{Xi}}$, $A_{\tau_{Li}}$, $B_{\tau_{Ii}}$, $B_{\tau_{Xi}}$, and $B_{\tau_{Li}}$ are always positive. Note that $C_{\tau_{Ii}} > 0$ (and therefore also $C_{\tau_{Xi}} > 0$ and $C_{\tau_{Li}} > 0$) when either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$. It then follows that $dC_{ij} > 0$ when either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$ and $d\tau_{Ii} < 0$, or $d\tau_{Xi} > 0$, or $d\tau_{Li} > 0$. ■

## C.3 Proof of Proposition 2

**Proof** We prove Proposition 2 point by point.

**(a)** When only production taxes are available $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Therefore, the consumption-efficiency wedges in (18) are absent. Hence, to prove this point it is sufficient to rewrite the term-of-trade effect as a function of $dL_{Ci}$ only, and then add it to the production-efficiency term.

For this purpose, we follow the same approach used in point (a) of Proof C.1. We use the differentials of the equilibrium conditions derived in Appendix B.2.2 to evaluate each component of the terms-of-trade effects as decomposed in (19) with the difference that in this case we do not only impose symmetry and $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ but also the restrictions $d\tau_{Ii} = d\tau_{Xi} = 0$. Moreover, given the system of 3 equations ((B-11), (B-16), and (B-18)) in 6 variables ($d\tau_{Li}$, $d\tau_{Ii}$, $d\tau_{Xi}$, $dL_{Ci}$, $dC_{ii}$, $dC_{ij}$) and given that here we are imposing $d\tau_{Ii} = d\tau_{Xi} = 0$, we are able to express $d\tau_{Li}$, $dC_{ii}$, $dC_{ij}$ as a function of $dL_{Ci}$ only. This allows us to obtain $C_{ji}dP_{ji} - C_{ij}dP_{ij} = \Sigma_i dL_{Ci}$ with $\Sigma_i \equiv \frac{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})[(\alpha(2\delta_{ii}-1)(1+\varepsilon(\tau_{Li}-1))-\varepsilon\tau_{Li})-2\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i]}{(\varepsilon-1)\Sigma_{di}}$
$\Sigma_{di} \equiv (\varepsilon-1)\left[(1-\delta_{ii})(1+2\delta_{ii}(\varepsilon-1))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li})\right]$
$+ 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i$.

Then, in this case condition (18) can be simplified as $dV_i = (\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1)dL_{Ci} + C_{ji}dP_{ji} - C_{ij}dP_{ij} = E_i dL_{Ci} + \Sigma_i dL_{Ci} = \Omega_i dL_{Ci}$ where $\Omega_i \equiv E_i + \Sigma_i$ and $E_i \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1$. This last condition leads to condition (21) in the main text.

**(b)** Characterizing the Nash problem when only production taxes are available means solving the constrained problem in (17) imposing $\tau_{Ii} = \tau_{Xi} = 1$. We follow the same steps explained in general terms in Appendix B.1.2. The problem can be reduced to a maximization problem in 25 variables (24 endogenous variables plus 1 policy instrument) subject to the equilibrium conditions (6)-(13). In the previous point we showed how to rewrite the total differential of

(17) as in (21) namely as a function of one total differential only, $dL_{Ci}$. The number of policy instruments available to the individual-country policy maker is also one. This implies that at the optimum condition (21) must be equal to zero, i.e., $\Omega_i = 0$. Note how we can rewrite $\Omega_i$ as $\Omega_i = \frac{\overline{\Omega}_i}{(\varepsilon-1)\Sigma_{di}}$ where $\overline{\Omega}_i \equiv (\varepsilon-1)[(1+\varepsilon(\tau_{Li}-1))((1-\delta_{ii})(1-\alpha+2\delta_{ii}(\varepsilon-(1-\alpha)))(\alpha+(1-\alpha)\tau_{Li})+(1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li}))-(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})\varepsilon\tau_{Li}]+2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-(1-\alpha))(\tau_{Li}-1)\Phi_i$. Given this last condition we can conclude that $\Omega_i = 0$ iff $\overline{\Omega}_i = 0$.

**(c)** First, note that $\overline{\Omega}_i$ is a function of 6 variables: $\tau_{Li}$, $\varphi_{ii}$, $\varphi_{ij}$, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and $\delta_{ii}$. Second, under symmetry and when $\tau_{Ii} = \tau_{Xi} = 1$, the equilibrium equations (6)-(9) give us 5 conditions, which provide a solution for $\varphi_{ii}$, $\varphi_{ji}$, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ji}$, and $\delta_{ii}$ independently from $\tau_{Li}$. Hence, condition $\overline{\Omega}_i = 0$ jointly with conditions (6)-(9) allows us to fully characterize the Nash equilibrium when only the production tax is available.

For what follows, note that $\overline{\Omega}_i$ is a quadratic polynomial in $\tau_{Li}$ (called $\overline{\Omega}_i(\tau_{Li})$). The symmetric Nash-equilibrium policy will not affect the profit-share from sales in the domestic market and thus $\delta_{ii}$ can be determined independently of $\tau_{Li}$. Moreover, $\overline{\Omega}_i(0) < 0$ for $0 < \delta_{ii} \leq 1$ and $\overline{\Omega}_i(0) = 0$ when $\delta_{ii} = 0$ since $\overline{\Omega}_i(0) = -(\varepsilon-1)^2\alpha\,[(1-\delta_{ii})(1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1))\ -(1-2\delta_{ii})(1-\alpha)] - 2\alpha(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i$ and both $1-\delta_{ii} > 1-2\delta_{ii}$ and $1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1) > 1-\alpha$. In addition, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = -(1-\delta_{ii})(\alpha+\varepsilon-1)\,[(\varepsilon-1)^2+2\delta_{ii}(\alpha+\varepsilon-1)\Phi_i]\,\varepsilon^{-1}$. Hence, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$ for $0 \leq \delta_{ii} < 1$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = 0$ when $\delta_{ii} = 1$. Moreover, observe that $\overline{\Omega}_i(1) = (2\delta_{ii}-1)(\varepsilon-1)\,[(1-\delta_{ii})(\varepsilon-1+\alpha)+\delta_{ii}(1-\alpha)]$. As a consequence, $\overline{\Omega}_i(1) \geq 0$ iff $\delta_{ii} \geq \frac{1}{2}$. Finally, taking into account that $\overline{\Omega}''_i(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon[(\varepsilon-1)\varpi_i(\delta_{ii})\ +2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$ is linear in $\delta_{ii}$ and can be characterized as follows: $\varpi_i(0) = 2\varepsilon+\alpha-3 \geq 0$ iff $\varepsilon \geq \frac{3-\alpha}{2}$, $\varpi_i(1) = 1-\alpha > 0$ and $\varpi_i(\delta_{ii}) \geq 0$ iff $\delta_{ii} \geq \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}$. Now, we are ready to prove points (i) and (ii) point by point.

**(i)** Consider the case $\delta_{ii} \geq \frac{1}{2}$. This implies that $\overline{\Omega}_i(1) \geq 0$. Recall that $\overline{\Omega}_i(\tau_{Li})$ is quadratic, implying that it has at most two zeros. Note that $\overline{\Omega}_i(0) < 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$. If $\overline{\Omega}''_i(\tau_{Li}) \geq 0$ then $\overline{\Omega}_i(\tau_{Li})$ is convex, and the zeros must be such that $\tau_L^1 < 0$ and $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^2 \leq 1$. However, $\tau_{Li} \geq 0$ by assumption. Hence, as long as $\delta_{ii} \geq \frac{1}{2}$ and $\overline{\Omega}''_i(\tau_{Li}) \geq 0$, there exist a unique symmetric Nash equilibrium, namely $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N = \tau_L^2 \leq 1$ . Therefore, what remains to show in order to prove point (c) (i) is that $\overline{\Omega}_i''(\tau_{Li}) \leq 0$ when $\delta_{ii} \geq \frac{1}{2}$. The second derivative is given by $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon\left[(\varepsilon-1)\varpi_i(\delta_{ii}) + 2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i\right]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$. Note that if $\varepsilon \geq \frac{3-\alpha}{2}$, then by linearity $\varpi_i(\delta_{ii}) \geq 0$ for all $0 \leq \delta_{ii} \leq 1$. Instead, if $\varepsilon < \frac{3-\alpha}{2}$, then $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} \leq 1$. However, we can show that $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ when $\varepsilon < \frac{3-\alpha}{2}$. Indeed, $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $\frac{2\varepsilon+\alpha-3}{\varepsilon+\alpha-2} < 1$ and $\varepsilon+\alpha-2 < 0$ when $\varepsilon < \frac{3-\alpha}{2}$. Therefore, in this case $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $2\varepsilon+\alpha-3 > \varepsilon+\alpha-2$. This inequality holds since $\varepsilon > 1$. As a consequence, $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{1}{2} \leq \delta_{ii} \leq 1$, which implies that $\overline{\Omega}_i(\tau_{Li})$ is convex in this parameter range.

**(ii)** Now consider the case $\delta_{ii} < \frac{1}{2}$. In this case $\overline{\Omega}_i(1) < 0$. In the previous point we have already argued that $\overline{\Omega}_i(\tau_{Li})$ is convex when either $\varepsilon \geq \frac{3-\alpha}{2}$ or when $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ and $\varepsilon < \frac{3-\alpha}{2}$. Since $\overline{\Omega}_i(\tau_{Li})$ is quadratic $\overline{\Omega}_i(0) \leq 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$, there exist two zeros of $\overline{\Omega}_i(\tau_{Li})$ such that $\tau_L^1 \leq 0$ and $\tau_L^2 > 1$. Again, we can exclude $\tau_L^1 \leq 0$ since $\tau_{Li} > 0$ by assumption. As a consequence, there exists a unique symmetric Nash equilibrium with $\tau_L^N = \tau_L^2 \geq 1$ . ■

## C.4 Proof of Lemma 2

**Proof** I We prove Lemma 2 point by point.

**(a)** According to Proposition 2, when $\delta_{ii} \geq \frac{1}{2}$ and only domestic policies are available any symmetric Nash equilibrium is such that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$. Hence, a sufficient condition for the Nash allocation to entail higher welfare than the free-trade allocation is that in a symmetric equilibrium individual-country welfare is monotonically decreasing in $\tau_{Li}$. Thus, we need to show that in a symmetric equilibrium $\frac{dU_i}{d\tau_{Li}} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. To show this, first observe that $\frac{dU_i}{d\tau_{Li}} = \frac{dU_i}{dL_{Ci}}\frac{dL_{Ci}}{d\tau_{Li}}$. Second, consider that the total differential of utility in (3) can be written as:

$$dU_i = \frac{1}{I_i}\sum_{j=H,F} P_{ij}dC_{ij} + \frac{1}{I_i}dZ_i, \quad i = H, F \tag{F-8}$$

To see why this is the case, substitute the definition of the consumption aggregator (4) into the utility function (3), to get:

$$U_i = \alpha\frac{\varepsilon}{\varepsilon-1}\log\left(\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\right) + (1-\alpha)\log Z_i, \quad i = H, F$$

Taking the total differential of this objective function, we obtain:

$$dU_i = \alpha \sum_{j=H,F} \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} dC_{ij} + \frac{1-\alpha}{Z_i} dZ_i, \quad i = H, F \tag{F-9}$$

Note that $\frac{1-\alpha}{Z_i} = \frac{1}{I_i}$ and $\alpha \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} \frac{P_i}{I_i} = \frac{P_{ij}}{I_i}$ since $\left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} = \frac{P_{ij}}{P_i}$ for $i, j = H, F$. As a result, condition (F-9) can be rewritten as in (F-8). Then, combining the total differential in (F-8) with the one of (12) and (13) departing from a symmetric allocation we get: $dU_i = \frac{P_{ii}}{I_i} dC_{ii} + \frac{P_{ij}}{I_i} dC_{ij} - \frac{1}{I_i} dL_{Ci}$. Moreover, it can be shown[27] that under symmetry $dC_{ij} = \frac{C_{ij}}{L_{Ci}} \frac{\varepsilon}{\varepsilon-1} dL_{Ci}$ for $i, j = H, F$. By substituting these conditions into the differential above and taking into account conditions (10) and (11) we obtain: $dU_i = \frac{1}{I_i}\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right) dL_{Ci}$. This last result follows directly from the fact that symmetric deviations of the production subsidy from a symmetric allocation do not have an impact on the cut offs $\varphi_{ij}$ and on the market shares $\delta_{ij}$, implying that terms-of-trade effects are zero. Moreover, consumption-efficiency wedges are also zero since import tariffs and export taxes are absent. Hence, changes in welfare in condition (18) are equal to the production-efficiency effects. Finally, it can be shown that $\frac{dL_{Ci}}{d\tau_{Li}} = -\frac{(1-\alpha)L_{Ci}}{\alpha+\tau_{Li}(1-\alpha)} < 0$ We conclude that $\frac{dU_i}{d\tau_{Li}} = -\frac{L_{Ci}}{I_i}\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)\frac{1-\alpha}{\alpha+\tau_{Li}(1-\alpha)} \leq 0$ if and only if $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. We know from Proposition 2 that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$ when $\delta_{ii} \geq \frac{1}{2}$ implying that whenever $\delta_{ii} \geq \frac{1}{2}$, the Nash equilibrium when countries con only set domestic policies strategically welfare dominates the laissez-faire allocation with $\tau_{Li} = 1$. From Proposition 2 we also know that when either $\delta_{ii} < \frac{1}{2}$ and $\varepsilon \geq \frac{3-\alpha}{\alpha}$ or $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ and $\varepsilon < \frac{3-\alpha}{3}$, then there exists a unique symmetric Nash equilibrium with $\tau_L^N > 1$. Therefore, in these cases the symmetric Nash equilibrium is welfare dominated by the laissez-faire allocation.

**(b)** Taking the differential of conditions (6), (7) and (8) with respect to $f_{ij}$ and $\tau_{ij}$, it can be shown that $d\delta_{ii} = \frac{(\varepsilon-1+\Phi_i)\delta_{ii}(1-\delta_{ii})}{\tau_{ij}} d\tau_{ij} + \frac{\Phi_i \delta_{ii}(1-\delta_{ii})\tilde{\varphi}_{ij}^{1-\varepsilon}\varphi_{ij}^{\varepsilon-1}}{(\varepsilon-1)f_{ij}} df_{ij}$, which confirms that $\delta_{ii}$ is monotonically increasing in both $\tau_{ij}$ and $f_{ij}$. ■

[27]The proof is available on request.