Trade and Domestic Policies Under Monopolistic Competition*

Alessia Campolmi†
Università di Verona

Harald Fadinger‡
University of Mannheim and CEPR

Chiara Forlati§
University of Southampton

This draft: July 2021

Should trade agreements also constrain domestic policies? We analyze this question from the perspective of models with monopolistic competition, potentially heterogeneous firms, and multiple sectors. We propose a welfare decomposition based on principles from welfare economics to show that, in a broad class of models, welfare changes induced by trade and domestic policies can be exactly decomposed into consumption-efficiency, production-efficiency and terms-of-trade effects. Using this decomposition, we compare trade agreements with different degrees of integration and show how their performance is affected by the interaction between firm heterogeneity and the relative importance of production efficiency versus terms-of-trade effects. We consider several forms of shallow trade agreements, modeled according to GATT-WTO rules, and show that they are not sufficient to achieve the full benefits of globalization that can be obtained with a deep trade agreement coordinating both trade and domestic policies. Moreover, the distortions arising from uncoordinated domestic policies under shallow free trade agreements increase when physical trade costs fall, thus raising the benefits of deep trade integration.

Keywords: Heterogeneous Firms, Trade Policy, Domestic Policy, Trade Agreements, Terms of Trade, Efficiency, Tariffs and Subsidies

JEL classification codes: F12, F13, F42

---

*We thank Costas Arkolakis, Paola Conconi, Jan Haaland, Eckhard Janeba, Ahmad Lashkaripour, Monika Mrazova, Gonzague Vanoorenberghe, and seminar participants in Glasgow, Mannheim, Norwegian School of Economics, Nottingham, Paris School of Economics, Sciences Po, Southampton and conference participants at EEA, ETSG, the CRC TR 224 conference, the Midwest International Trade Conference, ESEM, the ITSG, the workshop on Trade Policy and Firm Performance and the 5th CEPR Conference on Global Value Chains, Trade and Development for useful comments.

†Università di Verona, Department of Economics, via Cantarane 24, 37129 Verona, Italy. alessia.campolmi@univr.it. Part of this research was written while Alessia was a Visiting Fellow at the European University Institute, whose hospitality is gratefully acknowledged.

‡University of Mannheim, Department of Economics, L7 3-5, D-68131 Mannheim, Germany and Centre for Economic Policy Research (CEPR), harald.fadinger@uni-mannheim.de. Harald gratefully acknowledges the hospitality of CREI and funding by the German Research Foundation (DFG) through CRC TR 224 (project B06).

§University of Southampton, Social Science, S017 1BJ Southampton, UK. c.forlati@soton.ac.uk.

1 Introduction

We are witnessing a change in the way countries approach trade policy. In the past, regional and multilateral trade agreements were mostly "shallow", i.e. focused on the reduction of import tariffs and export taxes. More recently, there has been a shift to "deeper" agreements, which, in addition to traditional trade policies, cover various domestic policies, such as production subsidies, product and labor standards, intellectual property rights, competition policy, and many other subjects (e.g., Horn, Mavroidis and Sapir, 2010; Dür, Baccini and Elsig, 2014; Rodrik, 2018).[1] Despite these fundamental changes in countries' actual approach to trade agreements, much of the theoretical literature still focuses on classical trade policies: import and export taxes (see Bagwell and Staiger, 2016, for a survey). Moreover, it misses a common tool to analyze the incentives for trade and domestic policies and to design trade agreements within the standard trade framework featuring monopolistic competition and firm heterogeneity.

To fill this gap, in this paper we develop a welfare decomposition for policy analysis based on efficiency principles from welfare economics that is valid in a broad class of trade models and allows us to jointly analyze the incentives for trade and domestic policies. Our approach is inspired by Arkolakis, Costinot and Rodríguez-Clare (2012) and Costinot, Rodríguez-Clare and Werning (2020) who showed, respectively, that the effects of trade cost reductions and trade policy have a common aggregate representation within a wide class of one-sector trade models. We use a general version of the modern workhorse trade model with monopolistic competition and free entry Krugman (1980), firms that are potentially heterogeneous in terms of productivity Melitz (2003) and operate in multiple sectors with CES demand. This model is particularly well suited for studying domestic policies (which we model as sector-specific production taxes/subsidies)[2] and thus deep trade agreements because it features a clear motive

[1]To illustrate the increasing depth and complexity of trade agreements, Rodrik (2018) compares the US trade agreements with Israel and Singapore, signed two decades apart. The US-Israel Free Trade Agreement, which went into force in 1985, was the first bilateral trade agreement the US concluded in the postwar period. It contains 22 articles and three annexes, the bulk of which are devoted to free-trade issues such as tariffs, agricultural restrictions, import licensing, and rules of origin. The US-Singapore Free Trade Agreement went into effect in 2004 and contains 20 chapters (each with many articles), more than a dozen annexes, and multiple side letters. Of its 20 chapters, only seven cover conventional trade topics. Other chapters deal with behind-the-border topics such anti-competitive business conduct, electronic commerce, labor, the environment, investment rules, financial services, and intellectual property rights.

[2]While we model domestic policies in terms of production taxes/subsidies because they fit most naturally into the Melitz (2003) framework, conceptually one can think more broadly of any policies that aim at correcting a distortion between domestic social marginal costs and domestic social marginal benefits, such as market power,

for domestic regulation, even in the absence of international trade: without sector-specific production subsidies, market outcomes are distorted by monopolistic price setting due to multiple sectors with different markups. At the same time, our setup allows us to study to what extent policies might be affected by the presence of firm heterogeneity.

We proceed as follows. We first derive a welfare decomposition and use it to tackle the long-standing theoretical debate on the motives for trade policy and trade agreements. A key advantage of the decomposition is that it allows us to identify beggar-thy-neighbor incentives[3] and to separate them clearly from efficiency considerations of policies. We then use our welfare decomposition to study the relative performance of trade agreements with different levels of integration: several forms of *shallow trade agreements* (agreements on trade taxes without coordination of domestic policies) modeled according to GATT-WTO rules; a *deep trade agreement* (cooperation on trade taxes and domestic policies); and a *laissez-faire agreement* (free trade and a commitment to abstain from using domestic policies). We find that achieving the full benefits of integration requires signing a deep trade agreement and that firm heterogeneity crucially affects the costs and benefits of a shallow free trade agreement relative to a laissez-faire agreement.

The key idea of our approach is to rewrite the model in terms of aggregate CES bundles and to express welfare changes induced by policy instruments in terms of the wedges between market prices and those that would implement the allocation chosen by a social planner. These efficiency wedges are present whenever consumer prices of aggregate bundles deviate from marginal production costs. In the context of our model, such wedges can be either due to monopolistic markups or due to policy distortions. In the spirit of Meade (1955) and Harberger (1971), we split these efficiency effects into consumption-efficiency (given by wedges between consumer and producer prices due to trade taxes) and production-efficiency terms (given by wedges between the marginal value product of labor at producer prices and its marginal cost). The general-equilibrium welfare effects induced by trade or domestic policies can then be exactly decomposed into (i) consumption-efficiency and (ii) production-efficiency effects and (iii) terms-

or a consumption or production externality. This covers, e.g., issues such as competition policy, environmental and product standards or subsidies for research and development.

[3] A beggar-thy-neighbor or zero-sum incentive means that one player/country is made better off at the expense of the other player/country.

of-trade effects that operate via changes in international prices.[4][5]

Our welfare decomposition clarifies that – as long as a sufficient set of policy instruments is available – even in the presence of domestic policies the terms-of-trade motive is the only beggar-thy-neighbor incentive in our framework and thus the only reason to sign a trade agreement. If policy makers did not value the terms-of-trade effects of their policies, they would implement an efficient allocation as long as they dispose of the right set of tax instruments to do so. This result also implies that the delocation effect[6] (Venables, 1987) is not a policy motive on its own. Our approach extends and generalizes (Bagwell and Staiger, 1999, 2001)' concept of "politically optimal trade policies". As our welfare decomposition is valid independently of the number of policy instruments, it is particularly useful for studying policy in second-best environments, where the available instruments are not sufficient to separate production-efficiency and terms-of-trade motives and thus policy makers face a trade-off between them. By contrast, "politically optimal policies" generally do not allow identifying policy-makers' incentives in such a situation.[7]

To clarify the trade-off between production-efficiency and terms-of-trade effects associated with using individual policy instruments, and the impact of firm heterogeneity in governing this trade off, we first consider unilateral deviations from the (inefficient) laissez-faire equilibrium in the two-sector CES framework with an outside good. When starting from this equilibrium, production efficiency can be improved with a small import tariff, a small export subsidy or a small production subsidy in the differentiated sector that triggers entry of firms at home and increases the amount of labor allocated to this sector. However, this comes at the cost of worsening the terms of trade via the extensive margin by reducing the ideal price index of the exportable bundle. There exists a sufficient statistic, the variable profit share of the average active firm from sales in its domestic market, that determines which effect dominates. When

[4]The terms of trade are defined in terms of ideal price indices of exportables and importables. As a consequence, the terms of trade are affected both by changes in the international prices of individual varieties (intensive margin) and by changes in the set of firms active in foreign markets (extensive margin).

[5]A similar decomposition is also valid under perfect competition, see Helpman and Krugman (1989), Chapter 2. Our welfare decomposition can also be extended to applications not considered in the current paper, such as unilaterally optimal policies or to study the welfare implications of other tax instruments, or changes in fundamentals, such as trade costs.

[6]In models with free entry, the delocation effect (also called home-market effect) of policies can be used to attract a larger share of a sector's production by increasing local demand for the good.

[7]See, e.g. Bagwell and Staiger (2016), page 26.

the profit share from domestic sales is larger than the one from export sales, the terms-of-trade motive is weak relative to the production-efficiency motive: only relatively few firms select into exporting and most sales go to the domestic market. Thus, increasing production efficiency is the dominant motive and policy makers exploit the delocation effect to achieve this outcome. By contrast, when the profit share from domestic sales is smaller than the one from export sales the terms-of-trade motive dominates. Consequently, countries can benefit from a small unilateral import subsidy, a production tax, or an export tax that delocates firms to the foreign market (an anti-delocation effect).

With an understanding of the theoretical mechanisms that govern policy makers' incentives, we then study strategic policy setting in the absence of a trade agreement and the normative implications of trade agreements with different degrees of integration.

We first consider strategic trade and domestic policies in the absence of any type of trade agreement in order to have a benchmark for the distortions arising without international cooperation. In this case, the targeting principle applies and strategic outcomes are qualitatively independent of firm heterogeneity: in a symmetric Nash equilibrium, production subsidies are set at the first-best level and exactly offset monopolistic distortions, while trade policies consist of import subsidies and export taxes. Thus, Nash trade policies aim at delocating firms to the *other* economy in order to improve countries' terms of trade via the extensive margin (anti-delocation effect). This result confirms the insight gained from our welfare decomposition: when policy makers have sufficiently many instruments to deal with production efficiency and terms-of-trade effects separately, the terms-of-trade motive is the only international externality and thus the only reason to enter a trade agreement.

We then study a deep trade agreement that coordinates both trade and domestic policies. We show that, starting from the symmetric Nash equilibrium described above, countries can attain the world planner allocation in cooperative negotiations by reducing import subsidies and export taxes reciprocally to zero, while leaving the terms of trade unaffected and production subsidies unchanged at their first-best levels. Thus, a deep trade agreement is sufficient to achieve a globally efficient outcome. We then ask the question if a shallow agreement supplemented with tariff bindings and market access commitments that are implied by GATT-WTO rules achieve the same outcome, as argued by Bagwell and Staiger (2001). In fact, we show that in the context of our model framework such a shallow agreement is not enough to guarantee a globally efficient outcome: without a commitment to coordinate both trade *and* domestic policies, individual-country policy makers have incentives to unilaterally deviate from the previously negotiated globally efficient allocation, e.g., by reducing production subsidies or by subsidizing imports. Such deviations improve domestic terms of trade without reducing market access of foreign firms.

Next, we consider a more stringent scenario modelled along the lines of a shallow free trade agreement according to GATT Article XXIV: we consider strategic domestic policies in a situation where trade taxes are set to zero. In this case domestic policies are governed by the trade-off between improving production efficiency and manipulating the terms of trade, and are thus not set efficiently. When firms are heterogeneous, the relative importance of the two effects depends on whether the profit share from domestic sales is larger than the one from export sales. When it is larger, the production-efficiency effect dominates, and the Nash policy is an (inefficiently low) production subsidy. When it is smaller, the second effect dominates, and the Nash policy is a production tax. Due to endogenous selection into exporting, the average variable profit share from domestic sales is an increasing function of fixed and variable physical trade costs. When physical trade costs fall, uncoordinated domestic policies become more distortive. Thus, in a highly globalized world with low physical trade costs signing a deep trade agreement becomes more important. However, full coordination of domestic policies may not always be feasible. We thus consider as an alternative a laissez-faire agreement, which forbids both the use of trade and domestic policies. We show that whether or not this welfare dominates a shallow free trade agreement depends on whether the profit share from domestic sales is smaller or larger than the one from export sales.

The rest of the paper is structured as follows. In the next subsection we briefly discuss the related literature. In Section 2 we describe a multi-sector Melitz (2003) model expressed in terms of macro bundles. In Section 3 we discuss the solution to the world social planner problem, which allows us to identify the relevant efficiency wedges arising in the market allocation. In Section 4 we solve the problem of a world policy maker who maximizes global welfare and we derive a welfare decomposition that decomposes welfare effects of policies. We then turn to the problem of individual-country policy makers, derive individual-country welfare and discuss welfare effects of unilateral policy deviations (5). Finally, in Section 6 we consider strategic trade and domestic policies under various institutional arrangements. Section 7 presents our conclusions.

1.1 Related literature

Several theoretical contributions have studied the incentives for trade policy in specific versions of the Krugman (1980) and Melitz (2003) models and have identified numerous mechanisms through which trade policy affects outcomes.[8] We add to the literature on trade policy in the CES monopolistic competition framework by incorporating domestic policies and showing that – since they all have the same aggregate representation – these models share a common set of policy motives, which can be understood using our welfare decomposition. Studies investigating trade policy in the two-sector version of Krugman (1980) with homogeneous firms typically find that strategic tariffs are set due to a delocation motive (Venables, 1987; Helpman and Krugman, 1989; Ossa, 2011) that induces policy makers to increase the size of the domestic differentiated sector. Recently, Campolmi, Fadinger and Forlati (2014) have shown that this result is a consequence of limiting the number of policy instruments. When policy makers dispose of production, import and export taxes, the Nash equilibrium is characterized by the first-best level of production subsidies, import subsidies and export taxes that aim at delocating firms to the foreign economy. This paper generalizes their results to heterogeneous firms and interprets their finding in terms of production-efficiency and terms-of-trade effects. Also closely related to our study is Costinot et al. (2020), who consider unilateral trade policy in a generalized two-country version of Melitz (2003) with a single sector. They study optimal firm-specific and non-discriminatory policies and investigate how optimal trade taxes are affected by firm

[8] Gros (1987) and Helpman and Krugman (1989) examine the one-sector version of the Krugman (1980) model with homogeneous firms and identify a terms-of-trade motive for tariffs. Several studies have analyzed the incentives for trade policy in the Melitz (2003) model with a single sector. Demidova and Rodríguez-Clare (2009) and Haaland and Venables (2016) investigate optimal *unilateral* trade policy in a small-open-economy version of Melitz (2003) with Pareto-distributed productivity. While Demidova and Rodríguez-Clare (2009) identify a distortion in the relative price of imported varieties (markup distortion) and a distortion on the number of imported varieties (entry distortion) as motives for unilateral policy, Haaland and Venables (2016) single out terms-of-trade effects as the only reason for individual-country trade policy. Similarly, Felbermayr, Jung and Larch (2013), who consider strategic import taxes in a two-country version of this model, identify the same motives for tariffs as Demidova and Rodríguez-Clare (2009). Turning to models with firm heterogeneity, Haaland and Venables (2016) investigate unilateral policy in the two-sector small-open-economy variant of Melitz (2003) with Pareto-distributed productivities. They identify terms-of-trade externalities and monopolistic distortions as drivers of unilateral policy.

heterogeneity, emphasizing the role of terms-of-trade effects.

Our paper is also connected to the vast literature on trade policy in perfectly competitive trade models (Dixit, 1985). We show that many insights from this literature carry over to the framework with monopolistic competition (and firm heterogeneity). Specifically, we find that the result that (given a sufficient set of policy instruments) trade agreements solve a terms-of-trade externality, which has been forcefully argued by Grossman and Helpman (1995) and Bagwell and Staiger (1999) for perfect competition and by Bagwell and Staiger (2016) for monopolistic competition with homogeneous firms, also applies in the CES monopolistic competition framework with heterogeneous firms.[9] Moreover, we show that Bagwell and Staiger (2001)'s result from a perfectly competitive model that even in the presence of domestic policies and given a sufficient set of policy instruments trade agreements just solve a terms-of-trade externality also applies to monopolistic competition models.[10] Furthermore, also the Bhagwati-Johnson principle of targeting, which states that optimal policy should use the instrument that operates most effectively on the appropriate margin, remains valid. Finally, our welfare decomposition establishes a tight link between the policy incentives in the CES monopolistic competition framework and those in the neoclassical model. Meade (1955) has developed a partial-equilibrium decomposition of welfare incentives in neoclassical trade models that splits welfare effects of policies into efficiency wedges and terms-of-trade effects. We show how to apply this decomposition to general-equilibrium welfare effects of policies in CES monopolistic competition models.

Finally, we also contribute to the literature on trade and domestic policies. Within a model with perfect competition and a local production externality, Copeland (1990) discusses the idea that in the presence of a shallow trade agreement – that limits the strategic use of tariffs – individual-country policy makers may use domestic policies to manipulate the terms of trade. Bagwell and Staiger (2001) use a similar model to study the gains from integrating agreements on domestic policies into trade agreements within a two-stage setup. They argue against integrating rules on domestic policies into trade agreements since in their model GATT-

[9]Maggi and Rodríguez-Clare (1998) show that in the presence of political-economy motives for protection, trade agreements may additionally serve as commitment device to abstain from using distortive policies.

[10]Note that, as explained in detail above, an additional key contribution compared to the work of Bagwell and Staiger is our analysis of welfare incentives in scenarios with a limited set of instruments.

WTO rules are sufficient to sustain efficient levels of domestic policies: they prohibit changes in domestic policies that undo the market access commitment of previously granted tariff concessions and thus a shallow trade agreement can achieve the same level of efficiency as a deep agreement. We show that this is no longer the case in our setup because with monopolistic competition countries can use domestic policies to improve their terms of trade without reducing foreign market access.[11] In a recent contribution, Lashkaripour and Lugovsky (2019) analyze a quantitative multi-sector Krugman (1980) model with trade policies and domestic production subsidies to assess welfare gains from a deep trade agreement relative to unilaterally optimal policies. Finally, Grossman, McCalman and Staiger (2021) investigate deep trade agreements with a focus on harmonization of horizontal production standards in a monopolistic competition model with homogeneous firms when domestic and foreign consumers have different preferences over product characteristics.[12] A key difference between their setup and ours is that in their model regulation of standards play no role in the absence of international trade: in their main setup inefficiencies in standard setting arise only because policy makers manipulate standards to free ride on the other country, while laissez-faire standards are optimal. By contrast, in our model the laissez-faire free-trade allocation is inefficient due to monopolistic distortions, which provides a strong rationale for using and coordinating domestic policies. Also related is Ossa and Maggi (2019) who consider agreements on standards in the presence of consumption or production externalities and political-economy motives, from which we abstract.

2 The Model

The setup follows Melitz and Redding (2015). The world economy consists of two countries i: Home (H) and Foreign (F). The only factor of production is labor which is supplied inelastically in amount L in each country, perfectly mobile across firms and sectors and immobile across countries. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index corresponds to the location of consumption and the second sub-index to the location of production.

[11] Also related is Ederington (2001), who considers the optimal design of joint agreements on trade and domestic policies in the absence of commitment. He establishes that deep trade agreements should require full coordination of domestic policies, while allowing countries to set positive levels of tariffs in order to reduce deviation incentives. While our paper studies agreements with full commitment, it is the first to study shallow and deep trade agreements within the heterogeneous-firm model.

[12] Parenti and Vanoorenberghe (2019) consider a Ricardian model of deep trade agreement under preference heterogeneity.

2.1 Technology and Market Structure

Each country has either one or two sectors. The first sector produces a continuum of differentiated goods under monopolistic competition with free entry. If present, the other sector is perfectly competitive and produces a homogeneous good.[13] Labor markets are perfectly competitive. Differentiated goods are subject to iceberg transport costs. Firms in the differentiated sector pay a fixed cost in terms of labor, f_E, to enter the market and to pick a draw of productivity φ from a cumulative distribution $G(\varphi)$.[14] After observing their productivity draw, they decide whether to pay a fixed cost f in terms of domestic labor to become active and produce for the domestic market. Active firms then decide whether to pay an additional market access cost f_X (in terms of domestic labor) to export to the other country, or to produce only for the domestic market. Therefore, labor demand of firm φ located in market i for a variety sold in market j is given by:

$$l_{ji}(\varphi) = \frac{q_{ji}(\varphi)}{\varphi} + f_{ji}, \quad i, j = H, F \tag{1}$$

where $f_{ji} = f$ for $j = i$, $f_{ji} = f_X$ for $j \neq i$ and where $q_{ji}(\varphi)$ is the production of a firm with productivity φ located in country i for market j. Varieties sold in the foreign market are subject to an iceberg transport cost $\tau > 1$. We thus define $\tau_{ji} = 1$ for $j = i$ and $\tau_{ji} = \tau$ for $j \neq i$.

In case the homogeneous-good sector is present, labor demand L_{Zi} for the homogenous good Z, which is produced in both countries i with identical production technology, is given by:

$$L_{Zi} = Q_{Zi}, \tag{2}$$

where Q_{Zi} is the production of the homogeneous good. Since this good is sold in a perfectly competitive market without trade costs, its price is identical in both countries and equals the marginal cost of production W_i. We assume that it is always produced in both countries in equilibrium. This implies equalization of wages $W_i = W_j$ for $i \neq j$. We also consider a version of the model without the homogeneous sector. In this case, wages across the two countries will not necessarily be equalized.

[13] The generalization of the model to multiple monopolistically competitive sectors is straightforward.

[14] We assume that φ has support $[0, \infty)$ and that $G(\varphi)$ is continuously differentiable with derivative $g(\varphi)$.

2.2 Preferences

Households' utility function is given by:

$$U_i \equiv \alpha \log C_i + (1-\alpha) \log Z_i, \quad i = H, F, \tag{3}$$

where C_i aggregates over the varieties of differentiated goods and α is the expenditure share of the differentiated bundle. When α is set to unity, we go back to a one-sector economy (Melitz, 2003). Z_i represents consumption of the homogeneous good (Krugman, 1980). The differentiated varieties produced in the two countries are aggregated with a CES function given by:[15]

$$C_i = \left[\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i = H, F \tag{4}$$

$$C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H, F \tag{5}$$

Here, C_{ij} is the aggregate consumption bundle of country-i consumers of varieties produced in country j, $c_{ij}(\varphi)$ is consumption by country-i consumers of a variety φ produced in country j, N_j is the measure of varieties produced by country j. φ_{ij} is the cutoff-productivity level, such that a country-j firm with this productivity level makes exactly zero profits when selling to country i, while firms with strictly larger productivity levels make positive profits from selling to this market, so that all country-j firms with $\varphi \geq \varphi_{ij}$ export to country i. Finally, $\varepsilon > 1$ is the elasticity of substitution between domestic and foreign bundles and between different varieties.

2.3 Government

The government of each country disposes of the following fiscal instruments: a sector-specific production tax/subsidy (τ_{Li}) on the fixed and marginal costs of firms in the differentiated sector,[16] a sector-specific tariff/subsidy on imports in the differentiated sector (τ_{Ii}) and a sector-

[15]Notice that we can index consumption of differentiated varieties by firms' productivity level φ since all firms with a given level of φ behave identically. Note also that our definitions of C_{ij} imply $C_i = \left[N_i \int_{\varphi_{ii}}^{\infty} c_{ii}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) + N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e., the model is the standard one considered in the literature. However, it is convenient to define optimal consumption indices.

[16]Since the only production factor in the model is labor, this is equivalent to a sector-specific labor tax/subsidy. We impose that the same production tax is levied on both fixed and marginal costs (including also the fixed entry cost f_E). This assumption is necessary to keep firm size unaffected by production taxes, which turns out to be optimal, as we show in Appendix C.3.

specific tax/subsidy on exports in the differentiated sector (τ_{Xi}).[17] We model domestic policies in terms of sector-specific production taxes/subsidies because they fit most naturally into the Melitz (2003) framework. We show below that one can interpret them more broadly as any policies that aim at correcting a distortion between domestic social marginal costs and domestic social marginal benefits. Such distortions may arise due to market power, as in our framework, but may also be due, e.g., to local consumption or production externalities. Thus one can think of domestic policies as covering a wide range of issues, including competition policy, environmental and product standards or R&D subsidies. In terms of notation, τ_{mi} indicates a gross tax for $m \in \{L, I, X\}$, i.e., $\tau_{mi} < 1$ indicates a subsidy and $\tau_{mi} > 1$ indicates a tax. In what follows, we employ the word *tax* whenever we refer to a policy instrument without specifying whether τ_{mi} is smaller or larger than one and we use the notation $\tau_{Tij} = 1$ for $i = j$ and $\tau_{Tij} = \tau_{Ii}\tau_{Xj}$ for $i \neq j$. Moreover, we assume that taxes are paid directly by the firms[18] and that all government revenues are redistributed to consumers through a lump-sum transfer T_i. We use the term *laissez-faire allocation* to refer to the market allocation in which both countries refrain from using any of the policy instruments, i.e., $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$.

2.4 Equilibrium

Since the model is standard, we relegate a more detailed description of the setup and the derivation of the market equilibrium to Appendix A. Similarly to Arkolakis et al. (2012), Campolmi et al. (2014) and Costinot et al. (2020), we write the equilibrium in terms of sectoral aggregates. Specifically, the one-sector model can be represented in terms of three aggregate goods: a good that is domestically produced and consumed (non-tradable good); a domestic exportable good and a domestic importable good. The two-sector model additionally features a homogeneous good. This representation in terms of aggregate bundles (i) highlights that models with monopolistic competition and CES preferences have a common macro representation and (ii) makes the connection to standard neoclassical trade models visible. It will also be useful for interpreting the wedges between the planner and the market allocations and for our welfare de-

[17]Note that we could easily allow for tax instruments in the perfectly competitive sector but these would be completely redundant. We do not explicitly introduce sector-specific consumption taxes/subsidies but they can be replicated with a combination of production subsidies and import tariffs.

[18]In particular, following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

composition. Finally, the macro representation will make clear that the welfare-relevant terms of trade that policy makers consider in their objective are defined in terms of ideal price indices of sectoral exportables relative to importables.

The market equilibrium is described by the following conditions:

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1 - G(\varphi_{ji})} \right]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \tag{6}$$

$$\delta_{ji} = \frac{f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1}}{\sum\limits_{k=H,F} f_{ki}(1 - G(\varphi_{ki})) \left(\frac{\widetilde{\varphi}_{ki}}{\varphi_{ki}} \right)^{\varepsilon-1}}, \quad i,j = H,F \tag{7}$$

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left(\frac{f_{ii}}{f_{ij}} \right)^{\frac{1}{\varepsilon-1}} \left(\frac{\tau_{Li}}{\tau_{Lj}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{W_i}{W_j} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ij}^{-1} \tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i = H,F, \quad i \neq j \tag{8}$$

$$\sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})), \quad i = H,F \tag{9}$$

$$C_{ij} = \frac{\varepsilon - 1}{\varepsilon} (\varepsilon f_{ij})^{\frac{-1}{\varepsilon-1}} \tau_{ij}^{-1} \varphi_{ij} (\delta_{ij} L_{Cj})^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{10}$$

$$P_{ij} = \frac{\varepsilon}{\varepsilon - 1} (\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} W_j \varphi_{ij}^{-1} (\delta_{ij} L_{Cj})^{\frac{-1}{\varepsilon-1}}, \quad i,j = H,F \tag{11}$$

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{12}$$

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij} \tag{13}$$

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij} \quad i = H,F \tag{14}$$

Condition (6) defines $\tilde{\varphi}_{ji}$, the average productivity of country-i firms active in market j, which is given by the harmonic mean of productivity of those firms that operate in the respective market. Condition (7) defines δ_{ji}, the variable-profit share of a country-i firm with average productivity $\tilde{\varphi}_{ji}$ arising from sales in market j – henceforth called *domestic profit share*.[19] Equivalently, δ_{ji} is also the share of total labor used in the differentiated sector in country i that is allocated to production for market j. Condition (8) follows from dividing the zero-profit conditions defining the survival-productivity cutoffs – which imply zero profits for a country-i firm with the cutoff-productivity level φ_{ij} from selling in market j – for firms in their domestic market by the one for foreign firms that export to the domestic market. Condition (9) is the

[19] It can be shown that $f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1}$ are variable profits of a the average country-i firm active in market j.

free-entry condition combined with the zero-profit conditions. In equilibrium, expected variable profits (left-hand side) have to equal the expected overall fixed cost bill (right-hand side).

Condition (10) can be interpreted as a sectoral aggregate production function $C_{ij} = Q_{Cij}(L_{Cj})$ in terms of aggregate labor allocated to the differentiated sector, L_{Cj}, measuring the amount of production of the aggregate bundle produced in country j for consumption in market i. Condition (11) defines the equilibrium consumer price index P_{ij} of the aggregate differentiated bundle produced in country j and sold in country i.[20]

Importantly, condition (12) defines the trade-balance condition that states that the value of net imports of the homogeneous good plus the value imports of the differentiated bundle (left-hand side) must equal the value of exports of the differentiated bundle (right-hand side). Note that imports and exports of differentiated bundles are evaluated at international prices (before tariffs are applied). The model-consistent definition of the terms of trade then follows directly from this equation.[21] The international price of imports $\tau_{Ii}^{-1} P_{ij}$ defines the inverse of the terms of trade of the differentiated importable bundle (relative to the homogeneous good), while the international price of exports $\tau_{Ij}^{-1} P_{ji}$ defines the terms of trade of the differentiated exportable bundle (relative to the homogeneous good). In addition, the terms of trade of the differentiated exportable relative to the importable bundle are given by $(\tau_{Ij}^{-1} P_{ji})/(\tau_{Ii}^{-1} P_{ij})$, which is the only relevant relative price when $\alpha = 1$. Given that terms of trade are defined in terms of sectoral ideal price indices of exportables relative to importables, they will be affected not only by changes in the prices of individual varieties but also by changes in the measure of exporters and importers and their average productivity levels. We will discuss this in detail in Section 5.

Finally, (13) is the market-clearing condition for the homogeneous good[22] and condition (14) defines demand for the homogeneous good, presented here for future reference. We normalize the foreign wage, W_i, $i = F$, to unity.[23]

[20] More precisely, if $\alpha < 1$, P_{ij} should be interpreted as a relative aggregate price index in terms of the homogeneous good.

[21] This definition is also consistent with Campolmi, Fadinger and Forlati (2012) and Costinot et al. (2020), who also define terms of trade in terms of aggregate international price indices of exportables and importables.

[22] Alternatively if $\alpha = 1$ it states the domestic labor-market-clearing condition.

[23] We thus have a system of 24 equilibrium equations in 25 unknowns, namely δ_{ji}, φ_{ji}, $\widetilde{\varphi}_{ji}$, C_{ji}, P_{ij}, L_{Ci}, Z_i for $i, j = H, F$ and W_i for $i = H$. Note that if $\alpha < 1$, so that the homogeneous sector is present, $W_i = 1$ for $i = H$, since factor prices must be equalized in equilibrium; by contrast, if $\alpha = 1$, so that there is only a single sector, $L_{Ci} = L$ for $i = H, F$, since labor markets must clear. For more details and the equilibrium conditions characterizing the model with homogeneous firms see Appendix A.

3 Planner Allocation and Efficiency Wedges

In this section we discuss the problem of a social planner who maximizes total world welfare[24] given the constraints imposed by the production technology in each sector and the aggregate labor resources available to each country. The solution to this problem provides a benchmark against which one can compare any market allocation. Moreover, and more importantly, it identifies the efficiency wedges that need to be closed in order to implement the planner allocation in a market equilibrium. These wedges allow us to determine the efficiency effects of small policy changes. As explained in Section 4, they will exactly correspond to the ones in our welfare decomposition.

3.1 The Planner Problem and Efficiency Conditions

We solve the planner problem in three stages[25]. This approach allows us to derive an aggregate representation of production technologies and consumption indices and to identify the market distortions that arise at the micro and at the macro level. The formal setup of the problem and the derivations are relegated to Appendix C.

At the first stage, we determine the amount of consumption and labor allocated to each variety of the differentiated good in each location. The solution to this problem determines the consumption of individual varieties $c_{ij}(\varphi)$, the amount of labor allocated to the production of each variety $l_{ij}(\varphi)$, the optimal sectoral labor aggregator L_{Cij} and allows us to obtain a sectoral aggregate production function in terms of L_{Cij}.[26]

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1 - G(\varphi_{ij}))]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij}[N_j(1 - G(\varphi_{ij}))]^{\frac{\varepsilon}{\varepsilon-1}} \right\}, \quad i, j = H, F. \tag{15}$$

In the second stage, we solve for the optimal average productivity of firms active in the domestic and export markets $\tilde{\varphi}_{ij}$, the optimal allocation of consumption C_{ji} and labor L_{Cji} across aggregate bundles within sectors, and the measure of differentiated varieties in each

[24] World welfare is defined as the unweighted sum of individual countries' welfare. In this way we single out the symmetric point on the global Pareto frontier. This point is the relevant one from the perspective of cooperative trade negotiations because countries are symmetric in terms of preferences, technology and size.

[25] Our approach of solving the planner problem in several stages is similar to the unilateral policy-maker problem of Costinot et al. (2020). However, while they consider the problem of a policy maker who chooses taxes to maximize individual-country welfare taking as given the taxes of the other country, we consider a world planner who chooses an allocation that maximizes welfare of both countries.

[26] Note that in the case of homogeneous firms equation (15) holds with $\tilde{\varphi}_{ij} = 1$, $(1 - G(\varphi_{ij})) = 1$ and $f_{ij} = 0$.

sector N_i given the allocation of labor across sectors. From this problem, we obtain a sectoral production function for Q_{Cji} in terms of aggregate labor L_{Ci}:

$$Q_{Cji}(L_{Ci}) = \frac{\varepsilon - 1}{\varepsilon} (\varepsilon f_{ji})^{\frac{-1}{\varepsilon-1}} \tau_{ji}^{-1} \varphi_{ji} (\delta_{ji} L_{Ci})^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \tag{16}$$

Finally, in the third stage we find the optimal allocation of consumption C_i, Z_i and labor L_{Ci}, L_{Zi} across aggregate sectors.

We show in the Appendix that the optimality conditions of the first stage are satisfied in any market allocation and are independent of policy instruments. This implies that the relative production levels of individual varieties are optimal in any market allocation. By contrast, the optimality condition of the second and third-stage planner problems are not automatically satisfied in the market. The following Lemma states the conditions that are necessary and sufficient for the market allocation to coincide with the planner allocation. Here $u_i = \log C_i$ corresponds to the sub-utility function of the differentiated sector.

Lemma 1 ***Efficiency conditions for the market allocation***

The market allocation coincides with the planner allocation if and only if in the market equilibrium:[27]

(a)

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i, j = H, F \quad i \neq j \tag{17}$$

(b) and (for the multi-sector model only)

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, \quad j = F \tag{18}$$

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F \tag{19}$$

Proof See Appendix C.6. ■

Condition 17 requires that the marginal value product of labor of the domestic non-tradable

[27] $\frac{\partial Q_{Cji}}{\partial L_{Cji}}$ in (17) corresponds to the partial derivative of the aggregate production function derived in the second stage, as given by (15), while in (19) $\frac{\partial Q_{Cji}}{\partial L_{Ci}}$ corresponds to the one of the production function derived in the third stage, as given by (16).

bundle (measured in terms of domestic marginal utility) has to equal the marginal value product of labor of the domestic exportable bundle (measured in terms of foreign marginal utility).[28] Condition (18) states that the social marginal value product of each country's aggregate labor,[29] (evaluated with the marginal utility of the consuming country), has to be equalized across sectors. In the Appendix we also prove that – due to the countries' symmetry and equal welfare weights of both countries – the planner implements a symmetric allocation, which we show to be unique (see Lemma 8 in Appendix C.5)

3.2 Efficiency Wedges in the Market Equilibrium

We now investigate the distortions in the market equilibrium induced by (i) monopolistic competition, (ii) production taxes and (iii) trade policy at each side of the border. For this purpose we re-state the efficiency conditions of Lemma 1 – which are defined in terms of marginal rates of substitution and transformation – in terms of efficiency wedges between market prices and producers' aggregate marginal costs, corresponding to differences between social marginal costs and social marginal benefits. Our approach follows the public finance literature (Harberger, 1971) and clarifies how specific policy instruments affect these wedges. This strategy will allow us to give a clearcut interpretation of the terms in the welfare decomposition described in the next section.

Lemma 2 ***Efficiency wedges***

Conditions (17)*,* (18) *and* (19) *hold in the market equilibrium if and only if:*

(a) countries have the same level of income:

$$I_i = I_j \quad j \neq i, \tag{20}$$

where $\quad I_k = W_k L + T_k, \quad k = i, j.$

(b) the consumer price indices of the differentiated importable bundles correspond to the monopolistic markup over the aggregate marginal costs of the differentiated exportable bun-

[28] Equivalently, this condition states that the marginal rate of substitution (in terms of home versus foreign utility) between the domestic nontradable bundle and the domestic exportable bundle has to equal the marginal rate of transformation of these bundles.

[29] By construction, aggregate labor already incorporates the optimal split of labor in the differentiated sector between the domestically produced and consumed and the exportable bundles.

dles:

$$P_{ij} - \frac{\varepsilon}{\varepsilon - 1}\tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} = 0, \quad i = H, F, \quad j \neq i, \tag{21}$$

(c) and (for the multi-sector model only) the marginal value product of labor in the differentiated sector evaluated at producer prices equals the price of labor:

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} - W_i = 0, \qquad i = H, F, \tag{22}$$

Proof See Appendix C.7. ■

Condition (20) guarantees symmetry of the allocation. It is violated whenever countries pursue asymmetric policies. In general, conditions (21) and (22) are also not satisfied in a market allocation. The conditions corresponding to conditions (21) and (22) in the market equilibrium are:

$$P_{ij} = \tau_{Tij} \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}}, \;\; i, j = H, F \;\; j \neq i, \quad \sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} W_i, \;\; i = H, F \tag{23}$$

By comparing conditions (21) and (22) with their market analogues in (23), we can identify the causes for distortions in the market allocation. The presence of trade policies in the exportable and importable markets creates a wedge between the market value of the aggregate tradable bundles in their destination market and the producer price index, corresponding to the marginal cost of producing them multiplied by the markup. This induces a violation of (21), which happens whenever $\tau_{Tij} \neq 1$. This distortion is the only source of inefficiency in the one-sector model. By contrast, in the multi-sector model both monopolistic markups and production taxes create an additional distortion by inducing a wedge between the marginal value product of labor measured at producer prices and its cost, generating a violation of (22). Together, conditions (21) and (22) imply that consumer prices equal aggregate marginal costs in all markets, i.e., $P_{ij} = W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}}$ for $i, j = H, F$ and thus social marginal benefits are equal to social marginal costs for all aggregate bundles.[30]

The wedges in (21) and (22) represent distortions from the global perspective, capturing the

[30]Notice that in the market equilibrium $P_{ij} = \frac{\partial U_i}{\partial C_{ij}} \left(\frac{\partial U_i}{\partial I_i}\right)^{-1}$ for $i, j = H, F$. Then condition (21) and (22) equate the marginal benefits of consuming an additional unit of the differentiated bundles to the marginal costs of producing them.

joint effects of trade and domestic policies of both countries. However, our analysis requires to identify to what extent these wedges are affected by: (a) decisions of individual policy makers; (b) the impact of specific policy instruments; (c) interaction effects between policy instruments. We thus decompose them accordingly. As we show in Appendix C.8 the efficiency wedges in (21) and (22) can be decomposed into domestic and foreign components as follows:

$$P_{ij} - \frac{\varepsilon}{\varepsilon - 1}\tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} = \underbrace{(\tau_{Ii} - 1)}_{\substack{\textit{domestic} \\ \textit{consumption-efficiency} \\ \textit{wedge, home tariff}}} \tau_{Ii}^{-1} P_{ij} + \underbrace{(\tau_{Xj} - 1)\,\tau_{Tij}^{-1}}_{\substack{\textit{domestic} \\ \textit{consumption-effiency} \\ \textit{wedge, foreign export tax}}} P_{ij} = \underbrace{\left(1 - \tau_{Tij}^{-1}\right)}_{\substack{\textit{domestic} \\ \textit{consumption-efficiency} \\ \textit{wedge}}} P_{ij}, \qquad i = H, F \quad j \neq i \tag{24}$$

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} - W_i = \underbrace{\frac{\varepsilon}{\varepsilon - 1}\tau_{Li} - 1,}_{\substack{\textit{domestic} \\ \textit{production-efficiency} \\ \textit{wedge}}} \qquad i = H, F \tag{25}$$

Condition (24) decomposes the wedge between the consumer and the producer price indices of the differentiated importable bundle into two components: (i) the consumption-efficiency wedge induced by a domestic tariff, consisting of the difference between the domestic consumer price and the international price of the imported bundle; (ii) the consumption-efficiency wedge induced by a foreign export tax, consisting of the difference between the international price and the foreign producer price of the importable bundle. A domestic tariff, or a foreign export tax both reduce domestic imports of the foreign differentiated bundle inefficiently.

Condition (25) shows how the wedge between the marginal value product of aggregate labor in the domestic differentiated sector and the wage depends on the monopolistic markup and domestic production taxes. The domestic production-efficiency wedge is open whenever the monopolistic markup is not completely offset by a production subsidy.

Finally, the efficiency wedge induced by an export tax can be decomposed as:

$$(\tau_{Xi} - 1)\,\tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = \underbrace{(1 - \tau_{Xi})}_{\substack{\textit{domestic} \\ \textit{consumption-efficiency} \\ \textit{wedge, home export tax}}} P_{ii} \frac{\partial Q_{Cii}}{\partial L_{Ci}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon - 1}\tau_{Li}\tau_{Xi} - 1\right) - \left(\frac{\varepsilon}{\varepsilon - 1}\tau_{Li} - 1\right)}_{\substack{\textit{domestic} \\ \textit{production-efficiency} \quad \textit{wedge, home export tax}}}, \quad i = H, F \quad j \neq i \tag{26}$$

According to the left-hand side, an export tax induces a wedge between the marginal value product of labor in the differentiated exportable market evaluated at the international price and at the domestic producer price. The right-hand side of condition (26) decomposes this distortion into the sum of two wedges: First, the consumption-efficiency wedge induced by an export tax in the market of non-tradable differentiated goods. It implies a consumption distortion between the differentiated non-tradable bundle and the importable bundle. Second, a production-efficiency wedge induced by the domestic export tax τ_{Xi} on aggregate production in the differentiated sector.

Using condition (26) and summing the wedges in (24) and (25), we can then decompose the overall efficiency effects of a small policy change.[31]

Lemma 3 ***Global and individual-country efficiency effects of a small policy change***

(a) *The total efficiency effects of a small policy change can be decomposed as:*

$$\sum_{\substack{i=H,F\\ j\neq i}} \left[\left(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} \right) dC_{ij} + \left(\sum_{k=H,F} \tau_{Tki}^{-1} P_{ki} \frac{\partial Q_{Cki}}{\partial L_{Ci}} - 1 \right) dL_{Ci} \right]$$

$$= \sum_{\substack{i=H,F\\ j\neq i}} \underbrace{[\underbrace{(\tau_{Ii}-1)\tau_{Ii}^{-1} P_{ij} dC_{ij} + (1-\tau_{Xi}) P_{ii} dC_{ii}}_{\text{individual consumption-efficiency effect}} + \underbrace{(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1) dL_{Ci}}_{\text{individual production-efficiency effect}}]}_{dE_i} \tag{27}$$

$$= \underbrace{\sum_{\substack{i=H,F\\ j\neq i}} (\tau_{Tij}-1)\,\tau_{Tij}^{-1} P_{ij} dC_{ij}}_{\text{global consumption-efficiency effect}} + \underbrace{\sum_{i=H,F} \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1 \right) dL_{Ci}}_{\text{global production-efficiency effect}} \equiv \sum_{i=H,F} dE_i \tag{28}$$

(b) *The total efficiency effects of a small policy change are zero and the market allocation is efficient: (i) if each wedge on the right-hand side of condition* (27) *is zero; (ii) if and only if $I_i = I_j$ for $j \neq i$ and each wedge on the right-hand side of condition* (28) *is zero.*

Proof See Appendix C.9.2. ■

Lemma 3 provides a measure of the efficiency effects caused by a small policy change from the individual-country and from the global perspective. Specifically, condition (27) breaks down

[31] Note that the efficiency effects of small policy changes identified in Lemma 3 correspond exactly to Harberger's definition of general-equilibrium changes in consumer and producer surplus (measured in terms of normalized utility) induced by small changes in taxes (Harberger, 1971).

the efficiency effects of policy according to the efficiency gains and distortions a country imposes on itself by using its own policy instruments. By contrast, condition (28) accounts for the joint efficiency effects of both countries' policies and shows that in the aggregate some of the wedges of equations (24)-(26) cancel out. From the perspective of an individual country, both the use of a tariff and an export tax cause distortions whenever they are used. In the presence of a tariff ($\tau_{Ii} > 1$), the first consumption-efficiency wedge in (27) is positive. This implies that, in general, the consumption of importables is inefficiently low. Thus, an increase in C_{ij} ($dC_{ij} > 0$) improves efficiency. By contrast, in the presence of an export tax, the second consumption-effiency wedge in (27) is negative and then, everything else equal, the consumption of the domestically non-tradable bundle is inefficiently high. As a result, a reduction in C_{ii} ($dC_{ii} < 0$) increases consumption efficiency. Finally, even in the absence of trade taxes, in the multi-sector model the production-efficiency wedge is positive due to the monopolistic markup, which implies that too little labor is employed in the aggregate differentiated sector. Hence, an increase in L_{Ci} ($dL_{Ci} > 0$) improves production efficiency. Closing the production-efficiency wedge requires a production subsidy equal to the inverse of the markup. At the same time, when $\tau_{Xi} > 1$ the production-efficiency wedge in (27) is also positive (unless τ_{Xi} is more than compensated by a production subsidy) because an export tax shifts labor out of the differentiated sector. Then, an increase in L_{Ci} ($dL_{Ci} > 0$) again improves efficiency. By contrast, in the one-sector model, $dL_{Ci} = 0$, so production-efficiency effects are absent, as the monopolistic markup does not induce any distortions in this case.

When instead considering the perspective of a global policy maker, who can control policy instruments of both countries, the relevant efficiency effects of policies are given by (28). Indeed, the global policy maker – who can set all policy instruments at once – realizes that what matters in terms of consumption efficiency is the difference between the price in the producer country and the one paid by consumers in the other country, i.e., what matters is τ_{Tij}. Note that setting the wedges in (28) – which relies on conditions (21) and (22) – equal to zero traces out the entire global Pareto-efficiency frontier. Adding the condition that $I_i = I_j$ picks the symmetric point on this frontier, which corresponds to the allocation chosen by the planner.

4 Policy-Maker Problem and Welfare Decomposition

We now study two policy problems: the one faced by a benevolent world policy maker and the one faced by individual-country policy makers. In doing so, we derive a welfare decomposition which identifies policy makers' incentives and separates them into efficiency effects and terms-of-trade effects.

4.1 Policy and Welfare from the Global Perspective

We first solve the problem of the world policy maker who maximizes the sum of individual-country welfare and has all three policy instruments (production, import and export taxes in the differentiated sector) at her disposal.

The world policy maker sets domestic and foreign policy instruments τ_{Li}, τ_{Ii} and τ_{Xi} in order to solve the following problem:[32]

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}, \tau_{Li}, \tau_{Ii}, \tau_{Xi}\}_{i,j=H,F}}} \sum_{i=H,F} U_i \tag{29}$$

$$\textit{subject to conditions } (6)\text{-}(13).$$

We solve the world policy maker problem using the total-differential approach. This involves taking total differentials of (29) and the equilibrium equations. We then substitute the total differentials of the trade balance and some other equilibrium equations into the objective function to obtain the following representation of the world welfare effects induced by small changes in one or several policy instruments:[33]

Proposition 1 ***Decomposition of world welfare effects***[34]

The total differential of world welfare in (29) *in response to small domestic or foreign policy changes can be decomposed as follows:*

[32] U_i is defined in (3), (4) and (14) with the additional restrictions that $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$. In the case of homogeneous firms, conditions (6)-(9) need to be dropped and (10)-(11) are replaced by (A-21) and (A-22).

[33] See Appendix B for an explanation how to solve constrained optimization problems using total differentials.

[34] A predecessor of this welfare decomposition can be found in Meade (1955) and in chapter 2 of Helpman and Krugman (1989).

$$\sum_{i=H,F} dU_i = \underbrace{\sum_{i=H,F} \frac{dE_i}{I_i}}_{\text{global efficiency effects}} + \underbrace{\sum_{\substack{i=H,F \\ j\neq i}} \frac{C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}{I_i}}_{\text{terms-of-trade effects}} \tag{30}$$

which, if $I_i = I_j$, implies that

$$\sum_{i=H,F} dV_i = \sum_{i=H,F} dE_i = \underbrace{\sum_{\substack{i=H,F \\ j\neq i}} (\tau_{Tij} - 1)\, \tau_{Tij}^{-1} P_{ij} dC_{ij}}_{\text{global consumption-efficiency effect}} + \underbrace{\sum_{i=H,F} \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} - 1 \right) dL_{Ci}}_{\text{global production-efficiency effect}}, \tag{31}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$, dE_i is defined in Lemma 3, and $I_i = W_i L + T_i$ is household income.

Proof See Appendix D.1. ■

Changes in world welfare due to changes in policy instruments can be written as the sum of three terms: (i) a *consumption-efficiency effect*; (ii) a *production-efficiency effect* and (iii) *terms-of-trade effects*. We have already discussed the first two effects in detail. The only additional incentive – not driven by efficiency considerations – is the *terms-of-trade effect* of policies. An increase in the price of exportables raises domestic welfare and decreases welfare of the other country, while an increase in the price of importables has opposite effects. The domestic and foreign terms-of-trade effects are beggar-thy-neighbor effects, i.e., they exactly compensate each other and make one country better off at the expense of the other. Consequently, the differential of world welfare consists exclusively of the consumption-efficiency and the production-efficiency terms. Note that the welfare decomposition is valid (i) both for changes in world welfare induced by changes in all policy instruments or just a subset of them and (ii) for the cases of heterogeneous and homogeneous firms. The next Proposition characterizes the optimal policies from the global perspective.

Lemma 4 ***Optimal world policies and Pareto efficiency***

(a) When production, import and export taxes are available in the differentiated sector, solving the world-policy-maker problem in (29) *by using the total-differential approach is equivalent to setting $I_i = I_j$ and the efficiency wedges in* (31) *individually equal to zero.*

(b) As a result, the world policy maker implements the planner allocation and the global policy is optimal if and only if:

(i) when $\alpha = 1$ *(one-sector model):* $\tau_{Tij} = \tau_{Ii}\tau_{Xj} = 1$, $\tau_{Ii} = \tau_{Ij}$ *(or* $\tau_{Xi} = \tau_{Xj}$*) for* $i = H, F$ *and* $j \neq i$.

(ii) when $\alpha < 1$ *(multi-sector model):* $\tau_{Tij} = \tau_{Ii}\tau_{Xj} = 1$, $\tau_{Ii} = \tau_{Ij}$ *(or* $\tau_{Xi} = \tau_{Xj}$*) and* $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ *for* $i = H, F$ *and* $j \neq i$.

Proof See Appendix D.2. ■

When all policy instruments are available, global optimality requires that, for any small changes in C_{ij} and L_{C_i}, the term $\sum_{i=H,F} dE_i$, as given by (31) is zero. This requires closing all efficiency wedges. The global policy maker realizes that the distortion of a domestic import tariff can be completely offset with a foreign export subsidy, so that only $\tau_{Tij} = \tau_{Ii}\tau_{Xj}$ needs to be set to unity in order to avoid opening a consumption-efficiency wedge.[35] This guarantees that consumer price indices of importable differentiated bundles are equal to the corresponding producer price indices (condition (21)). Thus, zero trade taxes are sufficient but not necessary to achieve consumption efficiency. Finally, in the presence of multiple sectors the global policy maker implements global production efficiency by setting the production subsidy in both countries equal to the inverse of the markup. This guarantees that the marginal value product of labor in the differentiated sector evaluated at producer prices equals the price of labor (condition (22)).

4.2 Policy and Welfare from the Individual-country Perspective

We now turn to the welfare incentives of policy makers that are concerned with maximizing the welfare of individual countries and have either all policy instruments (production and trade taxes in the differentiated sector) or just a subset of them available.

The individual-country policy maker sets domestic policy instruments $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}\}_{i,j=H,F}, \mathcal{T}_i}} \quad U_i \tag{32}$$

subject to conditions (6)-(13),

[35] Our formulation of the trade balance implies that tariffs are applied to the international value of exports (including export taxes). This implies that also the *level* of trade taxes needs to be identical across countries to avoid distortions. With the alternative assumption that tariffs are applied to the producer value of exports just the product of the tariff and the export tax needs to equal unity, e.g. Costinot et al. (2020).

where $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ for $i = H, F$ and taking as given $\mathcal{T}_j \subseteq \{\tau_{Lj}, \tau_{Ij}, \tau_{Xj}\}$, with $j \neq i$.[36]

Again, as a first step for solving the individual-country policy maker problem given foreign policy instruments, we take total differentials of the objective function and the constraints and substitute them into the differential of the objective in order to obtain the welfare decomposition for individual countries. We will then use this decomposition to analyze unilateral deviations from the laissez-faire equilibrium in Section 5 and to interpret the outcomes of the policy games implied by different institutional arrangements in Section 6.

Proposition 2 ***Decomposition of individual-country welfare effects***

The total differential of individual-country welfare in (32) *in response to small policy changes can be decomposed as follows:*

$$dV_i = dE_i + \underbrace{C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}_{\text{domestic terms-of-trade effect}}, \quad j \neq i \tag{33}$$

$$= \underbrace{(1 - \tau_{Xi}) P_{ii} dC_{ii} + (\tau_{Ii} - 1)\tau_{Ii}^{-1} P_{ij} dC_{ij}}_{\substack{\text{domestic} \\ \text{consumption-efficiency effect}}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon - 1}\tau_{Li}\tau_{Xi} - 1\right) dL_{Ci}}_{\substack{\text{domestic} \\ \text{production-efficiency effect}}} + \underbrace{C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}_{\substack{\text{domestic} \\ \text{terms-of-trade effect}}},$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$, dE_i *is defined in Lemma 3 and* $I_i = W_i L + T_i$ *is household income.*

Proof See Appendix D.3. ■

This welfare decomposition is again valid both with homogeneous and heterogeneous firms and independently of the number of policy instruments that the individual-country policy makers have at their disposal. Like the world policy maker, they care about domestic consumption efficiency and production efficiency. Moreover, unlike the world policy maker, they also take into account the terms-of-trade effects of their policy choice, as these are not zero.

In the one-sector model, the laissez-faire allocation corresponds to the first-best one and production-efficiency effects of policies are always completely absent ($dL_{Ci} = 0$). Thus, individual-country policy makers may use trade policies to deviate from any given allocation exclusively to

[36] U_i is defined in (3), (4) and (14) with the additional restrictions that $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$. In the case of homogeneous firms, conditions (6)-(9) need to be dropped and (10)-(11) are replaced by (A-21) and (A-22).

change their terms of trade, which comes at the trade-off of changing consumption efficiency.[37] By contrast, in the presence of multiple sectors, individual-country policy makers may use trade or domestic policies to deviate from any given allocation either to change production efficiency or to change their terms of trade, potentially at the cost of changing consumption efficiency. Note that terms-of-trade manipulation is the only *beggar-thy-neighbor* policy in the model, i.e. an increase in domestic welfare due to the terms-of-trade improvement is always compensated by an equal fall in the foreign one (a zero-sum game). This follows from the global policy makers' welfare decomposition, where the foreign terms-of-trade effect of a policy equals the opposite of its domestic counterpart.[38]

The following Corollary summarizes these observations:

Corollary 1 ***Individual-country incentives***

(a) In the one-sector model, unilateral policy deviations of individual-country policy makers are driven by consumption-efficiency and terms-of-trade effects.

(b) In the multi-sector model, unilateral policy deviations of individual-country policy makers are driven by consumption-efficiency, production-efficiency and terms-of-trade effects.

(c) Terms-of-trade effects are the only beggar-thy-neighbor effects.

Thus, the welfare decomposition in (33) provides a common framework for analyzing the incentives for trade and domestic policies in a general CES monopolistic competition setup. It remains valid independently of the presence of firm heterogeneity and of the set of policy instruments available to the individual-country policy maker. It clearly identifies the terms-of-trade effect as the only beggar-thy-neighbor incentive.

At this point it seems warranted to compare our approach to the classical one of Bagwell and Staiger (1999) who identify the terms-of-trade motive as the reason to sign a trade agreement

[37] Costinot et al. (2020) also emphasize that in the one-sector heterogeneous-firm model terms-of-trade effects are the only externality driving the incentives of individual-country policy makers.

[38] By contrast, a policy that aims at improving domestic production efficiency is crucially different from a beggar-thy-neighbor policy: while such a policy may also impose an externality on the other country as a side effect, it is a not a zero-sum game. In fact, depending on the policy instrument used, it can be a positive or a negative-sum game. For example, a positive production subsidy that is set unilaterally by both countries improves global welfare compared to a situation without policy intervention because it improves global production efficiency, as we will show in Lemma 7 below.

in a wide class of models. [39]. These authors introduce the concept of *politically optimal policies*, as those policies that unilateral policy makers would choose if they did not value the terms-of-trade effects of their policy choices. Their approach involves writing individual-country welfare as a function of local and international prices. They prove that – in situations when sufficiently many instruments are available – if policy makers did not value the effect of their policies on international prices (the terms-of-trade effect), they would implement the allocation chosen by the world policy maker.[40] As we show now, our welfare decomposition is very similar in spirit. In the next section we demonstrate that it is more general than their concept, as it also applies to situations where theirs does not allow identifying policy incentives.

Consider first the one-sector model. Suppose individual-country policy makers did not value the terms-of-trade effect of their policies. If we disregard the terms-of-trade effect in condition (33), the welfare effects of individual-country policies are given exclusively by consumption-efficiency effects, corresponding exactly to the individual-country efficiency effects of policies in (27). These would be set to zero by abstaining from the use of trade taxes. If both countries pursued these policies they would implement the laissez faire allocation, which corresponds to the planner allocation. This extends the result of Bagwell and Staiger (2016) to models with heterogeneous firms.

Consider now the multi-sector model. If we ignore the terms-of-trade effect in condition (33), the welfare effects of individual-country policies are given by consumption-efficiency and production-efficiency effects. These equal the efficiency effects of policies in (27). If both production and trade taxes are available, individual-country policy makers would thus implement domestic production efficiency by using a production subsidy equal to the inverse of the monopolistic markup. Moreover, they would set consumption-efficiency wedges individually equal to zero by implementing free trade. If both countries pursued these policies they would achieve the planner allocation. Given that, in the absence of an agreement that limits the strategic use of policies, both production taxes and trade taxes can be set unilaterally, this proves that the only reason to sign a trade agreement in models with monopolistic competition (potentially

[39] Their original framework features perfect competition and trade policies. This framework has been extended to perfect competition and domestic policies by Bagwell and Staiger (2001), and to trade policy with monopolistic competition and homogeneous firms (but no domestic policies) in Bagwell and Staiger (2016)

[40] They show that when disregarding the terms associated with manipulating international prices, the first-order conditions of individual-country policy makers are identical to those of the world policy maker.

heterogeneous firms) and domestic policies is the terms-of-trade effect. This extends the result of Bagwell and Staiger (2001) to the case of monopolistic competition. Our result also implies that the delocation motive is not a reason to sign a trade agreement when a sufficient set of policy instruments is available. The following Corrolary summarizes this insight.

Corollary 2 ***The role of trade agreements***

In the one-sector and in the multi-sector model with monopolistic competition (and potentially heterogeneous firms), and in the presence of a sufficient set of policy instruments, the only reason for signing a trade agreement is the terms-of-trade effect of uncoordinated policies.

In the next section we show that the welfare decomposition also allows clearly identifying policy incentives in situations where the set of available policy instruments is not sufficient to implement the first-best allocation.

5 From Micro to Macro: Terms-of-Trade and Efficiency Effects of Policies

Before studying strategic policies in Section 6, we first analyze how unilateral policy choices affect the terms of trade and the efficiency wedges and thereby the welfare of individual countries. We are particularly interested in explaining the different micro channels through which policy instruments impact on them. As mentioned previously, the terms of trade can be influenced both through changes in the international prices of individual exportable and importable varieties and through changes in the number and composition of exporters and importers.

We discuss the impact of a small unilateral tariff in the multi-sector model starting from the laissez-faire equilibrium in detail and then show that any policy deviation in a single instrument creates a trade-off between production-efficiency and terms-of-trade effects.[41] In the multi-sector model, the terms-of-trade effect of a small tariff starting from the laissez-faire equilibrium[42] is *negative* and given by:[43]

[41]The case of the one-sector model is discussed in Appendix E.

[42]For the case of homogeneous firms, terms-of-trade effects can be signed globally. As we show in Campolmi et al. (2012), an improvement in the domestic terms of trade always requires a reduction in the number of domestic firms relative to the one of foreign firms. This can be achieved with an import subsidy.

[43]We define unilateral deviations as $dX_i/X_i = \frac{\partial X_i}{\partial \tau_{mi}} \frac{1}{X_i} d\tau_{mi}$ and consider $d\tau_{Ii} > 0$, $d\tau_{Li} = d\tau_{Xi} = 0$. In Appendix E, Lemma 12, we sign the contribution of each component to the terms-of-trade effect.

$$P_{ij}C_{ij}\left[(\varepsilon-1)^{-1}\underbrace{\left(\frac{dL_{Cj}}{L_{Cj}}-\frac{dL_{Ci}}{L_{Ci}}\right)}_{(ii)<0}+(\varepsilon-1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}}-\frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)<0\Leftrightarrow\delta_{ii}>1/2}+\underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}}-\frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)>0\Leftrightarrow\delta_{ii}>1/2}\right]<0 \qquad (34)$$

A small tariff increases home's demand for domestically produced varieties and thus, ceteris paribus, the profits of home firms and the demand for domestic labor. Since wages are pinned down by the linear outside sector and workers can freely move across sectors, labor supply is perfectly elastic. Therefore, home labor in the differentiated sector surges in response to the increase in labor demand, raising the number of domestic firms and reducing their equilibrium profits. At the same time, foreign firms experience a drop in demand and profits, leading to a reduction in foreign labor employed in the differentiated sector. These effects impact negatively on home's terms of trade via the extensive margin ((ii)<0).[44] Moreover, in the presence of heterogeneous firms there are two additional effects, the sign of which depends on whether firms make the larger share of profits in their domestic ($\delta_{ii}>1/2$) or in their export market ($\delta_{ii}<1/2$).[45]

In the first case, the tariff increases the profit share of home firms and decreases the profit share of foreign firm made in their respective export markets, which worsens home's terms of trade ((iii)<0) (more home exporters and less foreign exporters). In addition, the tariff leads to less stringent selection into exporting at home and more selection in the other country, which positively impacts on home's terms of trade ((iv)>0). When $\delta_{ii}<1/2$ the signs of the last two effects switch, but the overall terms-of-trade effect of a small tariff deviation from the laissez-faire equilibrium remains negative.

Because the tariff increases the amount of labor allocated to the differentiated sector, it induces a positive production-efficiency effect when starting from the laissez-faire allocation and thus

[44] An alternative decomposition splits the price index of exportables and importables into an extensive margin $[N_i(1-G(\varphi_{ji}))]^{\frac{1}{1-\varepsilon}}=\left(\frac{\delta_{ji}L_{Ci}}{\varepsilon f_{ji}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\varphi_{ji}}{\tilde{\varphi}_{ji}}\right)$ and an intensive margin $\tau_{Ij}^{-1}p_{ji}(\tilde{\varphi}_{ji})=\frac{\varepsilon}{\varepsilon-1}(\tau_{ji}\tau_{Xi}\tau_{Li})\left(\frac{W_i}{\tilde{\varphi}_{ji}}\right)$.

[45] As made clear by Lemma 11 in Appendix E.1 if we impose the standard assumption $f_{ji}>f_{ii}\tau_{ij}^{1-\varepsilon}$ then in the laissez-faire allocation the export cutoff φ_{ji} for $j\neq i$ must be larger than the domestic survival cutoff φ_{ii} and also δ_{ii} is always larger than 1/2. In general, in the presence of trade taxes, at a symmetric allocation exporters might still be more productive than firms serving only the domestic market even when $f_{ji}\leq f_{ii}\tau_{ij}^{1-\varepsilon}$. In this case $\varphi_{ji}>\varphi_{ii}$ as long as $f_{ji}>f_{ii}\tau_{ij}^{1-\varepsilon}\tau_{Tij}^{-\varepsilon}$. By contrast, when this condition is not satisfied $\delta_{ii}<1/2$ is possible. In general, at a symmetric allocation exporters are not necessarily more productive than firms serving only the domestic market even when $f_{ji}>f_{ii}\tau_{ij}^{1-\varepsilon}$. When τ_{Tij} is close to zero (high import or export subsidies) the export cutoff φ_{ji} for $j\neq i$ might smaller than the domestic survival cutoff φ_{ii} .

creates a trade-off between increasing production efficiency and worsening the terms of trade. Which of the two effects dominates in welfare terms depends again on δ_{ii}: when δ_{ii} is larger than one half, so that the domestic market is more important in terms of profits, production-efficiency effects are dominant. Intuitively, when firms sell mostly to their domestic market, welfare gains from improving the terms of trade are relatively small and policy makers care mostly about domestic production efficiency.

Analogous results hold for export and production taxes: they improve domestic terms of trade by shifting labor away from the differentiated sector, which simultaneously worsens domestic production efficiency. Again, the total welfare effect depends on the magnitude of δ_{ii}. The following Lemma summarizes these findings.

Lemma 5 ***Unilateral deviations from laissez-faire in multi-sector model*** *Consider a marginal unilateral increase in each policy instrument at a time starting from the laissez-faire equilibrium, i.e., a situation with* $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$*. Then:*

(a) the production-efficiency effect is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li}*.*

(b) the consumption-efficiency effect is zero for all policy instruments.

(c) the terms-of-trade effect is negative for τ_{Ii} *and positive for* τ_{Xi} *and* τ_{Li}*.*

(d) the welfare effect is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li} *if and only if* $1/2 < \delta_{ii} < 1$ *or when firms are homogeneous.*

Proof See Appendix E.5. ■

To summarize, when considering unilateral deviations in the multi-sector model, the qualitative impact of policy instruments on welfare depends on firm heterogeneity. In particular, whether individual-country policy makers benefit from a unilateral tariff or an import subsidy depends on the profit share from domestic relative to export sales (analogous statements hold for the other policy instruments). This results makes clear that policy makers may exploit the delocation effect to increase production efficiency when the set of policy instruments is limited and the positive production-efficiency effect of the policy dominates its negative terms-of-trade effect.[46]

[46] This is the case in Ossa (2011), who considers the homogeneous-firm model and an import tariff in the differentiated sector as the only available policy instrument.

Otherwise, policy makers want to delocate firms to the foreign economy (anti-delocation).

We now explain why our welfare decomposition also allows identifying policy-makers' incentives when the set of policy instruments is not sufficient to implement the first-best allocation and why in this situation the concept of politically optimal policies is not very useful. To make it concrete, assume that the sole instrument available to policy makers in the multi-sector model is an import tax and that policy makers do not value the terms-of-trade effect of their policies. As Bagwell and Staiger (2016) show for the homogeneous-firm model, in this case, even when disregarding the terms-of-trade effect of an import tax on welfare, the first-order conditions of the unilateral policy maker are different from those of the world policy maker and it remains unclear why this is the case. Our welfare decomposition clarifies this: when policy makers disregard terms-of-trade effects of policy on welfare, they care only about efficiency effects, as given by (27). If only an import tax is available, policy makers cannot set consumption and production-efficiency wedges simultaneously equal to zero. To improve production efficiency, all policy makers want to increase L_{Ci} in the domestic differentiated sector. However, the way to achieve this is different for the individual-country than for the world policy maker: the world policy maker understands that an import subsidy in the differentiated sector in both countries increases demand for the differentiated bundle and thus implies an increase in L_{Ci} in both countries. This comes at the cost of creating consumption-efficiency wedges. Differently, the individual-country policy maker takes the foreign import tax as given and realizes that a tariff induces delocation of firms from the foreign to the domestic economy and thereby increases domestic production efficiency at the cost of creating a domestic consumption wedge. Thus, with an import tax the individual-country policy maker induces a negative externality on the other country in her attempt to improve domestic production efficiency: delocation of firms from foreign to home reduces foreign and global production efficiency. However, delocation is a means rather than the motive (which is production efficiency) for the policy.

6 The Design of Trade Agreements in the Presence of Domestic Policies

After having analyzed individual-country incentives to set taxes in the absence of retaliation, we now move to strategic policies in order to study how trade agreements should be designed. We know from the planner problem that in the one-sector model the laissez-faire allocation is efficient so that in this case a trade agreement that forbids the use of trade and production taxes is optimal. We thus focus here on the more interesting case of the two-sector model in which production in the laissez-faire equilibrium is inefficient, so that there exists a motive for domestic policy intervention even in the absence of international trade.

We first consider strategic trade and domestic policies in the absence of any type of trade agreement in order to have a benchmark for the distortions arising without international cooperation. Next, we show that cooperative negotiations on trade and domestic policies under a deep trade agreement are sufficient to achieve the world planner allocation. In the remainder of the section, we consider various trade agreements with different levels of integration. First, we consider a shallow trade agreement modeled along the lines of GATT-WTO membership: in the first stage, countries negotiate reciprocal reductions in trade taxes, taking as given domestic policies. In the second stage, they can deviate unilaterally from the negotiation outcome subject to market access constraints and tariff bindings imposed by WTO rules. Second, we consider a more stringent scenario modelled along the lines of a shallow free trade agreement according to GATT Article XXIV: we consider strategic domestic policies in a situation where trade taxes are set to zero. Finally, we compare welfare under the previous scenario with a laissez-faire agreement, where countries commit to abstain from using both trade and domestic policies.

6.1 Trade and Domestic Policies in the Absence of a Trade Agreement

We first consider a situation without any type of agreement, so that individual-country policy makers can set both trade and domestic policies non-cooperatively. We thus allow domestic policies τ_{Li} and trade policies τ_{Ii}, τ_{Xi}, for $i = H, F$ to be set strategically and simultaneously by the policy makers of both countries. Individual-country policy makers solve the problem described in (32). The welfare decomposition in (33) holds independently of the number of instruments at the disposal of the individual-country policy maker and corresponds to the policy maker's objective. After substituting additional equilibrium conditions, this objective can be rewritten in terms of three wedges that are all individually equal to zero at the optimum. Proposition 3 states this more formally and characterizes the symmetric Nash equilibrium of this policy game.

Proposition 3 ***Strategic trade and domestic policies***

When production, import and export taxes are available in the differentiated sector,

(a) it is possible to rewrite (33) *as follows:*

$$dV_i = [\Omega_{Cii} dC_{ii} + \Omega_{Cij} dC_{ij} + \Omega_{LCi} dL_{Ci}] \tag{35}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and the wedges* Ω_{Cii}, Ω_{Cij} *and* Ω_{LCi} *are defined in Appendix F.1.*

(b) Solving the individual-country policy maker problem stated in (32) *by using the total-differential approach requires setting* $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$.

(c) As a result, any symmetric Nash equilibrium in the two-sector model with heterogeneous firms when both countries can simultaneously set all policy instruments entails the first-best level of production subsidies, and inefficient import subsidies and export taxes in the differentiated sector. Formally,

$$\tau_L^N = \frac{\varepsilon - 1}{\varepsilon},\ \tau_I^N < 1 \text{ and } \tau_X^N > 1.$$

Proof See Appendix F.1 ■

Our welfare decomposition allows us to interpret the Nash policy outcome stated in Proposition 3. Domestic policies are set fully efficiently even under strategic interaction and do not cause any beggar-thy-neighbor effects. By contrast, trade policy instruments are set with the intention to manipulate the terms of trade. As made clear in Section 5, an import subsidy or an export tax both aim at improving the terms of trade by delocating firms to the other economy (anti-delocation effect). Because there are two international relative prices (the one of the differentiated exportable bundle and the one of the differentiated importable bundle relative to the homogeneous good) two trade-policy instruments are necessary to target both. In the symmetric Nash equilibrium, policy makers do not achieve this objective and the trade taxes just create consumption and production-efficiency wedges.

The result that production subsidies are set so as to completely offset monopolistic distortions is an application of the Bhagwati-Johnson targeting principle in public economics (Dixit, 1985). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. If the policy maker disposes of sufficiently many instruments to deal with each incentive separately, she uses the production subsidy to address production efficiency. The trade policy instruments are instead used to exploit the terms-of-trade effect, which is the only remaining incentive.[47]

6.2 A Deep Trade Agreement – Globally Efficient Trade and Domestic Policies

Proposition 3 implies that some type of trade agreement is necessary to prevent countries from trying to exploit the terms-of-trade effects of their policies. Thus, the question arises how to design such an agreement and how much cooperation is necessary to achieve a globally efficient outcome.

Let us first address the question if countries can move from a situation of no cooperation, i.e. the situation described in Proposition 3, to a fully efficient outcome by negotiating cooperatively over trade taxes and production taxes and then to commit to the negotiation outcome. We call such a setup a *deep trade agreement*. Indeed, it is easy to show that this is possible: because countries are fully symmetric, the symmetric point on the production possibility frontier (the planner allocation) makes both countries better off than the Nash equilibrium described in Proposition 3. Moreover, moving from the Nash equilibrium to this point can be achieved without changes in the terms of trade (which would require compensating international transfers): a reciprocal reduction in import subsidies τ_{Ii} and export taxes τ_{Xi} for $i = H, F$ all the

[47] Proposition 3 extends the result of Campolmi et al. (2014) – who find that in the two-sector model with homogeneous firms strategic trade policy consists of first-best wage subsidies and inefficient import subsidies and export taxes – to the case of heterogeneous firms. This implies that firm heterogeneity neither adds further motives for signing a trade agreement beyond the classical terms-of-trade effect nor changes the qualitative results (import subsidies and export taxes in the differentiated sector) of the equilibrium outcome compared to the case with homogeneous firms.

way to zero does not change the terms of trade and leads to full consumption and production efficiency, as proved in Lemma 4.[48] Observe that domestic policies are left unchanged during this process, since the Nash production subsidies already correspond to the optimal ones. We have thus established the following result:

Corollary 3 ***Efficiency of a deep trade agreement***

In the two-sector model with heterogeneous firms, countries can negotiate a mutually beneficial deep trade agreement with cooperation on trade and domestic policies. This agreement implements the symmetric Pareto-efficient allocation by forbidding the use of trade policy instruments ($\tau_{Ii} = 1$ and $\tau_{Xi} = 1$ for $i = H, F$) and setting production subsidies in both countries equal to the inverse of the monopolistic markup ($\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$).[49]

Thus, a deep trade agreement is sufficient to achieve global efficiency. But is it also necessary to achieve it, or would a shallow trade agreement, which does not comprise coordination of domestic policies, achieve a similarly efficient outcome?

6.3 Shallow Trade Agreements

We now consider various forms of *shallow trade agreements*, which focus purely on coordination of trade taxes. We first consider a situation of trade negotiations under the current GATT-WTO rules. We follow Bagwell and Staiger (2001) in modeling the negotiation process of a shallow trade agreement in the presence of domestic policies as a two-stage process. In the first stage, countries negotiate cooperatively over trade taxes while keeping domestic policies constant at the Nash levels (which, in our setup, correspond to the first-best production subsidies). In the second stage, countries can deviate non-cooperatively from the stage-one outcome by setting trade taxes and/or production taxes non-cooperatively but subject to two additional constraints imposed by WTO rules: first, import taxes are subject to tariff bindings (they cannot be increased relative to the outcome negotiated in stage one); second, market access

[48] Changes in the terms of trade induced by simultaneous policy changes in both countries are given in Appendix E. They are zero for any symmetric cooperative policy changes because all policy instruments and variables move symmetrically.

[49] In principle, countries could alternatively continue to use tariffs and export taxes as long as they agree to set $\tau_{Tij} = 1$ and $\tau_{Ii} = \tau_{Ij}$ and $\tau_{Xi} = \tau_{Xj}$ for $i, j = H, F$. Since this is not very practical, we focus on zero trade taxes. In the one-sector model the laissez-faire allocation is Pareto optimal and individual-country policy makers' only incentive is to manipulate their terms of trade. Thus, the use of any type of policy instruments (both trade and production taxes) should be restricted by a trade agreement.

cannot be reduced: countries are not allowed to offset market access commitments (i.e., they cannot reduce imports) from the first stage by an unanticipated change in their policies.[50]

We have already shown above that the stage-one negotiation outcome leads to the first-best allocation. We now show that in stage two there exist unilateral deviations from the stage-one outcome that make the deviating country better off without violating the tariff-binding or market access constraints. As a consequence, a shallow trade agreement under the current WTO rules is not sufficient to obtain the first-best outcome.

To see this, note first that any deviations from the first-stage outcome are due to terms-of-trade effects because production efficiency is already guaranteed (this is established in Corrolary 1). Second, we now show that there exist unilateral deviations from the first-best outcome that improve domestic terms of trade and welfare and do not reduce market access. In particular, a reduction in the production subsidy below the first-best level, a small import subsidy or an export tax all improve the terms of trade, while also increasing imports in the differentiated sector.[51]

Lemma 6 ***Unilateral deviations from first-best allocation in multi-sector model*** *Consider a marginal unilateral increase in each policy instrument at a time starting from the first-best equilibrium of the two-sector model with heterogeneous firms, i.e., a situation with* $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$ *and let* $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\varepsilon-1)}$ *or* $\delta_{ii} > \frac{1}{2}$. *Then:*

(a) the domestic welfare effect is negative for τ_{Ii} *and positive for* τ_{Xi} *and* τ_{Li}.

(b) the volume of imports in the differentiated sector decreases in τ_{Ii} *and increases in* τ_{Xi} *and* τ_{Li}.

Proof See Appendix F.2. ■

Having shown that a shallow agreement in combination with WTO rules on tariff bindings and market access is not sufficient to replicate the outcome of a deep trade agreement, we now

[50] As Bagwell and Staiger (2001) argue, the legal basis for such "nonviolation" complaints is provided in GATT Article XXIII: countries are not allowed to reduce foreign countries' access to their markets with policy changes, even if these policy changes broke no explicit WTO rules.

[51] Note that imports in the homogeneous sector are zero in the first-best allocation, so that any changes in imports in this sector due to policy changes do not affect market access commitments. Lemma 6 also holds with homogeneous firms. The proof is available upon request.

analyze a situation that mimics the more stringent setup of a shallow free trade agreement under Article XXIV of GATT-WTO. Such an agreement requires full trade liberalization among its members (zero trade taxes), while leaving domestic policies uncoordinated. We characterize in detail the Nash equilibrium arising from strategic domestic policies (production taxes) under such an agreement. In this case, individual-country policy makers face a missing-instrument problem and consequently a trade-off between changing production efficiency (calling for a production subsidy) and the terms of trade (calling for a production tax). We have already discussed for unilateral deviations that in the presence of firm heterogeneity the relative weight of these motives depends on the profit share from sales in the domestic market. We now show that this intuition carries over to the Nash policies.

Proposition 4 ***Strategic domestic policies in the presence of a shallow trade agreement***

When only production taxes in the differentiated sector are available,

(a) it is possible to rewrite (33) *as follows:*

$$dV_i = \Omega_i dL_{Ci} \tag{36}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and where the wedge* Ω_i *is defined in Appendix F.3.*

(b) Solving the individual-country policy maker problem in (32) *by using the total-differential approach when* $\tau_{Ii} = \tau_{Xi} = 1$, $i = H, F$ *requires setting* $\Omega_i = 0$.

(c) As a result, the symmetric Nash equilibrium of the two-sector model with heterogeneous firms when trade taxes are not available and both countries can simultaneously set production taxes in the differentiated sector is characterized as follows: it exists, is unique and entails positive, but inefficiently low, production subsidies when the domestic profit share, δ_{ii}, *is larger or equal than 1/2. Otherwise, the Nash equilibrium entails positive production taxes. Formally:*

(i) If $\delta_{ii} \geq \frac{1}{2}$, *then there exists a unique symmetric Nash equilibrium with* $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$;

(ii) If either $0 < \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon \geq \frac{3-\alpha}{2}$ *or* $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon < \frac{3-\alpha}{2}$, *there exists a unique symmetric Nash equilibrium with* $\tau_L^N > 1$;

Proof See Appendix F.3. ■

The domestic profit share δ_{ii}, is a sufficient statistic for the impact of firm heterogeneity and selection. Proposition 4 states that if it is larger than the export profit share, strategic domestic policies feature positive production subsidies. From Lemma 5 we know that this outcome reflects that the (positive) production-efficiency effect dominates the (negative) terms-of-trade effect. However, these subsidies are inefficiently low due the trade-off between these motives.[52] By contrast, when the domestic profit share is smaller than the export profit share, strategic domestic policies feature production taxes, which worsen the allocation compared to the laissez-faire allocation.[53] In this case, the terms-of-trade effect dominates the production-efficiency effect because firms make the bulk of their profits from exporting, so that manipulating international prices is key. In the presence of firm heterogeneity, the relative importance of the two effects thus depends on the magnitude of the domestic profit share. Therefore, when the set of policy instruments is limited, firm heterogeneity plays a crucial role in shaping the equilibrium policies, and thus the desirability of specific institutional arrangements, as we show next.

6.4 A Laissez-faire Agreement

As shown above, a sufficient condition for reaping the full benefits of integration is to sign a deep trade agreement with cooperation on trade and domestic policies. However, full cooperation on domestic policies may not be feasible in practice. Alternatively, countries may be able to commit to free trade and not to use domestic policies at all. We thus consider as an alternative scenario a *laissez-faire agreement*, which forbids both the use of trade and domestic policies and we compare its performance with the one of a shallow free trade agreement. Whether or not such an arrangement dominates a shallow free trade agreement when firms are heterogeneous depends on whether the profit share from domestic sales is smaller or larger than the one from export sales. This is straightforward: a Nash production subsidy improves equilibrium production efficiency, and thus welfare, compared to the laissez-faire allocation, while a Nash

[52] Proposition 4 extends the result of Campolmi et al. (2014) – who find that in the two-sector model with homogeneous firms strategic domestic policies feature positive but inefficiently low production subsidies – to the case of heterogeneous firms.

[53] Observe that if we impose the assumption that the export cutoff φ_{ji} for $j \neq i$ must be larger than the domestic survival cutoff φ_{ii} at the symmetric Nash equilibrium, i.e. $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} > 1$, then δ_{ii} is always strictly greater than 1/2.

production tax worsens it. (Terms-of-trade effects of domestic policies offset each other in the symmetric Nash equilibrium.)

Finally, note that in the presence of firm heterogeneity and selection effects, the domestic profit share is endogenous to physical trade costs: one can show that δ_{ii} is increasing in τ_{ij} and f_{ij} for $j \neq i$. Thus, as physical trade barriers fall, the domestic profit share falls and may even become smaller than one half. Therefore, with sufficiently low physical trade barriers a laissez-faire agreement can be better than a shallow free trade agreement. These insights on the welfare effects of shallow vs. laissez-faire agreements are summarized by the following Proposition.[54]

Lemma 7 ***Welfare effects of strategic domestic policies in the presence of a shallow free trade agreement***

Assume that $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and let firms' average variable-profit share from sales in their domestic market be given by δ_{ii}.

(a) When $\delta_{ii} < \frac{1}{2}$ the symmetric Nash equilibrium of the two-sector model with heterogeneous firms when countries can only set domestic policies strategically is welfare-dominated by the laissez-faire allocation with $\tau_{Li} = 1$, $i = H, F$.

(b) δ_{ii} is increasing in τ_{ij} and f_{ij}, $j \neq i$.

Proof See Appendix F.4. ■

To summarize, when $\delta_{ii} \geq \frac{1}{2}$, a shallow free trade agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely welfare-dominates a laissez-faire agreement that forbids countries to use domestic and trade policies. When instead $\delta_{ii} < \frac{1}{2}$ a laissez-faire agreement welfare-dominates a shallow free trade agreement. Thus, a laissez-faire agreement is less distortive than a shallow free trade agreement when physical trade costs are sufficiently low.

[54]In numerical simulations with Pareto-distributed productivity we have obtained the robust result that when physical trade barriers fall Nash-equilibrium production subsidies decrease smoothly until they turn into positive taxes at a level of trade barriers that implies $\delta_{ii} = 1/2$. From that point on, production taxes strictly increase as trade barriers fall further. These results imply that the proportional welfare gains from moving from a shallow to a deep trade agreement rise as physical trade barriers fall.

7 Conclusion

In this paper we have made progress on several fronts. Starting with the observation that trade models with CES preferences and monopolistic competition have a common macro representation, we have shown that this class of models also has common welfare incentives for trade and domestic policies. Solving the problem of a world policy maker, we have derived a welfare decomposition that decomposes world welfare changes induced by trade and domestic policies into changes in consumption- and production-efficiency wedges. As long as the world policy maker disposes of a sufficient set of instruments, she closes these wedges one by one and implements the first-best allocation. In the multi-sector model this requires using production subsidies to offset monopolistic markups.

From the individual-country perspective, welfare incentives for trade and domestic policies are additionally governed by terms-of-trade incentives. This makes clear that the terms-of-trade motive is the only pure beggar-thy-neighbor externality in this class of models.

Then we have discussed that using individual policy instruments always leads to a trade-off between production-efficiency and terms-of-trade effects. Firm heterogeneity in combination with physical trade costs matter for unilateral policies because they determine the profit share from sales in each market. This variable governs how the trade-off between these motives plays out: when physical trade barriers are high, firms make most of their profits domestically, and thus production efficiency dominates.

Finally, we have studied the design of trade agreements from the perspective of the multi-sector heterogeneous-firm model. We have shown that in the absence of any trade agreement, the Nash equilibrium entails the first-best level of production subsidies and inefficient import subsidies and export taxes that aim at improving the terms of trade. Thus, even in the presence of firm heterogeneity and domestic policies terms-of-trade motives remain the only reason for signing a trade agreement. We have then considered trade negotiations under current WTO rules: countries first negotiate reciprocal reductions in trade taxes and can then adjust their policies unilaterally subject to tariff bindings and market access commitments. We have shown that such a setup is not sufficient to guarantee an efficient outcome. Moreover, when a shallow free trade agreement prevents countries from using trade policy strategically, strategic domestic policies are set to balance a trade-off between improving the terms of trade and increasing production efficiency. In this case, Nash-equilibrium domestic policies depend on firm heterogeneity via the profit share from domestic sales: when it is larger than the one from export sales, the Nash equilibrium features positive (albeit inefficiently low) production subsidies. By contrast, when it is smaller, the Nash equilibrium is characterized by positive production taxes. This result implies that achieving the full benefits of globalization requires a deep trade agreement that allows countries to coordinate both trade and domestic policies. Moreover, it means that shallow free trade agreements are more distortive when physical trade costs are lower and thus signing deep trade agreements becomes more desirable.

References

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (6), 94–130.

Bagwell, Kyle and Robert W. Staiger, "An Economic Theory of GATT," *American Economic Review*, 1999, *89* (1), 215–248.

_ and _ , "Domestic Policies, National Sovereignty, and International Economic Institutions," *Quarterly Journal of Economics*, 2001.

_ and _ , "Chapter 8 - The Design of Trade Agreements," in Kyle Bagwell and Robert W. Staiger, eds., *Handbook of Commercial Policy*, Vol. 1A, Elsevier, 2016.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati, "Trade Policy: Home Market Versus Terms of Trade Externality," *Center for Fiscal Policy Working Paper*, 2012.

_ , _ , and _ , "Trade Policy: Home Market Versus Terms of Trade Externality," *Journal of International Economics*, 2014, *93*, 92–107.

Copeland, Brian R., "Strategic Interaction among Nations: Negotiable and Non-negotiable Trade Barriers," *Canadian Journal of Economics*, 1990, *21* (1), 84–108.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning, "Micro to Macro: Optimal Trade Policy with Firm Heterogeneity," *Econometrica*, 2020, *88*, 2739–2776.

Demidova, Svetlana and Andrés Rodríguez-Clare, "Trade Policy Under Firm-level Heterogeneity in a Small Open Economy," *Journal of International Economics*, 2009, *78*, 100–112.

Dixit, Avinash K., "Tax policy in Open Economies," in Allan. J. Auerbach and Martin Feldstein, eds., *Handbook of Public Economics*, Vol. 1, Elsevier, 1985.

Dür, Andreas, Leornardo Baccini, and Martin Elsig, "The Design of International Trade Agreements: Introducing a New Dataset," *Review of International Organization*, 2014, *9* (3), 353–375.

Ederington, Josh, "International Coordination of Trade and Domestic Policies," *American Economic Review*, 2001, *91* (5), 1580–1593.

Felbermayr, Gabriel, Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation, and the Welfare Loss from Tariff Wars in the Melitz Model," *Journal of International Economics*, 2013, *89*, 13–25.

Gros, Daniel, "A Note on the Optimal Tariff Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade," *Journal of International Economics*, 1987, *23* (3-4), 357–367.

Grossman, Gene M. and Elhanan Helpman, "Trade Wars and Trade Talks," *Journal of Political Economy*, 1995, *103* (4), 675–708.

Grossman, Gene, Phillip McCalman, and Robert Staiger, "The "New" Economics of Trade Agreements: From Trade Liberalization to Regulatory Convergence?," *Econometrica*, 2021, *89* (1), 215–249.

Haaland, Jan I. and Anthony J. Venables, "Optimal Trade Policy with Monopolistic Competition and Heterogeneous Firms," *Journal of International Economics*, 2016, *102*, 85–95.

Harberger, Arnold, "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay," *Journal of Economic Perspectives*, 1971, *9* (3), 785–797.

Helpman, Elhanan and Paul Krugman, *Trade Policy and Market Structure*, MIT Press, 1989.

Horn, Henrik, Petros Mavroidis, and André Sapir, "Beyond the WTO? An Anatomy of EU and US Preferential Trade Agreements," *The World Economy*, 2010, *33* (11), 1565–1588.

Krugman, Paul, "Scale Economics, Product Differentiation, and Pattern of Trade," *American Economic Review*, 1980, *70* (5), 950–959.

Lashkaripour, Ahmad and Volodymyr Lugovsky, "Scale Economies and the Structure of Trade and Industrial Policy," *mimeo*, 2019.

Maggi, Giovanni and Andrés Rodríguez-Clare, "The Value of Trade Agreements in the Presence of Political Pressures," *Journal of Political Economy*, 1998, *106* (3), 574–601.

Meade, James, *The Theory of International Economic Policy Vol. 2 - Trade and Welfare: Mathematical Supplement*, Oxford University Press, 1955.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

_ **and Stephen J. Redding**, "Heterogeneous Firms and Trade," in Elhanan Helpman Gita Gopinath and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2015.

Ossa, Ralph, "A 'New Trade' Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 2011, *119* (1), 122–152.

_ **and Giovanni Maggi**, "Are Trade Agreements Good for You?," *mimeo*, 2019.

Parenti, Mathieu and Gonzague Vanoorenberghe, "A simple theory of deep trade integration," *mimeo*, 2019.

Rodrik, Dani, "What Do Trade Agreements Really Do?," *Journal of Economic Perspectives*, 2018, *32* (2), 73–90.

Venables, Anthony, "Trade and Trade Policy with Differentiated Products: A Chamberlinian-Ricardian Model," *The Economic Journal*, 1987, *97* (387), 700–717.

APPENDIX - FOR ONLINE PUBLICATION

A The Model

In this Appendix we first lay out the general model set-up. Then, we explain how to recover the set of equilibrium conditions (i) in the presence and in the absence of the homogeneous sector and (ii) for the cases of heterogeneous and homogeneous firms. Finally, we derive the laissez-faire allocation for the two-sector model.

A.1 Households

Given the Dixit-Stiglitz structure of preferences in (4), the households' maximization problem can be solved in three stages. At the first two stages, households choose how much to consume of each domestically produced and foreign produced variety, and how to allocate consumption between the domestic and the foreign bundles. The optimality conditions imply the following demand functions and price indices:

$$c_{ij}(\varphi) = \left[\frac{p_{ij}(\varphi)}{P_{ij}}\right]^{-\varepsilon} C_{ij}, \qquad C_{ij} = \left[\frac{P_{ij}}{P_i}\right]^{-\varepsilon} C_i, \quad i,j = H,F \tag{A-1}$$

$$P_i = \left[\sum_{j\in H,F} P_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \qquad P_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} p_{ij}(\varphi)^{1-\varepsilon} dG(\varphi)\right]^{\frac{1}{1-\varepsilon}}, \quad i,j = H,F \tag{A-2}$$

Here P_i is the price index of the differentiated bundle in country i, P_{ij} is the country-i price index of the bundle of differentiated varieties produced in country j, and $p_{ij}(\varphi)$ is the country-i consumer price of variety φ produced by country j.

In the last stage, households choose how to allocate consumption between the homogeneous good and the differentiated bundle. Thus, they maximize (3) subject to the following budget constraint:

$$P_i C_i + p_{Zi} Z_i = I_i, \quad i = H,F$$

where $I_i = W_i L + T_i$ is total income and T_i is a lump sum transfer which depends on the tax scheme adopted by the country-i government. The solution to the consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_i}{C_i} = \frac{P_i}{p_{Zi}}, \quad i = H,F \tag{A-3}$$

Then following Melitz and Redding (2015), we can rewrite the demand functions as

$$c_{ij}(\varphi) = p_{ij}(\varphi)^{-\varepsilon} A_i, \quad C_{ij} = P_{ij}^{-\varepsilon} A_i, \quad C_i = P_i^{-\varepsilon} A_i, \quad i,j = H,F, \tag{A-4}$$

where $A_i \equiv P_i^{\varepsilon-1}\alpha I_i$. A_i can be interpreted as an index of market (aggregate) demand.

A.2 Firms

A.2.1 Firms' behavior in the differentiated sector

Given the constant price elasticity of demand, optimal prices charged by country-i firms in their domestic market are a fixed markup over their perceived marginal cost ($\tau_{Li}\frac{W_i}{\varphi}$), and optimal prices charged to country-j consumers for varieties produced in country i equal country-i prices augmented by transport costs and trade taxes

$$p_{ji}(\varphi) = \tau_{ji}\tau_{Tji}\tau_{Li}\frac{\varepsilon}{\varepsilon-1}\frac{W_i}{\varphi}, \quad i,j = H,F \tag{A-5}$$

The optimal pricing rule implies the following firm revenues:

$$r_{ji}(\varphi) \equiv \tau_{Tji}^{-1} p_{ji}(\varphi) c_{ji}(\varphi) = \tau_{Tji}^{-1} p_{ji}(\varphi)^{1-\varepsilon} A_j = \varepsilon \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} \tau_{Li}^{1-\varepsilon} W_i^{1-\varepsilon} \varphi^{\varepsilon-1} B_j, \quad i,j = H,F, \tag{A-6}$$

where $B_i \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \frac{1}{\varepsilon} A_i$. Profits are given by:

$$\pi_{ji}(\varphi) \equiv B_j \left(\frac{\tau_{Li} W_i}{\varphi}\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} = \frac{r_{ji}(\varphi)}{\varepsilon} - \tau_{Li} W_i f_{ji}, \quad i,j = H,F \tag{A-7}$$

A.2.2 Zero-profit conditions

Firms choose to produce for the domestic (export) market only when this is profitable. Since profits are monotonically increasing in φ, we can determine the equilibrium productivity cutoffs for firms active in the domestic market and export market, φ_{ji}, by setting $\pi_{ji}(\varphi_{ji}) = 0$, namely:

$$\pi_{ji}(\varphi_{ji}) = 0 \Rightarrow \frac{r_{ji}(\varphi_{ji})}{\varepsilon} = \tau_{Li} W_i f_{ji}, \quad i,j = H,F \tag{A-8}$$

As in Melitz (2003), we call these conditions the *zero profit* (**ZCP**) conditions. Using (A-7) we rewrite (A-8) as follows:

$$B_j = \tau_{ji}^{\varepsilon-1} \tau_{Li}^{\varepsilon} \tau_{Tji}^{\varepsilon} W_i^{\varepsilon} \varphi_{ji}^{1-\varepsilon} \quad j = H,F, \quad i \neq j \tag{A-9}$$

A.2.3 Free-entry conditions (FE)

The *free entry* (**FE**) conditions require expected profits (before firms know the realization of their productivity) in each country to be zero in equilibrium:

$$\sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} \pi_{ji}(\varphi) dG(\varphi) = \tau_{Li} W_i f_E, \quad i = H,F$$

Substituting optimal profits (A-7), we obtain

$$\sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} \left[B_j \left(\frac{\tau_{Li} W_i}{\varphi}\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} \right] dG(\varphi) = \tau_{Li} W_i f_E, \quad i = H,F \tag{A-10}$$

A.2.4 Firms' behavior in the homogeneous sector

Since the homogeneous good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization in the presence of the homogeneous sector:

$$p_{Zi} = p_{Zj} = W_i = W_j = 1, \quad i = H, j = F$$

A.3 Government

The government is assumed to run a balanced budget. Hence, country-i government's budget constraint is given by:

$$\begin{aligned} T_i = {} & (\tau_{Ii} - 1) \tau_{Ii}^{-1} P_{ij} C_{ij} + (\tau_{Xi} - 1) \tau_{Tji}^{-1} P_{ji} C_{ji} + \\ & + (\tau_{Li} - 1) N_i W_i \left[\sum_{k=H,F} \int_{\varphi_{ki}}^{\infty} \left(\frac{q_{ki}(\varphi)}{\varphi} + f_{ki} \right) dG(\varphi) + f_E \right], \quad i = H,F, \quad j \neq i \end{aligned} \tag{A-11}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and foreign export taxes (thus, tariffs are charged on CIF values of foreign exports), export tax revenues charged on exports gross of transport costs, and production tax revenues.

A.4 Equilibrium

A.4.1 Equilibrium of the two-sector model

Substituting **ZCP** (A-9) into **FE** (A-10), we obtain:

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})), \quad i=H,F, \tag{A-12}$$

where

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1}\frac{dG(\varphi)}{1-G(\varphi_{ji})}\right]^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F, \tag{A-13}$$

which correspond to (9) and (6) in the main text. Moreover, dividing the **ZCP** conditions (A-9), we obtain condition (8) in the main text:

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left(\frac{f_{ii}}{f_{ij}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{W_i}{W_j}\right)^{\frac{\varepsilon}{\varepsilon-1}}\tau_{ij}^{-1}\tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}}, \quad i,j=H,F \tag{A-14}$$

The remaining equilibrium equations are then given as follows:

Consumption sub-indices, which can be determined using (A-4) jointly with (A-9):

$$C_{ij} = P_{ij}^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon-1}\varepsilon\tau_{Lj}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\tau_{Tij}^{\varepsilon}\varphi_{ij}^{1-\varepsilon}W_j^{\varepsilon}f_{ij}, \quad i,j=H,F \tag{A-15}$$

Price sub-indices, which emerge from substituting (A-5) into (A-2):

$$P_{ij}^{1-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}N_j(1-G(\varphi_{ij}))(\tau_{ij}\tau_{Tij}\tau_{Lj})^{1-\varepsilon}\widetilde{\varphi}_{ij}^{\varepsilon-1}W_j^{1-\varepsilon}, \quad i,j=H,F \tag{A-16}$$

Aggregate profits Π_i are given by $\Pi_i = R_i - \tau_{Li}W_iL_{Ci} + \tau_{Li}W_iN_if_E$, where R_i is aggregate revenue, $R_i \equiv N_i\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty} r_{ji}(\varphi)dG(\varphi)$. From the **FE** condition (A-10) it then follows that $\Pi_i = \tau_{Li}W_iN_if_E$ and thus $R_i = \tau_{Li}W_iL_{Ci}$. Substituting the definition of optimal revenues (A-6) into the previous condition, we get

$$\tau_{Li}W_iL_{Ci} = \varepsilon N_i\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty} B_j\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon}\tau_{Li}^{1-\varepsilon}W_i^{1-\varepsilon}\varphi^{\varepsilon-1}dG(\varphi), \quad i=H,F$$

Combining this condition with (9) and (A-9), we obtain:

Labor market clearing in the differentiated sector

$$L_{Ci} = \varepsilon N_i\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})) + \varepsilon f_E N_i, \quad i=H,F \tag{A-17}$$

This can be solved for the equilibrium level of N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) + \varepsilon f_E}, \quad i = H, F \tag{A-18}$$

Combining this last condition with (9), plugging into (A-15) and (A-16) and taking into account the definition (6), allows us to recover (10) and (11) in the main text.

In the presence of the homogeneous sector, the trade-balance condition is given by:[55]

$$Q_{Zi} - Z_i + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F \tag{A-19}$$

We can use the fact that $\sum_{j=H,F} P_{ij} C_{ij} = P_i C_i$ to rewrite (A-3) as:

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij}, \quad i = H, F$$

We can combine this equation with the trade-balance condition above and the aggregate labor market clearing $L = L_{Ci} + L_{Zi}$ to obtain:

Trade-balance condition

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F,$$

which corresponds to condition (12).

Finally, when the homogeneous sector is present, we also require equilibrium in the market for the homogenous good, i.e. $\sum_{i=H,F} Q_{Zi} = \sum_{i=H,F} Z_i$. Combining this condition with aggregate labor market clearing and demand for the homogeneous good (A-3) we obtain:

Homogeneous-good market clearing condition

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij},$$

which coincides with condition (13).

We thus have a system of 24 equilibrium equations in 25 unknowns, namely δ_{ji}, φ_{ji}, $\widetilde{\varphi}_{ji}$, C_{ji}, P_{ij}, L_{Ci}, Z_i for $i, j = H, F$ and W_i for $i = H$. Note that $W_i = 1$ for $i = H$, since factor prices must be equalized in equilibrium.

A.4.2 Equilibrium of the one-sector model

When there is no homogeneous sector, i.e., when $\alpha = 1$, then from (14) $Z_i = 0$ for $i = H, F$, and $L_{Ci} = L$ for $i = H$ from the labor market clearing $L = L_{Ci} + Q_{Zi}$. Conditions (6)-(11) remain the same. Condition (12) simplifies to:

$$\tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{A-20}$$

Then, $L_{Ci} = L$ for $i = F$ from (13).

Finally note that, as well known, in the one-sector model the allocation of labor is efficient. Thus, we assume that policy makers abstain from strategically using the labor subsidy. For convenience, we assume that in any symmetric allocation the labor cost is equal across countries i.e., labor subsidies are such that $\tau_{Li} W_i = \tau_{Lj} W_j$

[55]Import taxes are collected directly by the governments at the border so they do not enter into this condition.

for $i = H$ and $j = F$. As it will become clear in the following sections, this assumption will simplify the comparison between the planner and the market allocation.

A.4.3 From Melitz to Krugman (1980)

The Melitz model encompasses the Krugman (1980) model with homogeneous firms under the assumptions that there are no fixed market access costs (i.e, $f_{ij} = 0$ for $i, j = H, F$) and that $G(\varphi)$ is a degenerate distribution. Then, without loss of generality, we normalize $\tilde{\varphi} = \varphi = 1$. Under this parametrization, conditions (6), (7), (8) and (9) should be dropped from the set of equilibrium conditions.

In addition, the free-entry conditions are given by:

$$\sum_{j=H,F} \pi_{ji} = \tau_{Li} W_i f_E, \quad i = H, F,$$

and profits are given by:

$$\pi_{ji} \equiv B_j \left(\tau_{Li} W_i\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon}, \quad i, j = H, F$$

By combining these two last conditions we can solve for B_i and B_j as functions of W_i, W_j and the policy instruments:

$$B_i = f_E W_j^{\varepsilon} \frac{\tau_{Lj}^{\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \tau_{Tji}^{\varepsilon} \left(\frac{W_i}{W_j}\right)^{\varepsilon}}{\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1}}, \quad i = H, F, \quad j \neq i$$

Moreover, by substituting the optimal pricing decision into the definition of the price indices and observing that $N_j = L_{Cj}/(\varepsilon f_E)$ we get:

$$P_{ij} = \frac{\varepsilon}{\varepsilon - 1} (\varepsilon f_E)^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} W_j L_{Cj}^{\frac{-1}{\varepsilon-1}}, \quad i, j = H, F \tag{A-21}$$

At the same time, from the definition of C_{ij}, it follows that:

$$C_{ij} = P_{ij}^{-\varepsilon} \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\varepsilon-1} \varepsilon B_i, \quad i, j = H, F$$

Substituting the expressions above for P_{ij} and B_i into the above condition, leads to:

$$C_{ij} = \frac{\varepsilon - 1}{\varepsilon} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} \frac{(\tau_{ij}\tau_{Tij})^{-\varepsilon} \left[\left(\frac{W_k}{W_j}\frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j}\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon} \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon}\right]}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}}, \quad i, j = H, F, \quad k \neq i \tag{A-22}$$

Thus, if the homogeneous sector is present ($\alpha < 1$), the equilibrium is given by equations (A-21) and (A-22) together with (12),(13) and (14) and the fact that $W_i = 1$ for $i = H$. By contrast, in the absence of the homogeneous sector (i.e., when $\alpha = 1$), the equilibrium is determined by conditions (A-20), (A-21), (A-22) and the fact that $L_{Cj} = L$ for $j = H, F$.

A.4.4 The allocation under the laissez-faire agreement in the two-sector model

Using equations (10) and (11), we find that

$$P_{ij} C_{ij} = \delta_{ij} L_{Cj} \tau_{Tij} \tau_{Lj} W_j, \quad i, j = H, F$$

Substituting into the trade-balance condition (12), we obtain:

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} \delta_{ik} L_{Ck} \tau_{Tik} \tau_{Lk} W_k + \delta_{ji} L_{Ci} \tau_{Xi} \tau_{Li} W_i = \delta_{ij} L_{Cj} \tau_{Xj} \tau_{Lj} W_j, \quad i = H, j = F$$

Under the laissez-faire agreement, $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Since the countries are symmetric, the equilibrium is also symmetric and thus $L_{Ci} = L_{Cj}$, $W_i = W_j = 1$, $\delta_{ij} = \delta_{ji}$ for $i = H, F$ and $j \neq i$.

Substituting these conditions, we find that

$$L_{Ci}^{LF} = \alpha L, \quad i = H, F$$

Using this result together with (A-17) and (A-12), we obtain

$$N_i^{LF} = \frac{\alpha L}{\varepsilon \sum_{j=H,F} \left[f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon - 1} \right]}, \quad i = H, F$$

B The Total-Differential Approach

We use the total-differential approach to optimization to solve both the planner and the optimal-policy problems.[56] In this way, we can use the same methodology to derive all the main results of the paper: the welfare decomposition and the efficiency wedges; the world, the unilateral and the strategic policies.

We first discuss how we apply this approach to find the optimal deviations of domestic and trade policies. Then, we explain how to employ it to solve constrained optimization problems. Finally, we derive a number of preliminary results that we will use in the rest of the appendix.

B.1 How to apply the total-differential approach

B.1.1 Unilateral policy deviations

The unilateral deviations of each policy instrument can be determined following these steps:

(1) Take the total differential of the objective function and the equilibrium conditions.

(2) Use the total differential of the equilibrium conditions to solve for the total differentials of the endogenous variables as linear functions of the total differentials of the policy instruments. Since we consider each policy instrument at a time, set the total differentials of the policy instruments that are not of interest to zero.

(3) Substitute the solution of the total differentials of the endogenous variables into the total differential of the objective function and evaluate it at the laissez-faire allocation. Collect all the terms and sign the coefficient multiplying the total differential of the policy instrument to determine the direction of the optimal deviations from the laissez-faire allocation.

B.1.2 Constrained optimization problems

A constrained optimization problem in n variables given m constraints with $n > m$ can be solved using the total-differential approach according to the following steps:

(1) Take the total differential of the objective function and the constraints.

(2) Use the total differential of the constraints to solve for m total differentials as a function of the $n - m$ other total differentials.

(3) Substitute the solution of the m total differentials into the total differential of the objective function. Then the total differential of the objective function must be zero for *any* of the $n - m$ total differentials (i.e., for any *arbitrary* perturbation of the $n - m$ relevant variables). Collect the terms multiplied by the $n - m$ differentials to find the $n - m$ conditions that need to be zero at the optimum.

(4) The $n - m$ conditions found in (3) jointly with the m constraints determine the solution of the n variables.

[56] Observe that using this approach implies restricting our analysis to interior solutions.

B.2 Preliminary steps for applying the total-differential approach

In this section, we derive some preliminary results that will be useful to derive the results of Sections 3 to 6.

As explained above, the first steps to apply the total-differential approach – independently of whether the optimal policy problem or unilateral deviations are considered – is to take the total differential of the equilibrium equations (6)-(13), which we do in Section B.2.1 below. Then, we evaluate the total differentials at a symmetric allocation. Moreover, when considering policies from the individual-country perspective, as analyzed in Section 5, we set $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$, and combine the equations so as to be left with 3 equations, which are linear functions of 6 differentials: dL_{Ci}, dC_{ii}, dC_{ij}, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. We can then use these 3 equations to express 3 differentials as functions of the remaining 3. For the unilateral deviations considered in Section 5, we solve for dL_{Ci}, dC_{ii} and dC_{ij} as linear functions of the deviations of the policy instruments $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. Then, we allow only a single policy instrument to vary at a time, while setting the deviations for the other two to zero. Differently, for the cases of strategic interaction in Section 6 we use the 3 equations to write the differentials of the tax instruments, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ as linear functions of the other 3 differentials, dL_{Ci}, dC_{ii} and dC_{ij}. Finally, for the case of strategic interaction when only production taxes are available (shallow trade agreement) we set the deviations for $d\tau_{Ii}$ and $d\tau_{Xi}$ to zero. This allow us to express $d\tau_{Li}$ as a function of dL_{Ci} only.

B.2.1 Total differentials of some equilibrium conditions

Since the total differentials of the equilibrium equations (6)-(10) are extensively used in the proofs of Sections 3 to 6, and since they hold for both the one-sector and the two-sector models, we present them here for future reference in their general formulation.

The total differential of (6) gives:

$$d\widetilde{\varphi}_{ji} = \frac{1}{\varepsilon - 1} \frac{g(\varphi_{ji})}{[1 - G(\varphi_{ji})]} \widetilde{\varphi}_{ji} \left[1 - \left(\frac{\varphi_{ji}}{\widetilde{\varphi}_{ji}} \right)^{\varepsilon - 1} \right] d\varphi_{ji}, \quad i, j = H, F \tag{B-1}$$

Substituting this condition into the total differential of (9), we get:

$$d\varphi_{ji} = -\frac{f_{ii}[1 - G(\varphi_{ii})]\varphi_{ii}^{-\varepsilon} \widetilde{\varphi}_{ii}^{\varepsilon - 1}}{f_{ji}[1 - G(\varphi_{ji})]\varphi_{ji}^{-\varepsilon} \widetilde{\varphi}_{ji}^{\varepsilon - 1}} d\varphi_{ii}, \quad i = H, F, \quad i \neq j \tag{B-2}$$

Using (7) and (9), this condition can be rewritten as

$$d\varphi_{ji} = -\frac{\delta_{ii}}{1 - \delta_{ii}} \frac{\varphi_{ji}}{\varphi_{ii}} d\varphi_{ii}, \quad i = H, F, \quad i \neq j, \tag{B-3}$$

which expresses the total differential of the productivity cut-offs for the domestically produced goods in the export markets as a function of the cut-offs in the domestic markets. Taking the total differential of (7) combined with (9) and substituting (B-1) and (B-2) into the resulting condition, we get:

$$d\delta_{ji} = -\frac{\delta_{ji}}{\varphi_{ji}} \left(\Phi_i + (\varepsilon - 1) \right) d\varphi_{ji}, \quad i, j = H, F \tag{B-4}$$

where $\Phi_i \equiv \delta_{ii} \frac{g(\varphi_{ji})\varphi_{ji}^{\varepsilon} \widetilde{\varphi}_{ji}^{1-\varepsilon}}{1 - G(\varphi_{ji})} + \delta_{ji} \frac{g(\varphi_{ii})\varphi_{ii}^{\varepsilon} \widetilde{\varphi}_{ii}^{1-\varepsilon}}{1 - G(\varphi_{ii})} > 0$, $i = H, F$ and $j \neq i$. Condition (B-4) states that as the productivity cut-off rises, the corresponding variable-profit share shrinks.

Moreover, by totally differentiating (10), we obtain:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{C_{ij}} dC_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{\delta_{ij}} d\delta_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{L_{Cj}} dL_{Cj}, \quad i, j = H, F, \tag{B-5}$$

which, using the symmetric condition of (B-4) to substitute out $d\delta_{ij}$, becomes:

$$d\varphi_{ij} = \frac{\varepsilon\varphi_{ij}}{L_{Cj}(\varepsilon-1)\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_j\right)}dL_{Cj} - \frac{\varphi_{ij}}{C_{ij}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_j\right)}dC_{ij}, \quad i,j=H,F \tag{B-6}$$

For future use, we substitute the symmetric condition of (B-6) into (B-4):

$$d\delta_{ji} = \frac{\delta_{ji}(\varepsilon-1+\Phi_i)}{C_{ji}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}dC_{ji} - \frac{\delta_{ji}\varepsilon(\varepsilon-1+\Phi_i)}{L_{Ci}(\varepsilon-1)\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}dL_{Ci}, \quad i,j=H,F \tag{B-7}$$

Finally, taking the total differential of (8), we have:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}}d\varphi_{ii} + \frac{\varepsilon}{\varepsilon-1}\varphi_{ij}\left[\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} + \frac{dW_j}{W_j} - \frac{dW_i}{W_i} + \frac{d\tau_{Tij}}{\tau_{Tij}}\right], \quad i,j=H,F, \quad i\neq j \tag{B-8}$$

where $d\tau_{Tji}=0$ if $i=j$ while $d\tau_{Tji}=\tau_{Xi}d\tau_{Ij}+\tau_{Ij}d\tau_{Xi}$ if $i\neq j$.

B.2.2 Total differentials of the two-sector model

In this section we restrict ourself to the case $\alpha<1$. Hence we can use the simplification $W_i=1$ for $i=H,F$. Under this restriction we combine the total differentials of the equilibrium equations to find 3 conditions that can be expressed as functions of dL_{Ci}, dC_{ii}, dC_{ij}, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ only.[57]

(1) The first condition can be derived in the following way. Taking the symmetric condition of (B-8), using (B-3) to substitute out $d\varphi_{ji}$, solving for $d\varphi_{jj}$ and finally using (B-6) to substitute out $d\varphi_{ii}$, we obtain:

$$d\varphi_{jj} = -\frac{\varphi_{jj}}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right) - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) \tag{B-9}$$

Using (B-3) to substitute out $d\varphi_{jj}$ from (B-9) we find the following expression for $d\varphi_{ij}$:

$$d\varphi_{ij} = -\frac{\delta_{jj}\varphi_{ij}}{1-\delta_{jj}}\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Tji}}{\tau_{Tji}}\right) - \frac{\delta_{ii}}{1-\delta_{ii}}\frac{1}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right)\right] \tag{B-10}$$

Moreover, we combine (B-6), (B-8) and (B-10) to obtain:

$$-\frac{d\tau_{Tij}}{\tau_{Tij}}(1-\delta_{jj}) + \frac{d\tau_{Tji}}{\tau_{Tji}}\delta_{jj} + \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{1-\delta_{ii}-\delta_{jj}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0$$

Finally we impose symmetry as well as $d\tau_{Lj}=d\tau_{Xj}=d\tau_{Ij}=0$. This means that $d\tau_{Tji}=\tau_{Ij}d\tau_{Xi}$ and $d\tau_{Tij}=\tau_{Xj}d\tau_{Ii}$. Under these restrictions, we can rewrite the last equation as:

$$\frac{d\tau_{Li}}{\tau_{Li}} - (1-\delta_{ii})\frac{d\tau_{Ii}}{\tau_{Ii}} + \delta_{ii}\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{1-2\delta_{ii}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0 \tag{B-11}$$

(2) The second condition can be found as follows. First, we combine (10) and (11):

$$P_{ij}C_{ij} = L_{Cj}\delta_{ij}\tau_{Tij}\tau_{Lj} \quad i,j=H,F \tag{B-12}$$

[57]For the sake of brevity we omit to specify for which countries the equations hold.

Second, we use (B-12) to rewrite (12) as follows:

$$L_{Cj} = \frac{\alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{(1-\delta_{jj})\tau_{Lj}\tau_{Xj}(\alpha + (1-\alpha)\tau_{Ii})} \tag{B-13}$$

Third, using (B-4) to find an expression for $d\delta_{jj}$ and combining it with (B-9) we get:

$$d\delta_{jj} = \delta_{jj}(\varepsilon - 1 + \Phi_j)\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) - \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right)\right] \tag{B-14}$$

Taking the total differential of (B-13), using (B-7) and (B-14) to substitute out $d\delta_{ii}$ and $d\delta_{jj}$, imposing symmetry and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, we obtain:

$$\begin{aligned}\frac{dL_{Cj}}{L_{Cj}} =& \left(\frac{\alpha}{(1-\alpha)\tau_{Ii} + \alpha} + \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})\left(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}\right)}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dC_{ii}}{C_{ii}}\\ &- \frac{\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})\tau_{Li}\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} + \frac{\varepsilon\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii}(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dL_{Ci}}{L_{Ci}}\\ &- \frac{1-\alpha}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} - \left(\frac{(1-\alpha)\delta_{ii} - \alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Li}}{\tau_{Li}}\end{aligned} \tag{B-15}$$

In addition, we combine (B-12) with (13), we take its total differential, and then we substitute out $d\delta_{ii}$ and $d\delta_{jj}$ using (B-7) and (B-14), respectively. We then impose symmetry and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$ to get:

$$\begin{aligned}&-(1-\alpha)L_{Ci}\left(\delta_{ii} + (1-\delta_{ii})\tau_{II}\tau_{Xi} + \frac{\delta_{ii}\varepsilon(1-\tau_{Ii}\tau_{XI})(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right)d\tau_{Li}\\ &-(1-\alpha)L_{Ci}\left((1-\delta_{ii})\tau_{Li} + \frac{\delta_{ii}\varepsilon\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{(\varepsilon-1)\tau_{Ii}\tau_{Xi}}\right)\tau_{Ii}d\tau_{Xi} - (1-\alpha)L_{Ci}(1-\delta_{ii})\tau_{Li}\tau_{Xi}d\tau_{Ii}\\ &+\frac{(1-\alpha)L_{Ci}\delta_{ii}}{\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}}\left(\frac{\delta_{ii}\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{1-\delta_{ii}} - \tau_{Li}(1-\tau_{Ij}\tau_{Xi})(\varepsilon-1+\Phi_i)\right)\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right)\\ &-(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))dL_{Cj} - (\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\,dL_{Ci} = 0\end{aligned} \tag{B-16}$$

We can then use condition (B-15) to substitute out dL_{Cj} from (B-16) and to rewrite (B-16) as follows:

$$
\begin{aligned}
&- \frac{(1-\alpha)(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{\alpha+(1-\alpha)\tau_{Ii}} d\tau_{Ii} \\
&- \frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}} (\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi}) \frac{d\tau_{Li}}{\tau_{Li}} \\
&+ \left((1-\alpha)(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \frac{\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Li}}\right) d\tau_{Li} \\
&+ \frac{\alpha(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}} d\tau_{Xi} \\
&+ \left((1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii} + \frac{\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Xi}}\right) d\tau_{Xi} \\
&+ \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})} \left(-\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} + (1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(1-\delta_{ii})\right. \\
&- \left.\frac{(1-\alpha+\alpha\tau_{Xi})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right) \frac{dC_{ii}}{C_{ii}} \\
&- \left[\left(\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}} - 1\right)(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\right. \\
&- \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})} \left((\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{1-\alpha+\alpha\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right. \\
&+ \left.\left.\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} - (1-\delta_{ii})(1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})\right)\right] \frac{dL_{Ci}}{L_{Ci}} = 0
\end{aligned} \tag{B-17}
$$

(3) The third condition can be retrieved as follows. First, we use (10) to solve for φ_{ii}. Second, we substitute the expression for φ_{ii} into (8) and solve for φ_{ij}. Finally, we employ this expression for φ_{ij} together with $\delta_{ij} = 1-\delta_{ii}$, and (B-13) to rewrite (10) as follows:

$$
C_{ij} = C_{ii}\left(\frac{L_{Ci}\delta_{ii}\tau_{Ii}(L\alpha - L_{Ci}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}))}{\tau_{Li}(\alpha+(1-\alpha)\tau_{Ii})}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B-18}
$$

Taking the total differential of (B-18), using (B-7) to substitute out $d\delta_{ii}$ and (B-13) and (B-18) to define, respectively, L_{Cj} and C_{ij}, we have:

$$
\begin{aligned}
0 = {} & \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \left(\frac{dC_{ii}}{C_{ii}}\frac{\varepsilon-1}{\varepsilon} - \frac{dL_{Ci}}{L_{Ci}}\right)\left(1 - \frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1+\frac{L_{Ci}\delta_{ii}\tau_{Li}}{\Lambda_i}(1-\alpha+\alpha\tau_{Xi})\right)\right) \\
& + \frac{dL_{Ci}}{\Lambda_i}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}) - \frac{d\tau_{Ii}}{\tau_{Ii}}\frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}} - d\tau_{Xi}\alpha\frac{L_{Ci}(1-\delta_{ii})\tau_{Li}}{\Lambda_i} \\
& + d\tau_{Li}\left(\frac{L_{Ci}}{\Lambda_i}((1-\alpha)\delta_{ii}-(1-\delta_{ii})\alpha\tau_{Xi}) + \frac{1}{\tau_{Li}}\right)
\end{aligned}
$$

where $\Lambda_i \equiv \alpha L - L_{Ci}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})$. Using (B-13), under symmetry we can rewrite the previous expression as follows:

$$
\begin{aligned}
0 = {} & \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}}\left(\frac{d\tau_{Ii}}{\tau_{Ii}}+\frac{d\tau_{Xi}}{\tau_{Xi}}\right) + \left(1+\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})}\right)\frac{d\tau_{Li}}{\tau_{Li}} \\
& - \left(1 - \frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1+\frac{\delta_{ii}(1-\alpha+\alpha\tau_{Xi})}{(1-\delta_{ii})\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\right)\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) \\
& + \frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Xi}\tau_{Li}}{(1-\delta_{ii})\tau_{Xi}\tau_{Li}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}}
\end{aligned} \tag{B-19}
$$

Conditions (B-11), (B-17), and (B-19) can be used to find an explicit solution for either dL_{Ci}, dC_{ii} and dC_{ij} as linear functions of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ (i.e., the solution for the unilateral deviations) or for $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ as linear functions of dL_{Ci}, dC_{ii} and dC_{ij}(i.e., the solution for the Nash problem with all policy instruments). Conditions (B-11), (B-17), and (B-19) also allow us to retrieve the solution for the Nash problem with only the production tax. All these expressions are available upon request.

B.2.3 Total differentials of the one-sector model

The total differentials computed in this section will be used only to study unilateral deviations. For this reason we can apply some simplifications to the total differentials defined in B.2.1. As explained in section A.4.2, when $\alpha = 1$ we get $Z_i = 0$, $L_{Ci} = L$ and $d\tau_{Li} = 0$ for $i = H, F$ so that $dZ_i = dL_{Ci} = 0$ for $i = H, F$. Also, $W_j = 1$ so that $dW_j = 0$ for $j = F$ and $d\tau_{Ij} = d\tau_{Xj} = 0$ for $j = F$. After taking the differentials, we evaluate them at the laissez-faire allocation ($\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$)

First, consider the case of heterogeneous firms. In this case, our objective is to retrieve 3 conditions as a function of $dW_i, dC_{ii}, dC_{ij}, d\tau_{Ii}$ and $d\tau_{Xi}$.

(1) To find the first condition, recall that (B-8) simplifies to:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}} d\varphi_{ii} - \frac{\varepsilon}{\varepsilon - 1}\varphi_{ij} dW_i + \frac{\varepsilon}{\varepsilon - 1}\varphi_{ij} d\tau_{Tij}, \quad i, j = H, F, \quad i \neq j \tag{B-20}$$

Second, from (B-6) we have:

$$d\varphi_{ii} = -\frac{\varphi_{ii}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} dC_{ii}, \quad i = H, F \tag{B-21}$$

Third, from (B-20) we have $d\varphi_{jj} = \frac{\varphi_{jj}}{\varphi_{ji}} d\varphi_{ji} - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj} d\tau_{Tji}$ which, using (B-3), (B-21), and $d\tau_{Tji} = d\tau_{Xi}$ when $i = H, j = F$, can be written as:

$$d\varphi_{jj} = \frac{\varphi_{jj}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1 - \delta_{ii}} dC_{ii} - \frac{\varepsilon}{\varepsilon - 1}\varphi_{jj} d\tau_{Xi}, \quad i = H, j = F \tag{B-22}$$

Finally, using (B-3) to express $d\varphi_{ij}$ together with (B-22) to substitute out $d\varphi_{jj}$ we have:

$$d\varphi_{ij} = \frac{\delta_{jj}}{1 - \delta_{jj}}\varphi_{ij}\left(\frac{1}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1 - \delta_{ii}} dC_{ii} + \frac{\varepsilon}{\varepsilon - 1} d\tau_{Xi}\right), \quad i = H, j = F \tag{B-23}$$

Using (B-21), (B-23), $\delta_{jj} = \delta_{ii}$, and $d\tau_{Tij} = d\tau_{Ii}$ when $i = H, j = F$, we can rewrite (B-20) as follows:

$$(1 - \delta_{ii}) dW_i - (1 - \delta_{ii}) d\tau_{Ii} + \delta_{ii} d\tau_{Xi} + \frac{1 - 2\delta_{ii}}{(1 - \delta_{ii})\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\varepsilon - 1}{\varepsilon} \frac{dC_{ii}}{C_{ii}} = 0, \quad i = H, j = F \tag{B-24}$$

(2) To retrieve the second condition, recall that (12) simplifies to (A-20) which, using (10) and (11) together with $L_{Ci} = L$ for $i = H, F$, $\delta_{ji} = 1 - \delta_{ii}$ for $i, j = H, F$, $\tau_{Ij} = \tau_{Xj} = 1$ for $j = F$ can be rewritten as:

$$L(1 - \delta_{ii})\tau_{Xi}\tau_{Li} W_i - L(1 - \delta_{jj})\tau_{Lj} = 0, \quad i = H, j = F \tag{B-25}$$

Using (B-4) for $i = j$ together with (B-22) we can write:

$$d\delta_{jj} = \delta_{ii}(\varepsilon - 1 + \Phi_i)\left(\frac{\varepsilon}{\varepsilon - 1} d\tau_{Xi} - \frac{\delta_{ii}}{1 - \delta_{ii}} \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{dC_{ii}}{C_{ii}}\right) \tag{B-26}$$

Taking the total differential of (B-25) and using (B-26) to substitute out $d\delta_{jj}$, and (B-4) to substitute out $d\delta_{ii}$, and evaluating it at the laissez-faire, we have:

$$dW_i + \left(1 + \frac{\delta_{ii}\varepsilon(\varepsilon - 1 + \Phi_i)}{(\varepsilon - 1)(1 - \delta_{ii})}\right) d\tau_{Xi} - \frac{\delta_{ii}(\varepsilon - 1 + \Phi_i)}{C_{ii}(1 - \delta_{ii})^2(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1}\Phi_i)} dC_{ii} = 0, \quad i = H, j = F \tag{B-27}$$

(3) We can rewrite (B-5) by imposing $dL_{Cj} = 0$, using (B-23) to substitute out $d\varphi_{ij}$, and $\delta_{ij} = 1 - \delta_{jj}$ (implying $d\delta_{ij} = -d\delta_{jj}$) together with (B-26) to substitute out $d\delta_{ij}$ we obtain:

$$\frac{\varepsilon - 1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} + \frac{\delta_{ii}}{1 - \delta_{ii}}\left(\frac{\varepsilon(\varepsilon - 1 + \Phi_i)}{\varepsilon - 1} - 1\right) d\tau_{Xi} + \frac{\delta_{ii}^2}{(1 - \delta_{ii})^2}\frac{1}{\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1}\Phi_i}\left(\frac{\varepsilon - 1}{\varepsilon} - \varepsilon + 1 - \Phi_i\right)\frac{dC_{ii}}{C_{ii}} = 0$$
$$i = H, j = F \tag{B-28}$$

Conditions (B-24), (B-27), and (B-28) can be used to derive an explicit solution for W_i, dC_{ii} and dC_{ij} as linear functions of $d\tau_{Ii}$ and $d\tau_{Xi}$. These expressions are available upon request.

Finally, consider the case with homogeneous firms. In this case, we need to express dW_i as a function of $d\tau_{Ii}$ and $d\tau_{Xi}$. For this purpose, we can substitute conditions (A-21) and (A-22) into the trade balance (A-20). Taking the total differential of this condition and evaluating it at the free-trade allocation we get:

$$dW_i = \frac{\varepsilon\tau^\varepsilon}{\tau + (2\varepsilon - 1)\tau^\varepsilon} d\tau_{Ii} - \frac{\tau + (\varepsilon - 1)\tau^\varepsilon}{(\tau + (2\varepsilon - 1)\tau^\varepsilon)} d\tau_{Xi} \tag{B-29}$$

C The Planner Allocation

In this appendix we first set up the planner problem and solve it using a three-stage approach. Next, we prove the Lemmata of Section 3.

C.1 The Planner Problem

The full planner problem can be written as follows. The planner maximizes:

$$\sum_{i=H,F} U_i = \sum_{i=H,F}\left[\alpha \log\left(\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} + (1 - \alpha)\log Z_i\right]$$

with respect to C_{ij},L_{Cij}, Z_i, N_i, $c_{ij}(\varphi)$, $l_{ij}(\varphi)$, φ_{ij}, for $i, j = H, F$ and subject to:

$$C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F$$

$$l_{ij}(\varphi) = \frac{\tau_{ij}c_{ij}(\varphi)}{\varphi} + f_{ij}, \quad i, j = H, F$$

$$L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i, j = H, F$$

$$L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i = H, F$$

$$\sum_{i=H,F} L_i = \sum_{i=H,F} L_{Ci} + \sum_{i=H,F} Z_i$$

Notice that by combining L_{Cij} and $l_{ij}(\varphi)$ we get:

$$L_{Cij} = \tau_{ij} N_j \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} dG(\varphi) + N_j f_{ij}(1 - G(\varphi_{ij})), \quad i,j = H,F$$

This problem can be split into three separate stages. The proof that this approach is equivalent to solving the full planner problem in a single stage is available on request.

C.2 First stage

C.2.1 First-stage optimality conditions

At the first stage the planner chooses $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij} for $i,j = H,F$ by solving the following problem:

$$\begin{aligned} & \max u_{ij} \\ s.t. \quad & c_{ij}(\varphi) = q_{ij}(l_{ij}(\varphi)), \quad i,j = H,F \\ & L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i,j = H,F, \end{aligned} \tag{C-1}$$

where $u_{ij} \equiv C_{ij}$, $C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}$, $q_{ij}(l_{ij}(\varphi)) = (l_{ij}(\varphi) - f_{ij}) \frac{\varphi}{\tau_{ij}}$, and where N_j and L_{Cij} – defining the amount of labor allocated in country j to produce differentiated goods consumed by country i – are taken as given since they are determined at the second stage.

Taking total differentials with respect to $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij}:

$$du_{ij} = \int_{\varphi_{ij}}^{\infty} \frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} dc_{ij}(\varphi) dG(\varphi) + \frac{\partial u_{ij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{C-2}$$

$$dc_{ij}(\varphi) = \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} dl_{ij}(\varphi) \tag{C-3}$$

$$N_j \int_{\varphi_{ij}}^{\infty} dl_{ij}(\varphi) dG(\varphi) + \frac{\partial L_{Cij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{C-4}$$

for $i,j = H,F$. By using (C-3) and (C-4) to substitute out $d\varphi_{ij}$ from (C-2) we get:

$$\int_{\varphi_{ij}}^{\infty} \left(\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} - \frac{\frac{\partial u_{ij}}{\partial \varphi_{ij}}}{\frac{\partial L_{Cij}}{\partial \varphi_{ij}}} \frac{N_j}{\frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)}} \right) dc_{ij}(\varphi) dG(\varphi) = 0, \quad i,j = H,F \tag{C-5}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} = \frac{\partial u_{ij}}{\partial L_{Cij}} N_j, \quad i,j = H,F$$

for all $\varphi \in [\varphi_{ij}, \infty)$. As a consequence:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi_1)} \frac{\partial q_{ij}(\varphi_1)}{\partial l_{ij}(\varphi_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\varphi_2)} \frac{\partial q_{ij}(\varphi_2)}{\partial l_{ij}(\varphi_2)},$$

for any $\varphi_1 \in [\varphi_{ij}, \infty)$ and $\varphi_2 \in [\varphi_{ij}, \infty)$.

C.2.2 First-stage aggregate production function

Using the functional forms, we get:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} = N_j C_{ij}^{\frac{1}{\varepsilon}} c_{ij}(\varphi)^{-\frac{1}{\varepsilon}}$$
$$\frac{\partial u_{ij}}{\partial \varphi_{ij}} = \frac{\varepsilon}{\varepsilon-1} N_j C_{ij}^{\frac{1}{\varepsilon}} c_{ij}(\varphi_{ij})^{\frac{\varepsilon-1}{\varepsilon}}$$
$$\frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} = \frac{\varphi}{\tau_{ij}}$$
$$\frac{\partial L_{Cij}}{\partial \varphi_{ij}} = N_j l_{ij}(\varphi_{ij})$$

Plugging in these functional forms into (C-5) we obtain:

$$\int_{\varphi_{ij}}^{\infty} \left(c_{ij}(\varphi)^{-1/\varepsilon} - \frac{\varepsilon}{\varepsilon-1} \frac{\tau_{ij}}{\varphi} \frac{c_{ij}(\varphi_{ij})^{\frac{\varepsilon-1}{\varepsilon}}}{l_{ij}(\varphi_{ij})} \right) dc_{ij}(\varphi) dG(\varphi) = 0, \quad i,j = H,F \tag{C-6}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$c_{ij}(\varphi) = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} \frac{c_{ij}(\varphi_{ij})^{1-\varepsilon}}{l_{ij}(\varphi_{ij})^{-\varepsilon}} \tau_{ij}^{-\varepsilon} \varphi^{\varepsilon}, \quad i,j = H,F \tag{C-7}$$

Substituting (C-7) into the definition of C_{ij}, using the definition of $\widetilde{\varphi}_{ij}$, and noting that $N_{ij} = [1 - G(\varphi_{ij})]N_j$, we get:

$$c_{ij}(\varphi_{ij})^{1-\varepsilon} = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon} l_{ij}(\varphi_{ij})^{-\varepsilon} \tau_{ij}^{\varepsilon} \widetilde{\varphi}_{ij}^{-\varepsilon}, \quad i,j = H,F$$

If we substitute this back into (C-7) we obtain:

$$c_{ij}(\varphi) = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi}\right)^{-\varepsilon}, \quad i,j = H,F \tag{C-8}$$

Finally, we can aggregate the production function as follows:

$$\begin{aligned} L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi) &= \tau_{ij} N_{ij} \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} + f_{ij} N_{ij} \\ &= \tau_{ij} N_{ij}^{-\frac{1}{\varepsilon-1}} \frac{C_{ij}}{\widetilde{\varphi}_{ij}} + f_{ij} N_{ij}, \quad i,j = H,F \end{aligned} \tag{C-9}$$

This leads to the aggregate production function (15) in the main text:

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1 - G(\varphi_{ij})]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij} [N_j(1 - G(\varphi_{ij})]^{\frac{\varepsilon}{\varepsilon-1}} \right\}, \quad i,j = H,F,$$

where $Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) = C_{ij}$.

C.2.3 First-stage comparison between planner and market allocation

We want to verify that the consumption of individual varieties chosen by the planner coincides with the one of the market allocation conditional on C_{ij}, N_{ij} and $\tilde{\varphi}_{ij}$ being the same. Recall that the demand function is

$c_{ij}(\varphi) = \left(\frac{p_{ij}(\varphi)}{P_{ij}}\right)^{-\varepsilon} C_{ij}$. Since the price index is given by $P_{ij} = N_{ij}^{\frac{1}{1-\varepsilon}} p_{ij}(\tilde{\varphi}_{ij})$, it follows that $\frac{p_{ij}(\varphi)}{p_{ij}(\tilde{\varphi}_{ij})} = \frac{\tilde{\varphi}_{ij}}{\varphi}$. Thus, we can conclude that condition (C-8) holds also in the market equilibrium.

C.2.4 First-stage optimality conditions with homogeneous firms

In this case, the problem stated in (C-1) simplifies to choosing $c_{ij}(\omega)$ and $l_{ij}(\omega)$ for $i, j = H, F$ under the assumptions that $G(\varphi)$ is a degenerate distribution and that $f_{ij} = 0$ for $i, j = H, F$.

Solving this problem gives the same condition as derived with heterogeneous firms:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\omega_1)} \frac{\partial q_{ij}(\omega_1)}{\partial l_{ij}(\omega_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\omega_2)} \frac{\partial q_{ij}(\omega_2)}{\partial l_{ij}(\omega_2)}$$

This implies that all firms will employ the same quantity of labor and produce the same amount of each variety, i.e., $l_{ij}(\omega) = l_{ij}$ and $c_{ij}(\omega) = c_{ij} \quad \forall \omega \in [0, N_j]$.

C.2.5 First-stage aggregate production function with homogeneous firms

Following the same steps as with heterogeneous firms we can derive the aggregate level of consumption

$$C_{ij} = N_j^{\frac{\varepsilon}{\varepsilon-1}} c_{ij}, i, j = H, F, \tag{C-10}$$

Hence, the aggregate production now simplifies to:

$$Q_{Cij}(N_j, L_{Cij}) \equiv \frac{1}{\tau_{ij}} N_j^{\frac{1}{\varepsilon-1}} L_{Cij}, \quad i, j = H, F, \tag{C-11}$$

where $Q_{Cij}(N_j, L_{Cij}) = C_{ij}$.

C.2.6 First-stage comparison between planner and market allocation with homogeneous firms

As for the case of heterogeneous firms, it is sufficient to recall that when firms are homogeneous $c_{ij} = \left(\frac{p_{ij}}{P_{ij}}\right)^{-\varepsilon} C_{ij}$ and $P_{ij} = N_{ij}^{\frac{1}{1-\varepsilon}} p_{ij}$, which implies that condition (C-10) holds also in the market equilibrium.

C.3 Second Stage

C.3.1 Second-stage optimality conditions

At the second stage, the planner chooses C_{ij}, L_{Cij}, N_i and $\widetilde{\varphi}_{ij}$ for $i, j = H, F$ in order to solve the following problem:

$$\max \sum_{i=H,F} u_i \tag{C-12}$$

$$\begin{aligned} s.t. \quad & L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i = H, F \\ & C_{ij} = Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij}), \quad i, j = H, F, \end{aligned}$$

where $u_i = \log C_i$, C_i is given by (4) and $Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij})$ is defined in (15).

Taking total differentials:

$$\sum_{i=H,F}\sum_{j=H,F}\frac{\partial u_i}{\partial C_{ij}}dC_{ij}=0$$

$$dN_i=-\frac{1}{f_E}\sum_{j=H,F}dL_{Cji},\quad i=H,F$$

$$dC_{ij}=\frac{\partial Q_{Cij}}{\partial N_j}dN_j+\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}d\tilde{\varphi}_{ij}+\frac{\partial Q_{Cij}}{\partial L_{Cij}}dL_{Cij},\quad i,j=H,F$$

Substituting the differentials of the constraints into the objective, we obtain:

$$\sum_{i=H,F}\sum_{j=H,F}\frac{\partial u_i}{\partial C_{ij}}\left[\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}d\tilde{\varphi}_{ij}+\frac{\partial Q_{Cij}}{\partial L_{Cij}}dL_{Cij}-\frac{\partial Q_{Cij}}{\partial N_j}\frac{1}{f_E}\sum_{k=H,F}dL_{Ckj}\right]=0$$

Collecting terms:

$$\sum_{j=H,F}\sum_{i=H,F}\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}d\tilde{\varphi}_{ij}+\sum_{j=H,F}\sum_{i=H,F}\left[\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}}-\sum_{k=H,F}\frac{\partial u_k}{\partial C_{kj}}\frac{\partial Q_{Ckj}}{\partial N_j}\frac{1}{f_E}\right]dL_{Cij}=0 \tag{C-13}$$

Since (C-13) should hold for any $d\tilde{\varphi}_{ij}$ and dL_{Cij} it follows that:

$$\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}=0,\quad i,j=H,F \tag{C-14}$$

$$\sum_{k=H,F}\frac{\partial u_k}{\partial C_{kj}}\frac{\partial Q_{Ckj}}{\partial N_j}=f_E\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}},\quad i,j=H,F,$$

The first-order conditions can be rewritten as:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}}=\frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}},\quad i,j=H,F,\quad i\neq j \tag{C-15}$$

$$f_E=\sum_{j=H,F}\frac{\partial Q_{Cji}/\partial N_i}{\partial Q_{Cji}/\partial L_{Cji}},\quad i=H,F \tag{C-16}$$

$$\frac{\partial Q_{Cji}}{\partial\tilde{\varphi}_{ji}}=0,\quad i,j=H,F \tag{C-17}$$

C.3.2 Second-stage aggregate production function

Using the functional forms, we obtain the following derivatives:

$$\frac{\partial u_i}{\partial C_{ij}} = \frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F} C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}} C_i^{-1}, \quad i,j = H,F \tag{C-18}$$

$$\frac{\partial Q_{Cji}}{\partial N_i} = \frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji}))]^{\frac{2-\varepsilon}{\varepsilon-1}} \frac{L_{Cji}}{(\varepsilon-1)}(1-G(\varphi_{ji})) - \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}} \left(\frac{\varepsilon}{\varepsilon-1}\right)(1-G(\varphi_{ji})), \quad i,j = H,F$$

$$\frac{\partial Q_{Cji}}{\partial \tilde{\varphi}_{ji}} = \frac{1}{\tau_{ji}}\left\{[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}} L_{Cji} - f_{ji}[N_i(1-G(\varphi_{ji})]^{\frac{\varepsilon}{\varepsilon-1}}\right\} - \frac{[N_i(1-G(\varphi_{ji})]^{\frac{2-\varepsilon}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ij}^{\varepsilon-1})} L_{Cji}(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1} N_i$$

$$+ \frac{f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})}\varepsilon(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1} N_i, \quad i,j = H,F$$

$$\frac{\partial Q_{Cji}}{\partial L_{Cji}} = \frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F$$

Then, these conditions can be substituted into (C-17) to obtain:

$$L_{Cji}\left(1 - \frac{\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})}\right) = f_{ji}[N_i(1-G(\varphi_{ji}))]\left(1 - \frac{\varepsilon\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})}\right), \quad i,j = H,F$$

It follows that:

$$L_{Cji} = f_{ji}N_i(1-G(\varphi_{ji}))\left(\frac{\varphi_{ji}^{\varepsilon-1} + (\varepsilon-1)\tilde{\varphi}_{ji}^{\varepsilon-1}}{\varphi_{ji}^{\varepsilon-1}}\right), \quad i,j = H,F \tag{C-19}$$

Moreover, combining the derivatives above with condition (C-16) we obtain:

$$f_E = \sum_{j=H,F}\left[\frac{L_{Cji}}{N_i(\varepsilon-1)} - \frac{\varepsilon}{(\varepsilon-1)} f_{ji}(1-G(\varphi_{ji}))\right] \tag{C-20}$$

This implies that

$$\varepsilon N_i f_E + \sum_{j=H,F} \varepsilon(1-G(\varphi_{ji}))N_i f_{ji} = f_E N_i + \sum_{j=H,F} L_{Cji}, \quad i = H,F. \tag{C-21}$$

Using (C-19) and $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out L_{Cji} and $f_E N_i$ in (C-21), we find:

$$L_{Ci} = \sum_{j=H,F} \varepsilon f_{ji} N_i (1-G(\varphi_{ji}))\left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}, \quad i = H,F \tag{C-22}$$

We use this last condition to solve for N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F}\left[f_{ji}(1-G(\varphi_{ji}))\left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}\right]}, \quad i = H,F \tag{C-23}$$

We now substitute (C-19) and (C-23) into the definition (15) to obtain (16) in the main text.

C.3.3 Second-stage comparison between planner and market allocation

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation. First, consider condition (C-15). Plugging the relevant derivatives in (C-18), we obtain:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ii}}{\tau_{ii}}[N_i(1-G(\varphi_{ii})]^{\frac{1}{\varepsilon-1}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}}, \quad i=H,F, \quad j\neq i \qquad \text{(C-24)}$$

Now consider the market allocation. Using (7) jointly with (8), (10) and (A-18) after some manipulations we get:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ii}}{\tau_{ii}}[N_i(1-G(\varphi_{ii})]^{\frac{1}{\varepsilon-1}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}}\left(\frac{C_j}{C_i}\frac{\tau_{ji}}{\tau_{ii}}\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{-1}{\varepsilon}} i=H,F, j\neq i$$

Thus, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \Omega_{P2ji}\frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i=H,F, \quad j\neq i, \qquad \text{(C-25)}$$

where Ω_{P2ji} is the wedge between the planner and the market allocation. Under symmetry:

$$\Omega_{P2ji} = \left(\frac{\tau_{ji}}{\tau_{ii}}\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{-1}{\varepsilon}}, \quad i=H,F, \quad j\neq i$$

Using condition (8), this can be written as $\Omega_{P2ji} = \tau_{Tji}^{-1}$.

Next, consider the planner's optimality condition (C-16). Using the functional forms from (C-18), this corresponds to (C-20).

We now want to check if this condition is also fulfilled by the market allocation. Recalling the labor market clearing condition in (A-17) and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (C-20) and this proves that (C-16) is satisfied in any market allocation.

Finally, consider the planner's optimality condition (C-17). As shown in Section C.3.2, this condition can be rewritten as (C-19). Now consider the market allocation. Appendix C.2.3 shows that condition (C-8) holds in the market equilibrium. As a consequence, also condition (C-9) holds in the market equilibrium. We can then use (C-9) and substitute it in equation (10) to obtain (C-19). This confirms that this condition and then (C-17) always holds both in the planner and in the market allocation.

C.3.4 Second-stage optimality conditions with homogeneous firms

In this case, the problem is stated in (C-12) where $C_{ij} = Q_{Cij}(N_j, L_{Cij})$ simplifies to (C-11) and the planner chooses C_{ij}, L_{Cij}, N_i for $i,j=H,F$ only, leading to conditions (C-15) and (C-16).

C.3.5 Second-stage aggregate production function with homogeneous firms

We can use the functional forms to find the aggregate production function. As a first step, we obtain the following derivatives:

$$\frac{\partial u_i}{\partial C_{ij}} = \frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F} C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}} C_i^{-1}, \quad i,j=H,F$$

$$\frac{\partial Q_{Cji}}{\partial N_i} = \frac{1}{\tau_{ji}} N_i^{\frac{2-\varepsilon}{\varepsilon-1}}\frac{L_{Cji}}{\varepsilon-1}, \quad i,j=H,F$$

$$\frac{\partial Q_{Cji}}{\partial L_{Cji}} = \frac{1}{\tau_{ji}} N_i^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F$$

Substituting these functional forms into (C-15) and (C-16), we obtain:

$$C_{ji} = \tau_{ij}^{-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} C_{ii}, \quad i,j = H,F \tag{C-26}$$

and

$$f_E = \sum_{j=H,F} \frac{L_{Cji}}{N_i(\varepsilon-1)}, \quad i = H,F \tag{C-27}$$

Using $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out the term $\sum_{j=H,F} L_{Cji}$ we obtain:

$$N_i = \frac{L_{Ci}}{\varepsilon f_E}, \quad i = H,F \tag{C-28}$$

We can then substitute the first-stage aggregate production function (C-11) into (C-26) to get:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Cii}, \quad i = H,F, j \neq i \tag{C-29}$$

Substituting this into the labor market clearing $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ and using condition (C-28), we find that:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Ci} \frac{\varepsilon-1}{\varepsilon} \left[\sum_{k=H,F} \tau_{ki}^{1-\varepsilon} \left(\frac{C_i}{C_k}\right)^{\varepsilon-1}\right]^{-1}, \quad i,j = H,F$$

Using again the definition of the first-stage aggregate production function (C-11), we get

$$Q_{ij}(L_{Ci}, L_{Cj}) = \frac{\varepsilon-1}{\varepsilon} \tau_{ij}^{-\varepsilon} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{C_j}{C_i}\right)^{\varepsilon-1} \left[\sum_{k=H,F} \tau_{kj}^{1-\varepsilon} \left(\frac{C_j}{C_k}\right)^{\varepsilon-1}\right]^{-1}, \quad i,j = H,F \tag{C-30}$$

where $Q_{ij}(L_{Ci}, L_{Cj}) = C_{ij}$.

C.3.6 Second-stage comparison between planner and market allocation with homogeneous firms

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation.

First, consider condition (C-26), which can be written as:

$$\frac{1}{C_i} \left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j} \left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}} \frac{1}{\tau_{ji}}, \quad i = H,F, \quad j \neq i \tag{C-31}$$

Now consider the market allocation. From the demand functions we get

$$\frac{C_{ii}}{C_{ji}} = \left(\frac{P_{ii}}{P_{ji}}\right)^{-\varepsilon} \left(\frac{C_i}{C_j}\right)^{1-\varepsilon} \left(\frac{P_i C_i}{P_j C_j}\right)^{\varepsilon}, \quad i = H,F, \quad j \neq i$$

This can also be written as:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}} \frac{1}{\tau_{ji}}\tau_{ji}\frac{P_{ii}}{P_{ji}}\frac{P_jC_j}{P_iC_i}, \quad i = H, F, \quad j \neq i$$

In other words, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}\Omega_{P2ji}, \quad i = H, F, \quad j \neq i$$

where $\Omega_{P2ji} \equiv \tau_{Tji}^{-1}\frac{P_jC_j}{P_iC_i}$ is the wedge between the planner and the market allocation. Under symmetry $\Omega_{P2ji} = \tau_{Tji}^{-1}$.

Next, consider the planner's optimality condition (C-27). We now want to check if this condition is also fulfilled in the market allocation. Recalling the labor market clearing requires $L_{Ci} = \varepsilon f_E N_i$ and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (C-27) and this proves that this condition is satisfied in any market allocation.

C.4 Third stage

C.4.1 Third-stage optimality conditions

The third stage is present only in the case of multiple sectors ($\alpha < 1$). In this stage, the planner chooses C_{ij} and Z_i for $i, j = H, F$, and the amount of aggregate labor allocated to the differentiated sector L_{Ci} to solve the following maximization problem:[58]

$$\begin{aligned} \max \sum_{i=H,F} U_i \qquad\qquad (\text{C-32}) \\ s.t. \quad C_{ij} = Q_{Cij}(L_{Cj}), \quad i, j = H, F \\ Q_{Zi} = Q_{Zi}(L - L_{Ci}), \quad i = H, F \\ \sum_{i=H,F} Q_{Zi} = \sum_{i=H,F} Z_i, \end{aligned}$$

where U_i is given by (3) and (4), $Q_{Zi}(L - L_{Ci}) = L - L_{Ci}$ and $Q_{Cij}(L_{Cj})$ is defined in (16). Taking total differentials of the objective function and of the constraints, we get:

$$\begin{aligned} \sum_{i=H,F} dU_i &= \sum_{i=H,F}\sum_{j=H,F}\frac{\partial U_i}{\partial C_{ij}}dC_{ij} + \sum_{i=H,F}\frac{\partial U_i}{\partial Z_i}dZ_i \\ dC_{ij} &= \frac{\partial Q_{Cij}}{\partial L_{Cj}}dL_{Cj}, \quad i, j = H, F \\ dQ_{Zi} &= \frac{\partial Q_{Zi}}{\partial L_{Ci}}dL_{Ci}, \quad i = H, F \\ \sum_{i=H,F} dQ_{Zi} &= \sum_{i=H,F} dZ_i \end{aligned}$$

[58]We state the third stage of the planner problem as a choice between C_{ij} and Z_i (instead of a choice between C_i and Z_i) because this enables us to identify the efficiency wedges in the welfare decomposition, as will become clear below.

Substituting the total differentials of the constraints into the total differential of the objective and rearranging terms, we obtain:

$$\sum_{k=H,F} dU_k = \sum_{k=H,F} \left[\sum_{l=H,F} \frac{\partial U_l}{\partial C_{lk}} \frac{\partial Q_{Clk}}{\partial L_{Ck}} + \frac{\partial U_k}{\partial Z_k} \frac{\partial Q_{Zk}}{\partial L_{Ck}} \right] dL_{Ck} + \left[\frac{\partial U_i}{\partial Z_i} - \frac{\partial U_j}{\partial Z_j} \right] dZ_j, \quad i = H, \quad j = F$$

It follows that at the optimum each term needs to equal zero, which leads to the following optimality conditions:

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, j = F \tag{C-33}$$

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F \tag{C-34}$$

C.4.2 Third-stage comparison between planner and market allocation

Here, we compare the market allocation with the allocation emerging from the third stage of the planner problem. Using the functional forms, we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial Z_i} &= \frac{1-\alpha}{Z_i}, \quad i = H, F \\ \frac{\partial U_j}{\partial C_{ji}} &= \alpha C_{ji}^{\frac{-1}{\varepsilon}} C_j^{-\frac{\varepsilon-1}{\varepsilon}} \quad i, j = H, F \\ \frac{\partial Q_{Cji}}{\partial L_{Ci}} &= \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Ci}}, \quad i, j = H, F \\ \frac{\partial Q_{Zi}}{\partial L_{Ci}} &= -1, \quad i = H, F \end{aligned} \tag{C-35}$$

First consider condition (C-33). Using (C-35) we get that $(1-\alpha)Z_j = (1-\alpha)Z_i$. This condition is satisfied in any symmetric market allocation.

Next consider condition (C-34). Using (C-35) we obtain:

$$\sum_{j=H,F} \frac{\alpha}{1-\alpha} \frac{Z_i}{L_{Ci}} \frac{1}{C_j} \left(\frac{C_{ji}}{C_j} \right)^{-\frac{1}{\varepsilon}} \frac{\varepsilon}{\varepsilon - 1} C_{ji} = 1, \quad i = H, F \tag{C-36}$$

From (A-1) and (A-3) the price of the differentiated bundle in the market allocation is given by:

$$P_{ji} = \frac{\alpha}{1-\alpha} Z_j \left(\frac{C_{ji}}{C_j} \right)^{-\frac{1}{\varepsilon}} \frac{1}{C_j}$$

Substituting the price into (C-36) we have:

$$\sum_{j=H,F} \frac{\varepsilon}{\varepsilon - 1} \frac{P_{ji} C_{ji}}{L_{Ci}} \frac{Z_i}{Z_j} = 1, \quad i = H, F \tag{C-37}$$

Finally recall that from (10) and (11) in the two-sector model we have:

$$P_{ji} C_{ji} = \delta_{ji} L_{Ci} \tau_{Tji} \tau_{Li}$$

so that (C-37) can be further rewritten as:

$$\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\left[1+\delta_{ji}\tau_{Tji}\frac{Z_i}{Z_j}\right]=1, \quad i=H,F, \quad j\neq i$$

We can thus define the third stage wedge between the planner and the market allocation as follows:

$$\Omega_{3Pi}\equiv\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\left[\delta_{ii}+\delta_{ji}\tau_{Tji}\frac{Z_i}{Z_j}\right]$$

In the symmetric allocation $\Omega_{3Pi}=1$ if $\tau_L=\frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij}=1$.

C.4.3 Third-stage optimality conditions with homogeneous firms

In the third stage, the planner chooses C_{ij}, Z_i and L_{Ci} for $i,j=H,F$ to solve a problem akin to problem (C-32) with the only difference that $Q_{Cij}(L_{Ci},L_{Cj})$ is implicitly defined in (C-30). Taking total differentials of the objective function and of the constraints we obtain conditions (C-33) and (C-34), like in the heterogeneous-firm case.

C.4.4 Third-stage comparison between planner and market allocation with homogeneous firms

As a first step, we show that at the optimum the derivatives implied by the functional forms are identical to those of the case with heterogeneous firms. While this is obvious for the first, the second and the fourth condition in (C-35), it needs to be proven for $\partial Q_{Cji}/\partial L_{Ci}$.

Taking total differentials of condition (C-30):

$$\begin{aligned} dQ_{Cij} &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ij}}{L_{Cj}}dL_{Cj} \\ &\quad + C_{ij}(\varepsilon-1)\left[\left(\frac{C_j}{C_i}\right)^{-1}d\left(\frac{C_j}{C_i}\right)-\left[\sum_{k=H,F}\tau_{kj}^{1-\varepsilon}\left(\frac{C_j}{C_k}\right)^{\varepsilon-1}\right]^{-1}\sum_{k=H,F}\tau_{kj}^{1-\varepsilon}\left(\frac{C_j}{C_k}\right)^{\varepsilon-2}d\left(\frac{C_j}{C_k}\right)\right], \quad i,j=H,F, \end{aligned}$$

$$d\left(\frac{C_i}{C_j}\right)=\left(\frac{C_i}{C_j}\right)^{\frac{1}{\varepsilon}}C_j^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{ii}^{\frac{-1}{\varepsilon}}dC_{ii}+C_{ij}^{\frac{-1}{\varepsilon}}dC_{ij}\right)-\left(\frac{C_i}{C_j}\right)^{\frac{2\varepsilon-1}{\varepsilon}}C_i^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{jj}^{\frac{-1}{\varepsilon}}dC_{jj}+C_{ji}^{\frac{-1}{\varepsilon}}dC_{ij}\right), \quad i,j=H,F$$

Notice that at the planner optimum, where the allocation is symmetric, this last condition equals zero not only for $i=j$ but also for $i\neq j$.

It follows that under symmetry

$$\frac{\partial Q_{Cji}}{\partial L_{Ci}}=\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ji}}{L_{Ci}}, \quad i,j=H,F,$$

while $\partial Q_{Cji}/\partial L_{Cj}=0$ as in the heterogeneous-firm case. We can now turn to the comparison between the planner and the market allocation.

Condition (C-33) is satisfied like in the case for heterogeneous firms. For condition (C-34) we have to compare the expression

$$Z_i\frac{\alpha}{1-\alpha}=L_{Ci}\frac{\varepsilon-1}{\varepsilon}, \quad i=H,F$$

with the corresponding condition in the market allocation. We know that in the market allocation the following holds:

$$Z_i \frac{\alpha}{1-\alpha} = P_i C_i = \sum_{j=H,F} P_{ij} C_{ij}, \quad i = H, F$$

Moreover, from (A-21) and (A-22), we get:

$$P_{ij} C_{ij} = L_{Cj} W_j \left(\tau_{ij} \tau_{Tij}\right)^{1-\varepsilon} \tau_{Lj} \frac{\left(\frac{W_k}{W_j}\frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j}\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon} \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}}, \quad i, j = H, F, \quad k \neq i$$

Hence:

$$\begin{aligned} Z_i \frac{\alpha}{1-\alpha} &= L_{Ci} \frac{\varepsilon-1}{\varepsilon} \sum_{j=H,F} \frac{\varepsilon}{\varepsilon-1} \frac{L_{Cj}}{L_{Ci}} W_j \left(\tau_{ij}\tau_{Tij}\right)^{1-\varepsilon} \tau_{Lj} \frac{\left(\frac{W_k}{W_j}\frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j}\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon} \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}} \\ &= L_{Ci} \frac{\varepsilon-1}{\varepsilon} \Omega_{3Pi}, \quad i = H, F, \quad k \neq i \end{aligned}$$

where Ω_{3Pi} is the wedge between the planner and the market allocation. In any symmetric allocation:

$$\Omega_{3Pi} = \frac{\varepsilon}{\varepsilon-1} \tau_L \sum_{j=H,F} \left(\tau_{ij}\tau_{Tij}\right)^{1-\varepsilon} \frac{1 - \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}}, \quad i = H, F, \quad k \neq i$$

which implies that $\Omega_{3P} = 1$ if $\tau_L = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij} = 1$ for $i, j = H, F$ since:

$$\Omega_{3Pi} = \frac{1 - \tau^{\varepsilon-1}}{\tau^{1-\varepsilon} - \tau^{\varepsilon-1}} \sum_{j=H,F} \tau_{ij}^{1-\varepsilon} = \frac{1 - \tau^{\varepsilon-1}}{\tau^{1-\varepsilon} - \tau^{\varepsilon-1}} (1 + \tau^{1-\varepsilon}) = 1, \quad i = H, F$$

C.5 Characterizing the planner allocation

The following lemma characterizes the properties of the planner allocation.

Lemma 8 ***The planner allocation***
The planner allocation is unique and symmetric.

Proof For future convenience note that the minimum set of conditions determining the Pareto efficient allocation for the multi-sector model consists of: i) the conditions that hold in both the homogeneous and the heterogeneous firm model, namely conditions (4), (C-33), (C-34), and the labor constraint, $Z_i + Z_j = 2L - L_{Ci} - L_{Cj}$; ii) the conditions which are model specific, namely conditions (C-30) and (C-31) in the case of homogeneous firms, and conditions (9) (obtained properly combining (C-21) and (C-22)), (6) and (7), and the following the zero cut-off condition:

$$\frac{\varphi_{ii}}{\varphi_{ji}} = \left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{1}{\varepsilon-1}} \frac{C_i}{C_j} \frac{1}{\tau_{ji}} \quad i = H, F \quad j \neq F \tag{C-38}$$

recovered by first using the first constraint in (C-1) and condition (C-7) evaluated at the cut-offs to substitute out $c(\varphi_{ij})$ and $l(\varphi_{ij})$ in condition (C-8) and then combining this condition with (15) and (C-24). When there is only one sector we drop (C-33) and (C-34) while the labor constraint simplifies to $L_{Ci} = L_{Cj} = L$.

What we need to show is that the planner problem has a unique and symmetric solution. We do that for all model versions considered, i.e., homogeneous and heterogeneous firms models with either one or multiple sectors. It is easy to verify that the symmetric allocation is always a solution of the above conditions. Thus, we only need to prove uniqueness.

Homogenous firms - one-sector model

First, note that by substituting (C-28) into (C-11) the second-stage aggregate production function can be written as:

$$C_{ji} = \tau_{ji}^{-1} L_{Ci}^{\frac{1}{\varepsilon-1}} (\varepsilon f_E)^{\frac{1}{1-\varepsilon}} L_{Cji}, \quad i,j = H,F \tag{C-39}$$

Substituting this into (C-26), we obtain:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii} \left(\frac{C_i}{C_j} \right)^{\varepsilon-1}, \quad i = H,F \quad j \neq i \tag{C-40}$$

Using (4) and substituting again (C-39), we find:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii} \left[\frac{L_{Ci}^{\frac{1}{\varepsilon}} L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cj}^{\frac{1}{\varepsilon}} L_{Cij}^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cj}^{\frac{1}{\varepsilon}} L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Ci}^{\frac{1}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\varepsilon}, \quad i = H,F \quad j \neq i \tag{C-41}$$

Combining the labor resource constraint with (C-27) and recalling that with a single sector $L_{Ci} = L$, we have

$$L_{Cji} = \frac{\varepsilon - 1}{\varepsilon} L - L_{Cii}, \quad i = H,F, \quad j \neq i$$

This last equation can be used to substitute out L_{Cji} and L_{Cij} from (C-41) in order to obtain a system of two equations in two variables:

$$F_i(L_{Cii}, L_{Cjj}) \equiv \left(\frac{L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{\varepsilon-1}{\varepsilon} L - L_{Cjj}\right]^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{\varepsilon-1}{\varepsilon} L - L_{Cii}\right]^{\frac{\varepsilon-1}{\varepsilon}}} \right)^{\varepsilon} - \frac{\frac{\varepsilon-1}{\varepsilon} L - L_{Cii}}{\tau_{ji}^{1-\varepsilon} L_{Cii}} = 0, \quad i = H,F \quad j \neq i \tag{C-42}$$

Note that $F_H()$ is monotonically increasing in L_{CHH} and monotonically decreasing in L_{CFF}, while exactly the opposite is true for $F_F()$. This implies that the functions $F_H()$ and $F_F()$ cross only once, i.e., there is a unique solution. More specifically, the unique solution is given by

$$L_{Cji} = \frac{\varepsilon - 1}{\varepsilon} \frac{\tau_{ji}^{1-\varepsilon}}{1 + \tau^{1-\varepsilon}} L, \quad i,j = H,F \tag{C-43}$$

The remaining variables and their symmetry follow immediately.

Homogenous firms - multi-sector model

For the multi-sector model, we also need to consider the third-stage optimality conditions (C-33) and (C-34). Using (C-35) they can be written as follows:

$$Z_i = Z, \quad i = H,F \tag{C-44}$$

$$L_{Ci} = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon - 1} Z \sum_{j=H,F} \left(\frac{C_{ji}}{C_j} \right)^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H,F \tag{C-45}$$

The second-stage aggregate production function (C-39) can be substituted in order to express this equation as:

$$C_i^{\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon - 1} Z (\varepsilon f_E)^{-\frac{1}{\varepsilon}} L_{Ci}^{-\frac{\varepsilon-1}{\varepsilon}} \left[L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_i}{C_j} \right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}} \right], \quad i = H,F \quad j \neq i \tag{C-46}$$

Taking the ratio of this expression for both countries:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cj}}{L_{Ci}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \frac{L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_j}{C_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cij}^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i = H, F \quad j \neq i \tag{C-47}$$

From (C-40) we get:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cji}}{L_{Cii}}\right)^{\frac{1}{\varepsilon}} \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-48}$$

Substituting this into (C-47), and using the fact that from (C-27) and (C-28) we have $\sum_{j=H,F} L_{Cji} = \frac{\varepsilon-1}{\varepsilon} L_{Ci}$, we get:

$$L_{Cji} = \tau_{ij}^{1-\varepsilon} \frac{L_{Ci}}{L_{Cj}} L_{Cjj}, \quad i = H, F \quad j \neq i \tag{C-49}$$

Substituting this expression again into (C-48) to write this equation in terms of L_{Cii} and L_{Cjj}, we get:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Ci}}{L_{Cj}} \frac{L_{Cjj}}{L_{Cii}}\right)^{\frac{1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-50}$$

Combining instead (C-47) with (C-49), we obtain:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cj}}{L_{Ci}} \frac{L_{Cii}}{L_{Cjj}}\right)^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-51}$$

From (C-50) and (C-51) it follows that $\frac{L_{Cii}}{L_{Cjj}} = \frac{L_{Ci}}{L_{Cj}}$ for $i = H, F$, $j \neq i$. Substituting this back into (C-50) it follows that $C_i = C$ for $i = H, F$.

Then from (C-49), we find:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii}, \quad i = H, F \quad j \neq i \tag{C-52}$$

Using $L_{Cii} + L_{Cji} = \frac{\varepsilon-1}{\varepsilon} L_{Ci}$ together with (C-49) and $\frac{L_{Cii}}{L_{Cjj}} = \frac{L_{Ci}}{L_{Cj}}$, we get:

$$L_{Cji} = \frac{\varepsilon-1}{\varepsilon} \frac{\tau_{ji}^{1-\varepsilon}}{1+\tau^{1-\varepsilon}} L_{Ci}, \quad i, j = H, F \tag{C-53}$$

Using this with (C-39):

$$C_{ji} = \frac{\varepsilon-1}{\varepsilon} (\varepsilon f_E)^{\frac{1}{1-\varepsilon}} \frac{\tau_{ji}^{-\varepsilon}}{1+\tau^{1-\varepsilon}} L_{Ci}^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \tag{C-54}$$

Substituting (C-54) into (4) and using the fact that $C_i = C$ for $i = H, F$, we have that $L_{Ci} = L_C$ for $i = H, F$. It then follows easily that $L_{Cii} = L_{Cjj}$, $L_{Cij} = L_{Cji}$, $C_{ii} = C_{jj}$, and $C_{ij} = C_{ji}$ for $i = H, F$ and $j \neq i$. From the aggregate resource constraint it then follows that $Z = L - L_{Ci}$. From (C-54) is also follows that $\frac{C_{ii}}{C_{ji}} = \tau_{ji}^{\varepsilon}$ for $j \neq i$.

Imposing $C_i = C$, $C_{ij} = C_{ji}$, and $\frac{C_{ii}}{C_{ji}} = \tau_{ij}^{\varepsilon}$ for $i = H, F$ and $j \neq i$ in (C-45) we have:

$$L_{Ci} = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon-1} Z \left[\frac{1}{1+\tau^{1-\varepsilon}} + \frac{1}{1+\tau^{\varepsilon-1}}\right] = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon-1} Z, \quad i = H, F$$

Combining this last equation with $Z = L - L_{Ci}$, we find that $L_{Ci} = \frac{\alpha\varepsilon}{\varepsilon+\alpha-1} L$ for $i = H, F$, i.e., there exists a unique symmetric solution for L_{Ci}. Uniqueness and symmetry of the remaining variables then follow immediately.

Heterogeneous firms - one-sector model
Using the first constraint in (C-1) and condition (C-7) evaluated at the cut-offs to substitute out $c(\varphi_{ij})$ and $l(\varphi_{ij})$ in condition (C-8), and then combining this condition with (15) and (C-24) we obtain the following 2 equations:

$$\frac{C_i}{C_j} = \tau_{ji}\frac{\varphi_{ii}}{\varphi_{ji}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \quad j \neq i \tag{C-55}$$

Combining these two equations to eliminate C_i/C_j we obtain the following set of two equations in four variables:

$$\frac{\varphi_{ii}}{\varphi_{ji}}\frac{\varphi_{jj}}{\varphi_{ij}}\tau_{ji}^2\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{2}{\varepsilon-1}} - 1 = 0, \quad i = H,F \quad j \neq i \tag{C-56}$$

Using the definition of C_i, (4), and the second-stage aggregate production function, (16), we obtain:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{f_{ii}^{\frac{-1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ii} + f_{ij}^{\frac{-1}{\varepsilon}}\tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{ij}}{f_{jj}^{\frac{-1}{\varepsilon}}\varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{jj} + f_{ji}^{\frac{-1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ji}}, \quad i,j = H,F \quad j \neq i \tag{C-57}$$

Combining (C-55) with (C-57) we obtain:

$$\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} = \frac{f_{ii}^{\frac{-1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ii} + f_{ij}^{\frac{-1}{\varepsilon}}\tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{ij}}{f_{jj}^{\frac{-1}{\varepsilon}}\varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{jj} + f_{ji}^{\frac{-1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ji}}, \quad i,j = H,F \quad j \neq i \tag{C-58}$$

Given that $\delta_{ji} = 1 - \delta_{ii}$, $\delta_{ij} = 1 - \delta_{jj}$, that from (C-56) $\varphi_{ij} = \frac{\varphi_{ii}}{\varphi_{ji}}\varphi_{jj}\tau_{ji}^2\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{2}{\varepsilon-1}}$, and that in the one-sector model $L_{Ci} = L$ for $i = H,F$, (C-58) implies that:

$$\frac{2\delta_{ii}-1}{2\delta_{jj}-1} - \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon]}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} = 0, \quad i,j = H,F \quad j \neq i \tag{C-59}$$

We now want to show that there is a unique symmetric solution to (C-56) and (C-59). We do this by expressing these equations as implicit functions of φ_{ii} and φ_{jj} and showing that one relationship has a positive slope and the other one a negative slope, so that there is a unique intersection. In order to do this we use equation (B-3) that relates $d\varphi_{ji}$ to $d\varphi_{ii}$ for $i = H,F$, $j \neq i$, and equation (B-4) that relates $d\delta_{ji}$ to $d\varphi_{ji}$, for $i,j = H,F$.

Taking the total differential of (C-56) and using (B-3) we obtain:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = -(1-\delta_{ii})\left(\frac{\delta_{jj}}{1-\delta_{jj}}\frac{\varphi_{ij}\varphi_{ji}}{\varphi_{jj}^2\tau_{ji}^2}\left(\frac{f_{ji}}{f_{ii}}\right)^{-\frac{2}{\varepsilon-1}} + \frac{\varphi_{ii}}{\varphi_{jj}}\right) < 0, \quad i,j = H,F \quad j \neq i$$

Similarly, taking the total differential of (C-59) and using (B-3) and (B-4) we obtain:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = \frac{\frac{\varphi_{ii}}{\varphi_{jj}}\frac{1-\delta_{ii}}{\delta_{ii}}\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}}\left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\varepsilon - 1 + 2\delta_{jj}((\varepsilon-1)^2 + \varepsilon\Phi_j)\right)}{2(1-\delta_{ii})\varepsilon(\varepsilon - 1 + \Phi_i) + (\varepsilon-1)(2\delta_{jj}-1)\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ij}}{f_{jj}}\right)^{\frac{1}{\varepsilon}}\left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i,j = H,F, \quad j \neq i$$

The numerator is unambiguously positive. As for the denominator, it is also positive, as becomes clear when further simplifying it using (C-59):

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = \frac{\frac{\varphi_{ii}}{\varphi_{jj}}\frac{1-\delta_{ii}}{\delta_{ii}}\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}}\left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\varepsilon-1+2\delta_{jj}((\varepsilon-1)^2+\varepsilon\Phi_j)\right)}{(1-\delta_{ii})((\varepsilon-1)(2\varepsilon-1)+2\varepsilon\Phi_i)+\delta_{jj}(\varepsilon-1)} > 0, \quad i,j=H,F \quad j\neq i$$

Hence, while (C-56) is continuous and monotonically decreasing in the $(\varphi_{ii}, \varphi_{jj})$ space, the opposite is true for (C-59), implying that there exists a unique intersection and thus a unique combination of φ_{ii} and φ_{jj} consistent with the planner solution. From (B-3) and (B-4) we know that there is a monotonic relationship between φ_{ji} and φ_{ii} and between δ_{ij} and φ_{ij}. Therefore, there is a unique and symmetric solution for φ_{ij} and δ_{ij} with $i,j=H,F$. From (C-57) it then follows that $C_i = C_j$ for $i=H, j=F$. Uniqueness and symmetry of the remaining variables follows immediately.

Heterogeneous firms - multi-sector model
Observe that (C-56) holds also in the case of multiple sectors. Instead, this is not the case for (C-59) which was derived under the assumption that $L_{Ci} = L$ for $i = H, F$. Thus, we need to consider the third-stage optimality conditions, (C-44) and (C-45), to derive a second relationship between φ_{ii} and φ_{jj}.

Combining them with the second-stage aggregate production function C_{ij} (16), we find:

$$C_i^{\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha}{1-\alpha}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{\varepsilon}} Z\varepsilon^{-\frac{1}{\varepsilon}}\left[f_{ii}^{-\frac{1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{ii} + f_{ji}^{-\frac{1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{ji}\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right], \quad i=H,F \quad j\neq i$$

Dividing this by the corresponding equation for the other country:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{f_{ii}^{-\frac{1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{ii} + f_{ji}^{-\frac{1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{ji}\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{f_{jj}^{-\frac{1}{\varepsilon}}\varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{jj} + f_{ij}^{-\frac{1}{\varepsilon}}\tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\delta_{ij}\left(\frac{C_j}{C_i}\right)^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i=H,F \quad j\neq i \tag{C-60}$$

Substituting (C-55) into (C-60) and using the fact that $\delta_{ii} = 1-\delta_{ji}$ for $i = H, F$, $j \neq i$ we find:

$$\frac{\varphi_{ji}}{\varphi_{jj}} = \tau_{ji}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}, \quad i=H,F \quad j\neq i \tag{C-61}$$

Taking the total differential of (C-61) and using (B-3) we have:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = -\tau_{ji}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}\frac{1-\delta_{ii}}{\delta_{ii}}\frac{\varphi_{ii}}{\varphi_{jj}} < 0, \quad i,j=H,F \quad ,j\neq i \tag{C-62}$$

Similarly to the one-sector model, the planner solution needs to satisfy two equations, (C-56) and (C-61), both of which can be expressed as implicit functions of $(\varphi_{ii}, \varphi_{jj})$. We showed that both functions monotonically decrease in the $(\varphi_{ii}, \varphi_{jj})$ space, implying that they cross at most once, i.e., there is a unique solution. The remaining steps are the same as in the one-sector model. ■

C.6 Proof of Lemma 1

Proof We prove this proposition in two steps.

First, observe that conditions (C-15) and (C-33) and (C-34) (when $\alpha < 1$) are optimality conditions of the planner problem, and therefore are necessary conditions for the market equilibrium to coincide with the planner

allocation.
Second, we prove that if (C-15) and (C-33) and (C-34) (when $\alpha < 1$) hold, then the market allocation coincides with the planner allocation. If (C-15) holds, then as shown in Appendices C.2.3 and C.3.3 for the heterogeneous-firm model and Appendices C.2.4 and C.3.6 for the homogeneous-firm model, all the optimality conditions of the first and second stage of the planner problem are satisfied in the market equilibrium. Moreover, if for the case $\alpha < 1$ also conditions (C-33) and (C-34) are satisfied, then – as shown in Appendices C.4.2 and C.4.4 – all the optimality conditions of the third stage hold. As a consequence, the market equilibrium coincides with the planner allocation. ■

C.7 Proof of Lemma 2

Proof We prove Lemma 2 in two steps.

First we show that conditions (20) and (21) and – for the case of the multi-sector model – condition (22) are sufficient conditions for (17), (18) and (19) to hold in the market equilibrium. It is evident that with log utility condition (20) ($I_i = I_j$, $j \neq i$) implies condition (18). Moreover, utility maximization implies

$$P_{ij} = \frac{\partial U_i}{\partial C_{ij}} / \frac{\partial U_i}{\partial I_i} = \frac{\partial U_i}{\partial C_{ij}} I_i, \qquad i,j = H,F \tag{C-63}$$

Using this result with (21), we get:

$$\frac{\partial U_i}{\partial C_{ij}} I_i = \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} \quad i,j = H,F.$$

Taking ratios of this condition for $i \neq j$ and using condition (20), we obtain:

$$\frac{\partial u_j}{\partial C_{jj}} \frac{\partial Q_{Cjj}}{\partial L_{Cjj}} = \frac{\partial u_i}{\partial C_{ij}} \frac{\partial Q_{Cij}}{\partial L_{Cij}} \quad j = H,F, \quad i \neq j$$

which proves that (17) holds.

Finally, by condition (21), condition (22) can be rewritten as follows:

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = W_i, \Leftrightarrow \sum_{j=H,F} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = 1 \Rightarrow \sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H,F$$

where the last implication follows from conditions (20) and (C-63). This proves that (19) holds.

Second, we show that (20) and (21) and – in the multi-sector model – condition (22) are necessary conditions for (17), (18) and (19).

First, we consider condition (20). For the multi-sector model, it is straightforward to see that this is a necessary condition for the market equilibrium to be efficient: if condition (20) is not satisfied, condition (18) cannot be satisfied either. In the one sector model, showing necessity of condition (20) is a bit more involved. Suppose the market allocation is efficient. Then, by Lemma (8), this allocation must be symmetric. This implies that we can use the assumption for the one-sector model that $\frac{\tau_{Li} W_i}{\tau_{Lj} W_j} = 1$ for $i \neq j$. Consider first the heterogeneous-firm case: it must be that by condition (8) $\tau_{Tij}^{-1} = 1$ for $i = H,F$ and $j \neq i$ since only under these conditions the market cutoffs correspond to the efficient cutoffs determined by conditions (7), (6), (9), and (C-38) under symmetry. At the same time, it must be that $\delta_{ij} = \delta_{ji}$ for $i,j = H,F$. This allows us to conclude that:

$$\begin{aligned} I_i = \sum_{k=H,F} P_{ik} C_{ik} &= \tau_{Li} W_i \delta_{ii} L + \tau_{Lj} W_j \delta_{ij} L = \tau_{Li} W_i L = \\ &= \tau_{Lj} W_j L = \tau_{Li} W_i \delta_{ji} L + \tau_{Lj} W_j \delta_{jj} L = \sum_{k=H,F} P_{jk} C_{jk} = I_j \end{aligned}$$

Consider now the homogeneous firm case. By conditions (C-39) and (C-43) if the market allocation is efficient it must be that in equilibrium $C_{ji} = \frac{\varepsilon-1}{\varepsilon}(\varepsilon f_E)^{\frac{1}{1-\varepsilon}} \frac{\tau_{ji}^{-\varepsilon}}{1+\tau^{1-\varepsilon}} L^{\frac{\varepsilon}{\varepsilon-1}}$ for $i,j = H,F$. Then, by conditions (A-22) it must also be that:

$$\frac{1}{1+\tau^{1-\varepsilon}} = \frac{\tau_{Tij}^{-\varepsilon}\left[1-\tau_{Tki}^{\varepsilon}\tau^{\varepsilon-1}\right]}{\tau_{Tik}^{-\varepsilon}\tau^{1-\varepsilon} - \tau_{Tki}^{\varepsilon}\tau^{\varepsilon-1}}, \quad i,j = H,F \quad k \neq i$$

As a consequence, $1+\tau^{1-\varepsilon} = \tau_{Tik}^{-\varepsilon}\tau^{1-\varepsilon} + \tau_{Tki}^{\varepsilon}$ and $\tau_{Tik}^{-\varepsilon} = \tau_{Tki}^{\varepsilon}$ for $i = H,F$ and $k \neq i$ Hence we can conclude that the market equilibrium is efficient only if $\tau_{Tij} = 1$ for $i = H,F$ and $j \neq i$. Therefore, by condition (A-21) and (A-22) $P_{ij}C_{ij} = \tau_{Lj}W_j \frac{\tau_{ij}^{1-\varepsilon}}{1+\tau^{1-\varepsilon}} L$ for $i,j = H,F$ and thus $I_i = \sum_{k=H,F} P_{ik}C_{ik} = \sum_{k=H,F} P_{jk}C_{jk} = I_j$ for $i \neq j$.

We next prove that condition (21) is necessary for condition (17) to hold in the market equilibrium. Without loss of generality, at this point we can assume that $I_i = I_j$ in the market equilibrium. From (C-25) and (8), the following condition must hold in a symmetric market allocation:

$$\frac{\partial u_j}{\partial C_{jj}}\frac{\partial Q_{Cjj}}{\partial L_{Cjj}} = \frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}}\tau_{Tij}^{-1} \qquad j = H,F \quad i \neq j \tag{C-64}$$

Using condition (C-63), this equation can be written as $\frac{P_{jj}}{P_{ij}} = \frac{\partial L_{Cij}/\partial Q_{Cij}}{\partial L_{Cjj}/\partial Q_{Cjj}}\,\tau_{Tij}^{-1}$. Imposing that (21) must hold, it follows that condition (17) is satisfied in the market equilibrium only if $\tau_{Tij}^{-1} = 1$ for both $j = H,F$ and $i \neq j$. Thus, condition (17) holds only if conditions (21) is satisfied in equilibrium. Finally, suppose that (20) and (21) hold in the market equilibrium. Then, in the multi-sector model it follows that:

$$\sum_{j=H,F} P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon-1}\tau_{Li}W_i, \Leftrightarrow \sum_{j=H,F} P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon-1}\tau_{Li} \Leftrightarrow \sum_{j=H,F}\frac{\partial U_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\frac{\partial U_i}{\partial Z_i}\frac{\partial Q_{Zi}}{\partial L_{Ci}}$$

with $i = H,F$. Hence, condition (18) holds in the market equilibrium only if $\frac{\varepsilon}{\varepsilon-1}\tau_{Li} = 1$ for both $j = H,F$. Put differently, condition (18) holds only if conditions (22) is satisfied in equilibrium. ■

C.8 Decomposition of efficiency wedges

To prove (24) it suffices to add and subtract $\tau_{Ii}^{-1}P_{ij}$ and then use (23):

$$\begin{aligned} P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{\partial Q_{Cij}} &= P_{ij} - \tau_{Ii}^{-1}P_{ij} + \tau_{Ii}^{-1}P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{\partial Q_{Cij}} \quad i = H,F \quad j \neq i \\ &= (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij} + \tau_{Ii}^{-1}P_{ij} - \tau_{Tij}^{-1}P_{ij} \quad i = H,F \quad j \neq i \\ &= (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij} + (\tau_{Xj}-1)\,\tau_{Tij}^{-1}P_{ij} \quad i = H,F \quad j \neq i \\ &= \left(1-\tau_{Tij}^{-1}\right)P_{ij} \quad i = H,F \quad j \neq i \end{aligned}$$

Condition (25) follows directly from (23) and the fact that in the multi-sector model $W_i = 1$. Finally, to prove (26) first notice that from (23) we have:

$$\tau_{Tji}^{-1}P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -P_{ii}\frac{\partial Q_{Cii}}{\partial L_{Ci}} + \frac{\varepsilon}{\varepsilon-1}\tau_{Li}W_i \quad i = H,F \quad j \neq$$

If we multiply everything by $\tau_{Xi} - 1$ and recall that in the multi-sector model $W_i = 1$, we obtain (26).

C.9 Two Lemmata and the Proof of Lemma 3

We first introduce two lemmata that will be useful for several proofs below and then we prove Lemma 3.

C.9.1 Lemmata 9 and 10 and their proofs

Lemma 9 *In the market equilibrium:*

$$\frac{\tau_{Xi}P_{ii}C_{ii}}{L_{Ci}} + \frac{\tau_{Ij}^{-1}P_{ji}C_{ji}}{L_{Ci}} = \tau_{Xi}\tau_{Li}W_i, \quad i = H, F, \quad j \neq i \tag{C-65}$$

Proof In the case of heterogeneous firms, using (10) and (11), we obtain:

$$\frac{P_{ji}C_{ji}}{L_{Ci}} = \tau_{Tji}\tau_{Li}\delta_{ji}W_i, \quad i, j = H, F,$$

which leads to C-65 once you recall that $\delta_{ii} = 1 - \delta_{ji}$. Similarly, for the case of homogeneous firms, one can use (A-21) and (A-22) to compute $P_{ii}C_{ii}$ and $P_{ji}C_{ji}$ and recover (C-65). ■

Lemma 10 *In the market equilibrium the following condition holds:*

$$\tau_{Xi}P_{ii}dC_{ii} + \tau_{Ij}^{-1}P_{ji}dC_{ji} - \frac{\varepsilon}{\varepsilon - 1}\tau_{Li}\tau_{Xi}dL_{Ci} = 0, \quad i = H, F \quad j \neq i \tag{C-66}$$

Proof we show that in equilibrium condition (C-66) is always satisfied. We first consider the case of firm heterogeneity and then turn to the case of homogeneous firms.

With heterogeneous firms, first, notice that equation (10) implies:

$$dC_{ji} = \frac{\partial C_{ji}}{\partial L_{Ci}}dL_{Ci} + \frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji} + \frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji} \quad i, j = H, F$$

Therefore, we can write

$$\begin{aligned}\tau_{Xi}P_{ii}dC_{ii} + \tau_{Ij}^{-1}P_{ji}dC_{ji} = &\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial L_{Ci}} + \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial L_{Ci}}\right)dL_{Ci} + \tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii} + \tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii} + \\ &+ \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji} + \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji} \quad i = H, F \quad j \neq i\end{aligned} \tag{C-67}$$

Notice that by condition (C-65) and the fact that by (10) $\frac{\partial C_{ji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon-1}\frac{C_{ji}}{L_{Ci}}$, we get:

$$\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial L_{Ci}} + \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial L_{Ci}}\right)dL_{Ci} = \frac{\varepsilon}{\varepsilon - 1}\tau_{Li}\tau_{Xi}dL_{Ci}, \quad i = H, F \quad j \neq i \tag{C-68}$$

Therefore, in order for (C-66) to hold for the case of heterogeneous firms, it must be that in equilibrium:

$$\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii} + \tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii} + \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji} + \tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji} = 0, \quad i = H, F \quad j \neq i$$

To prove this result, first consider that by (B-4):

$$\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji} + \frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji} = \frac{C_{ji}}{\varphi_{ji}}\left[1 - \frac{\varepsilon}{\varepsilon - 1}\left(\Phi_i + (\varepsilon - 1)\right)\right]d\varphi_{ji}, \quad i, j = H, F$$

Hence:

$$\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii}+\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}$$
$$=\left[1-\frac{\varepsilon}{\varepsilon-1}\left(\Phi_i+(\varepsilon-1)\right)\right]\left(\tau_{Xi}P_{ii}\frac{C_{ii}}{\varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{C_{ji}}{\varphi_{ji}}d\varphi_{ji}\right),\quad i=H,F\quad j\neq i,$$

which by (10) and (11) can be rewritten as:

$$\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii}+\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}$$
$$=\tau_{Xi}\tau_{Li}W_iL_{Ci}\left[1-\frac{\varepsilon}{\varepsilon-1}\left(\Phi_i+(\varepsilon-1)\right)\right]\left(\frac{\delta_{ii}}{\varphi_{ii}}d\varphi_{ii}+\frac{1-\delta_{ii}}{\varphi_{ji}}d\varphi_{ji}\right)\quad i=H,F\quad j\neq i$$

Finally, recalling (B-3), we can conclude that, as postulated, this last condition is equal to zero in equilibrium for all $0\leq\alpha\leq1$.

Similarly, in the presence of homogeneous firms first condition (A-22) leads to:

$$dC_{ji}=\frac{\partial C_{ji}}{\partial L_{Ci}}dL_{Ci}+\frac{\partial C_{ji}}{\partial W_j}dW_j+\frac{\partial C_{ji}}{\partial \tau_{Li}}d\tau_{Li}+\frac{\partial C_{ji}}{\partial \tau_{Lj}}d\tau_{Lj}$$
$$+\frac{\partial C_{ji}}{\partial \tau_{Ii}}d\tau_{Ii}+\frac{\partial C_{ji}}{\partial \tau_{Xj}}d\tau_{Xj}+\frac{\partial C_{ji}}{\partial \tau_{Ij}}d\tau_{Ij}+\frac{\partial C_{ji}}{\partial \tau_{Xi}}d\tau_{Xi}\quad i,j=H,F$$

where we already used the normalization $W_i=1$. Hence, in this case

$$\tau_{Xi}P_{ii}dC_{ii}+\tau_{Ij}^{-1}P_{ji}dC_{ji}=\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial L_{Ci}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial L_{Ci}}\right)dL_{Ci}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial W_j}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial W_j}\right)dW_j$$
$$+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Li}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Li}}\right)d\tau_{Li}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Lj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Lj}}\right)d\tau_{Lj}$$
$$+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ii}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ii}}\right)d\tau_{Ii}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ij}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ij}}\right)d\tau_{Ij}$$
$$+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xi}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xi}}\right)d\tau_{Xi}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xj}}\right)d\tau_{Xj}\quad i=H,F\quad j\neq i$$

Note that condition (C-65) and by (A-22) $\frac{\partial C_{ji}}{\partial L_{Ci}}=\frac{\varepsilon}{\varepsilon-1}\frac{C_{ji}}{L_{Ci}}$ hold in equilibrium also in the case of homogeneous firms, implying that (C-68) is valid too. Thus, (C-66) hold since by (A-21) and (A-22):

$$\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial W_j}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial W_j}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Li}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Li}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Lj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Lj}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ii}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ii}}$$
$$=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ij}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ij}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xi}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xi}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xj}}=0\quad i=H,F\quad j\neq i$$

■

C.9.2 Proof of Lemma 3

Proof We prove Lemma 3 point by point.

(a) To show why condition (27) holds first consider that by conditions (24) and (25) (26):

$$(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{Q_{Cij}})dC_{ij} = (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + (\tau_{Xj}-1)\tau_{Tij}^{-1}P_{ij}dC_{ij} \quad i=H,F \quad j\neq i \tag{C-69}$$

$$\begin{aligned}\sum_{i=H,F}(\tau_{Tij}^{-1}P_{ij}\frac{\partial Q_{Cij}}{L_{Cij}}-1)dL_{Cj} &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Lj}-1)dL_{Cj} \\ &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} + (1-\tau_{Xj})\frac{\varepsilon}{\varepsilon-1}\tau_{Lj}dL_{Cj} \\ &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} - (\tau_{Xj}-1)\tau_{Tij}^{-1}P_{ij}dC_{ij} + (1-\tau_{Xj})P_{jj}dC_{jj},\end{aligned} \tag{C-70}$$

where last equality follows from condition (C-66). Summing (C-69) and (C-70) we obtain:

$$\begin{aligned}&(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{Q_{Cij}})dC_{ij} + \sum_{i=H,F}(\tau_{Tij}^{-1}P_{ij}\frac{\partial Q_{Cij}}{L_{Cij}}-1)dL_{Cj} \\ &= (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} + (1-\tau_{Xj})P_{jj}dC_{jj}, \quad j=H,F \quad i\neq j\end{aligned} \tag{C-71}$$

Finally, we can sum the two conditions in (C-71) to obtain condition (27).
To show that condition (28) holds, recall that by condition (C-66) it follows that

$$(1-\tau_{Xi})P_{ii}dC_{ii} = (1-\tau_{Xi})\frac{\varepsilon}{\varepsilon-1}\tau_{Li}dL_{Ci} - (1-\tau_{Xi})\tau_{Tji}^{-1}P_{ji}dC_{ji}, \quad i=H,F \quad j\neq i \tag{C-72}$$

Substituting this condition into condition (27) we get:

$$\sum_{i=H,F}dE_i = \sum_{\substack{i=H,F\\ j\neq i}}(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} - \sum_{\substack{i=H,F\\ j\neq i}}(1-\tau_{Xj})\tau_{Tij}^{-1}P_{ij}dC_{ij} + \sum_{i=H,F}(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1)dL_{Ci}$$

which then leads to condition (28).

(b)(i) If all wedges in (27) are zero, i.e., $\tau_{Ii} = \tau_{Xi} = 1$ – and in the multi-sector model also $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ – for $i = H, F$.[59] the allocation determined by the conditions listed at the beginning of Appendix C.5 is also the solution of the set of equilibrium conditions listed in Section 2.4. As a consequence, if $\tau_{Ii} = \tau_{Xi} = 1$ and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$ the market allocation is efficient.

(b)(ii) Conditions (23) state the equations that correspond to (21) and (22) in the market equilibrium. It is obvious from these equations that (21) and (22) are satisfied in the market equilibrium if and only if $\tau_{Tij} = 1$ and - in the multiple sector model - $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i, j = H, F$, namely if and only if all wedges in (28) are zero. Then by Proposition 2, it must be that the market allocation is efficient if and only if $I_i = I_j$ and all the wedges in (28) are zero. ■

D Policy-Maker Problem and Welfare Decomposition

Here we prove the Propositions and Lemma of Section 4. For these proofs it is useful to recall that $\varepsilon > 1$, $0 < \alpha \leq 1$, $0 < \delta_{ii} < 1$, $\Phi_i > 0$, and $L_{Ci} > 0$.

D.1 Proof of Proposition 1

Proof The proof is organized in two steps. First, we derive the total differential of individual-country welfare by using the total differential of the trade-balance condition (12) and we show that this total differential leads to condition (30) given Lemma 10 Second, we show that if $I_i = I_j$ for $i \neq j$, condition (30) leads to condition (31).

[59]Note that if $\alpha = 1$ then $dL_C = 0$ for $= H, F$ and only the wedges in dC_{ij} are present in (27).

(1) Substituting the definition of the consumption aggregator (4) into the utility function (3), we get:

$$U_i = \alpha \frac{\varepsilon}{\varepsilon - 1} \log \left[\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right] + (1-\alpha) \log Z_i, \quad i = H, F$$

Taking the total differential of this objective function, we obtain:

$$dU_i = \alpha \sum_{j=H,F} \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} dC_{ij} + \frac{1-\alpha}{Z_i} dZ_i, \quad i = H, F \tag{D-1}$$

Note that $\frac{1-\alpha}{Z_i} = \frac{1}{I_i}$ and $\alpha \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} \frac{P_i}{I_i} = \frac{P_{ij}}{I_i}$ since $\left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} = \frac{P_{ij}}{P_i}$ for $i, j = H, F$. As a result, condition (D-1) can be rewritten as:

$$dU_i = \frac{1}{I_i} \sum_{j=H,F} P_{ij} dC_{ij} + \frac{1}{I_i} dZ_i, \quad i = H, F \tag{D-2}$$

Then, we can take the total differential of condition (12) and of its foreign counterpart[60] and use the fact that $Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij}$ to get:

$$-dZ_i - dL_{Ci} + C_{ji} d(\tau_{Ij}^{-1} P_{ji}) + \tau_{Ij}^{-1} P_{ji} dC_{ji} - C_{ij} d(\tau_{Ii}^{-1} P_{ij}) - (\tau_{Ii}^{-1} P_{ij}) dC_{ij} = 0, \quad i = H, F \quad j \neq i$$

Dividing this condition by I_i and adding it to (D-2), we obtain:

$$\begin{aligned} dU_i &= \frac{P_{ii}}{I_i} dC_{ii} + \frac{P_{ij}}{I_i} dC_{ij} + \frac{1}{I_i} dZ_i - \frac{1}{I_i} dZ_i - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji} - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) - \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} \\ &= \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji}, \quad i = H, F \quad j \neq i \end{aligned}$$

Adding and subtracting terms, this can be rewritten as:

$$\begin{aligned} dU_i = &(1 - \tau_{Xi}) \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \tau_{Ii}^{-1} \frac{P_{ij}}{I_i} dC_{ij} + \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} - 1 \right) \frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) \\ &+ \tau_{Xi} \frac{P_{ii}}{I_i} dC_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{dC_{ji}}{I_i} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} \frac{dL_{Ci}}{I_i}, \quad i = H, F \quad j \neq i \end{aligned}$$

Recall that by Lemma 10 in equilibrium the following condition holds:

$$\tau_{Xi} P_{ii} dC_{ii} + \tau_{Ij}^{-1} P_{ji} dC_{ji} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci} = 0, \quad i = H, F \quad j \neq i$$

If this is true, then:

$$\begin{aligned} dU_i &= (1 - \tau_{Xi}) \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \tau_{Ii}^{-1} \frac{P_{ij}}{I_i} dC_{ij} + \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} - 1 \right) \frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) \\ &= \frac{dE_i}{I_i} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}), \quad i = H, F \quad j \neq i \end{aligned} \tag{D-3}$$

where dE_i is defined in Lemma 3. Summing the total differentials for both countries condition (D-3) leads to condition (30). It also leads to the decomposition of individual-country welfare in (33). Notice that if condition (C-66) holds, condition (D-3) holds even with homogeneous firms and when considering the one-sector model in which $\alpha = 1$ and $dL_{Ci} = 0$.

[60] This condition can be recovered by combining (12) with (13).

(2) Finally, if $I_i = I$ for $i = H, F$ so that (30) leads to:

$$I \sum_{i=H,F} dU_i = \sum_{i=H,F} dE_i + \sum_{\substack{i=H,F\\ j\neq i}} \Big(C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij}) \Big)$$
$$= \sum_{i=H,F} dE_i$$

which by Lemma 3 point (a) corresponds to condition (31) and where the last equality follows from the fact that terms of trade effects exactly cancel out. ■

D.2 Proof of Lemma 4

Proof We prove Lemma 4 point by point.

(a) In appendix B.1.2 we explained how to apply the total differential approach to solve a constrained optimization problem. In this case we have 28 variables (22 endogenous variables plus 6 policy instruments) and 22 constraints (conditions (6) -(13)).[61]. To show point (a) we the proceed as follows: (i) we show how to express the total differential in (29) in terms of 6 differentials and then 6 wedges. Setting these wedges to zero gives us 6 additional conditions to determine the optimal policies; (ii) we make clear that these conditions correspond to setting $I_i = I_j$ and the wedges in (31) individually equal to zero.

(i) In order to rewrite the differential in (29), we combine it with condition (27) to obtain

$$\sum_{i=H,F} dU_i = \sum_{\substack{i=H,F\\ j\neq i}} \frac{(\tau_{Ii}-1)\tau_{Ii}^{-1} P_{ij} dC_{ij} + (1-\tau_{Xi}) P_{ii} dC_{ii} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1\right) dL_{Ci} + C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}{I_i}$$

Then, we use this condition and condition (C-72) to get

$$\sum_{i=H,F} dU_i = \sum_{\substack{i=H,F\\ j\neq i}} \frac{(\tau_{Ii}-1)\tau_{Ii}^{-1} P_{ij} dC_{ij} - (1-\tau_{Xi})\tau_{Tji}^{-1} P_{ji} dC_{ji} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right) dL_{Ci} + C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}{I_i}$$
$$= \sum_{\substack{i=H,F\\ j\neq i}} \left(\frac{1-\tau_{Ij}^{-1}}{I_j} - \frac{\tau_{Tji}^{-1} - \tau_{Ij}^{-1}}{I_i} \right) P_{ji} dC_{ji} + \sum_{i=H,F} \frac{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)}{I_i} dL_{Ci} + \sum_{\substack{i=H,F\\ j\neq i}} \frac{(I_i - I_j)}{I_i I_j} C_{ij} d(\tau_{Ii}^{-1} P_{ij}) \quad \text{(D-4)}$$

which confirms that the differential in (29) can be expressed as a function of dC_{ij}, dL_{Ci}, $d(\tau_{Ij}^{-1} P_{ji})$, $d(\tau_{Ii}^{-1} P_{ij})$ for $i = H, F$ and $j \neq i$ only.
(ii) Setting the wedges in (D-4) individually equal to zero leads to:

$$\tau_{Li} = \frac{\varepsilon - 1}{\varepsilon} \quad i = H, F \qquad I_i = I_j \quad i = H \quad j = F \qquad \tau_{Tji} = 1 \quad i = H, F \quad j \neq i$$

which, as claimed, is equivalent to imposing $I_i = I_j$ and to setting to zero the wedges in (31). Finally, notice how by (D-4) we can impose only 5 restrictions, and we are thus left with 1 degree of freedom in the choice of the 6 policy instruments, consistently with point (b).

(b) By point (a) above, the global policy $\{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}_{i=H,F}$ solves the world-policy-maker problem *if and only if* all the following conditions hold for $i = H, F$, $j \neq i$: (1) $I_i = I_j$; (2) $\tau_{Tij} = 1$; (3) $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ when $\alpha < 1$.

[61]When $\alpha = 1$ there are 26 endogenous variables and 22 constraints. Indeed, as made clear in Appendix A.4.2, in the one-sector model we assume that policy makers abstain from using labor subsidies. In the homogeneous firm model, there are 16 variables and 10 constraints if $\alpha < 1$ and 14 variables and 10 constraints if $\alpha = 1$.

At the same time, by Lemma 3 point (b) (ii) the market allocation is efficient *if and only if* all conditions (1) to (3) hold. Thus, at the optimum the global policy maker implements the planner allocation.

What remains to prove is that the global policy is optimal if and only if $\tau_{Ii} = \tau_{Ij}$ (or equivalently if and only if $\tau_{Xi} = \tau_{Xj}$). Put differently, we need to show that condition $I_i = I_j$ can be substituted away with condition $\tau_{Ii} = \tau_{Ij}$ (or equivalently condition $\tau_{Xi} = \tau_{Xj}$). More specifically, we need to prove that:

(b1) If $I_i = I_j$, $\tau_{Tij} = 1$, and when $\alpha < 1$ $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_{Ii} = \tau_{Ij}$;

(b2) If $\tau_{Ii} = \tau_{Ij}$, $\tau_{Tij} = 1$, and when $\alpha < 1$ $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then $I_i = I_j$.

We start from (b1). If $I_i = I_j$, $\tau_{Tij} = 1$, and, when $\alpha < 1$, $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then by Lemma 3 point (b) (ii) the market allocation is efficient and by Lemma (8) also symmetric. Under these restrictions condition 11 (and condition(A-21) for the case of homogeneous firms) implies that $P_{ij} = P_{ji}$ both in the one sector[62] and in the multiple sector model. As a consequence, condition (13) can be simplified as $L - L_{Ci} = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij}C_{ij}$. Thus, since $P_{ij} = P_{ji}$ and $C_{ij} = C_{ji}$ the trade balance condition (12) can hold in equilibrium only if $\tau_{Ii} = \tau_{Ij}$.

We now move to (b2). First, recall that from the second stage of the Planner's problem we know that when $\tau_{Tij} = 1$ and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ when $\alpha < 1$ (or $\frac{\tau_{Li}W_i}{\tau_{Lj}W_j} = 1$ when $\alpha = 1$) then in equilibrium the cutoffs φ_{ij} for $i, j = H, F$ are efficient. When this is the case, conditions (6) to (9) can be used to find the efficient allocation for φ_{ij}, $\tilde{\varphi}_{ij}$, δ_{ij} for $i, j = H, F$. Recall again from Lemma 8 that the efficient allocation is unique and symmetric. For the case of homogeneous firms we simply have $\varphi_{ij} = 1$ for $i, j = H, F$. When $\alpha = 1$ we have $L_{Ci} = L$ for $i = H, F$ and it thus follows from (10) and (11) (and from (A-21) and (A-22) for the case of homogeneous firms) that the solution for P_{ij} and C_{ij} is also symmetric. This implies that $I_i = P_iC_i = P_jC_j = I_j$. When $\alpha < 1$ instead, we can use (10) and (11) to get $P_{ij}C_{ij} = \delta_{ij}L_{Cj}$ when firms are heterogeneous, and (A-21) and (A-22) to get $P_{ij}C_{ij} = L_{Cj}\tau_{ij}^{1-\varepsilon} \frac{1-\tau_{ki}^{\varepsilon-1}}{\tau_{ki}^{1-\varepsilon} - \tau_{ki}^{\varepsilon-1}}$ with $i, j = H, F$ and $k \neq i$ when firms are homogeneous. In both case we can think of $P_{ij}C_{ij}$ as being a linear function of L_{Cj}. We can thus use the two equations (12) and (13) to solve for L_{Ci} and L_{Cj}. Note that this is a linear system in the two variables, and thus has a unique solution. It thus suffices to recall that the symmetric allocation is a possible solution when $\tau_{Tij} = 1$ and $\tau_{Ii} = \tau_{Ij}$. Therefore, the unique solution is symmetric and $L_{Ci} = L_{Cj}$. It then follows symmetry of P_{ij}, C_{ij}, P_i, C_i, Z_i and thus $I_i = P_iC_i + Z_i$. ■

D.3 Proof of Proposition 2

Proof We derived the total differential of the individual-country policy maker (condition (33)) in the proof of Proposition 1. More specifically see point 1 of Proof D.1. ■

E How Policy Instruments affect the Terms of Trade and Production Efficiency

First we state and prove two Lemmata. The first one identifies conditions for $\delta_{ij} \geq 1/2$ with $i, j = H, F$. The second one signs the contribution of each component to the terms-of-trade effect of condition (33).

Next, we discuss the different channels through which unilateral changes in the policy instruments affect the terms of trade. Finally, we sign efficiency effects, terms-of-trade effects and welfare effects for unilateral deviations from the laissez-faire equilibrium in the one-sector model (Lemma 13) and in the multi-sector model (Lemma 5).

E.1 Lemma 11 and its proof

Lemma 11 *Let $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}$ for $i \neq j$ and $i = H, F$. Then at any symmetric allocation:*

[62] Recall that in A.4.2 we assumed $\frac{\tau_{Li}W_i}{\tau_{Lj}W_j} = 1$ in any symmetric allocation.

(i) $\delta_{ii} \geq 1/2$ *if trade taxes are not used, namely such that* $\tau_{Tij} = 1$ *for* $i, j = H, F$.

(ii) $\delta_{ii} < 1/2$ *only if there are export or import subsidies such that* $\tau_{Tij} < 1$ *for* $i, j = H, F$.

Proof We prove this lemma point by point.

(i) Using equations (6), (7) and (8), imposing symmetry of the allocation and of taxes and $\tau_{Tij} = 1$, we obtain $\delta_{ii} = \left[1 + \tau_{ij}^{1-\varepsilon} \frac{\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}{\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}\right]^{-1}$ and $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij}^{\frac{\varepsilon}{\varepsilon-1}}$. Since $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} > 1$ by assumption it follows that $\frac{\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}{\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)} < 1$. Thus $1 + \tau_{ij}^{1-\varepsilon} \frac{\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}{\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)} < 2$ and $\delta_{ii} > 1/2$.

(ii) We prove this point by contradiction. Suppose that $\tau_{Tij} \geq 1$. Combining again conditions (6), (7) and (8) and imposing symmetry we get $\delta_{ii} = \left[1 + \tau_{ij}^{1-\varepsilon} \tau_{Tij}^{-\varepsilon} \frac{\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}{\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}\right]^{-1}$ and $\frac{\varphi_{ji}}{\varphi_{ii}} = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij}^{\frac{\varepsilon}{\varepsilon-1}}$. Then if $\tau_{Tij} \geq 1$, $\varphi_{ji} > \varphi_{ii}$ since $f_{ji} > f_{ii} \tau_{ij}^{1-\varepsilon}$. As a result, $\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi) < \int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)$ and thus $1 + \tau_{ij}^{1-\varepsilon} \frac{\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)}{\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)} < 2$ and hence $\delta_{ii} > 1/2$. Therefore, $\delta_{ii} < 1/2$ only if $\tau_{Tij} < 1$. ∎

E.2 Lemma 12 and its proof

Lemma 12 *Consider a marginal unilateral increase in each trade policy instrument at a time, starting from the laissez-faire equilibrium, i.e., with* $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$. *Then:*

(a) In the one-sector model deviating from the laissez-faire equilibrium induces:

(i) $\frac{dW_i}{W_i} - \frac{dW_j}{W_j} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{dW_i}{W_i} - \frac{dW_j}{W_j} < 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(ii) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(iii) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$.

(b) In the multi-sector model deviating from the laissez-faire equilibrium induces:

(i) $\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} > 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(ii) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} < 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(iii) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} > 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$.

Proof We prove Lemma 12 point by point.

(a) In the case of the one-sector model we can prove points (i), (ii) and (iii) as follows:

(i) Combining conditions (B-28), (B-27) and (B-24) we find at the laissez-faire equilibrium:

$$dW_i = A_{\tau Ii} d\tau_{Ii} + A_{\tau Xi} d\tau_{Xi} \tag{E-1}$$

where $A_{\tau Ii} = \frac{\delta_{ii} d\tau_{Ii} \varepsilon (\Phi_i + \varepsilon - 1)}{\delta_{ii} \Phi_i \varepsilon + (\varepsilon-1)(1 - \delta_{ii} + \delta_{ii}(\varepsilon-1))} > 0$ and $A_{\tau Xi} = -1 < 0$;

(ii) Recall that $d\delta_{ji} = -d\delta_{ii}$ for $i, j = H, F$ and $j \neq i$. Then we can use (B-26) and its symmetric counterpart to obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = B_{\tau Ii} d\tau_{Ii}, \tag{E-2}$$

where $B_{\tau Ii} = -\frac{\delta_{ii}\varepsilon\phi_{ij}(\varepsilon-1+\Phi_i)}{\delta_{ii}\Phi_i\varepsilon+(\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$;

(iii) Using the solution for dC_{ii} found in point (a), condition (B-23) and their symmetric counterparts we obtain:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = \Gamma_{\tau Ii} d\tau_{Ii} \tag{E-3}$$

where $\Gamma_{\tau Ii} = -\frac{\delta_{ii}\varepsilon\phi_{ij}}{\delta_{ii}\Phi_i\varepsilon+(\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$

(b) In the case of the two-sector model we can show points (i), (ii) and (iii) in the following way.

(i) Combining (B-15), (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we obtain:

$$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} = \Delta_{\tau Ii} d\tau_{Ii} + \Delta_{\tau Xi} d\tau_{Xi} \tag{E-4}$$

where $\Delta_{\tau Ii} \equiv -\frac{(1-\delta_{ii})[(\varepsilon-1)(1-\alpha+2\delta_{ii}(\varepsilon-1+\alpha))+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} < 0$ and $\Delta_{\tau Xi} \equiv \frac{(1-\delta_{ii})[(1-\alpha\delta_{ii}+\alpha(1-\delta_{ii})+2\delta_{ii}(\varepsilon-1))(\varepsilon-1)+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} > 0$.

(ii) Recall that $\delta_{ji} = 1 - \delta_{ii}$, implying that $d\delta_{ji} = -d\delta_{ii}$ and $d\delta_{ij} = -d\delta_{jj}$. Using (B-7) and (B-14) to compute $d\delta_{ii}$ and $d\delta_{jj}$, and combing them with (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = Z_{\tau Ii} d\tau_{Ii} + Z_{\tau Xi} d\tau_{Xi} \tag{E-5}$$

where $Z_{\tau Ii} = -Z_{\tau Xi} \equiv -\frac{(1-\delta_{ii})\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\delta_{ij}(\varepsilon-1)(2\delta_{ii}-1)} < 0$ iff $\delta_{ii} > 1/2$;

(iii) First, we use (B-3) and (B-6) to compute $d\varphi_{ji}$ and (B-10) to compute $d\varphi_{ij}$ and we impose symmetry. Second, combining these conditions with (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we get:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = H_{\tau Ii} d\tau_{Ii} + H_{\tau Xi} d\tau_{Xi} \tag{E-6}$$

where $H_{\tau Ii} = -H_{\tau Xi} \equiv \frac{\delta_{ii}\varepsilon}{(2\delta_{ii}-1)(\varepsilon-1)} > 0$ iff $\delta_{ii} > 1/2$. ■

E.3 Decomposing the terms-of-trade effect of unilateral deviations from laissez-faire

When starting from a symmetric allocation, the impact of a unilateral policy change on the terms of trade can be written as:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \tag{E-7}$$

$$\tau_{Ii}^{-1}P_{ij}C_{ij}\left[\frac{d\tau_{Li}}{\tau_{Li}} + \frac{d\tau_{Xi}}{\tau_{Xi}} + \underbrace{\left(\frac{dW_i}{W_i} - \frac{dW_j}{W_j}\right)}_{(i)} + \frac{1}{\varepsilon-1}\left(\underbrace{\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}}}_{(ii)} + \underbrace{\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}}_{(iii)}\right) + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)}\right],$$

where deviations are defined as $dX_i/X_i = \frac{\partial X_i}{\partial \tau_{mi}} \frac{1}{X_i} d\tau_{mi}$. We discuss the impact of tariffs (i.e., $d\tau_{Ii} > 0, d\tau_{Li} = d\tau_{Xi} = 0$) in more detail and then provide results for the other instruments. A domestic tariff influences the terms of trade (i) by changing the relative wage; (ii) by affecting the amount of labor allocated to the differentiated sector in both countries; (iii) by impacting on the average variable profit share of domestic and foreign firms in their respective export markets; (iv) by moving the cutoff productivity levels of domestic and foreign exporters. Here, (i) corresponds to a a change in the price of individual varieties, while (ii)-(iii) correspond to changes in the measure of exportables and importables. Finally, (iv), the change in the cutoff productivity levels, impacts both on the average price of individual varieties and the measure of domestic and foreign exporters. In particular, an increase in the domestic relative wage raises the price of exported varieties relative to imported ones and improves the terms of trade. By contrast, an increase in the amount of labor allocated to the domestic differentiated sector worsens the terms of trade by reducing the price index of exportables via an increase in the number of varieties, while an increase in foreign labor in this sector improves them by reducing the price index of importables. Domestic terms of trade worsen with an increment in the profit share of domestic firms from exports and improve in the corresponding share of foreign firms by changing the measure of firms that export to each market. Finally, an increase in the domestic cutoff-productivity level for exports worsens the terms of trade both by making the average exportable variety cheaper and by affecting the measure of exporters, whereas an increase in the foreign productivity cutoff has the opposite effect.

Here we discuss the impact of a small unilateral tariff (i.e., $d\tau_{Li} = d\tau_{Xi} = 0$) in the one-sector model (i.e., $dL_{Cj} = dL_{Ci} = 0$), starting from the laissez-faire equilibrium while the discussion for the multi-sector model is in section 5. In the presence of a single sector, the terms-of-trade effects of a small tariff are *positive* and given by

$$P_{ij}C_{ij}\left[\underbrace{\left(\frac{dW_i}{W_i} - \frac{dW_j}{W_j}\right)}_{(i)>0} + (\varepsilon - 1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)>0} + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)<0}\right] > 0. \tag{E-8}$$

A tariff raises home's demand for domestically produced varieties and thus, ceteris paribus, home firms' profits and the demand for domestic labor. Since labor supply is completely inelastic in this model, home's relative wage needs to adjust upward in response ((i)>0), thereby reducing equilibrium profits of domestic firms. Moreover, the increase in relative domestic income increases the share of profit firms from both countries make in home's domestic market, which improves home's terms of trade via the extensive margin by reducing the measure of domestic exporters and increasing the measure of foreign exporters ((iii)>0). Finally, the increase in the relative domestic wage leads to tougher selection into exporting at home and less selection in the other country, which negatively impacts on home's terms of trade ((iv)<0). In the absence of firm heterogeneity, the tariff exclusively raises home's relative wage. Firm heterogeneity leads to two additional and opposing effects: if heterogeneity mostly affects the profit share from exports, terms of trade respond more to tariffs compared to the case of homogeneous firms; by contrast, if selection effects are large, firm heterogeneity tends to reduce the response of the terms of trade by reducing the average price of exported varieties relative to the one of imported varieties. Note also that in the one-sector model production efficiency is always guaranteed, so the only incentive to deviate from the laissez-faire equilibrium is the positive terms-of-trade effect of the tariff.

Lemma 13 summarizes the results for import tariffs as well as for the other tax instruments.

E.4 Lemma 13 and its proof

Lemma 13 ***Unilateral deviations from laissez-faire in one-sector model***

Consider a marginal unilateral increase in each trade policy instrument at a time, starting from the laissez-faire equilibrium, i.e., with $\tau_{Ii} = \tau_{Xi} = 1$ and $\tau_{Li} = 1$ for $i = H, F$. Then:

(a) the production-efficiency effect is zero for all policy instruments.

(b) the consumption-efficiency effect is zero for all policy instruments.

(c) the terms-of-trade effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

(d) the total welfare effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

Proof We prove Lemma 13 point by point.

(a) In the one-sector model $dL_{Ci} = 0$ in (33) so that the production-efficiency effect is zero for all policy instruments.

(b) When $\tau_{Ii} = \tau_{Xi} = 1$, the consumption-efficiency effect in (33) is zero for any dC_{ii} and dC_{ij}.

(c) In the case of heterogeneous firms, we can substitute conditions (E-1), (E-2) and (E-3) into (E-7) and impose $dL_{Ci} = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and $dW_j = 0$ for $j \neq i$ to obtain:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \Theta_{\tau Ii}\ d\tau_{Ii},$$

where $\Theta_{\tau Ii} = \frac{L(1-\delta_{ii})\delta_{ii}\varepsilon((\varepsilon-1)^2+\varepsilon\Phi_i)}{(\varepsilon-1)[(\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-2))+\varepsilon\delta_{ii}\Phi_i]} > 0$.

Similarly, in the case of homogeneous firms, condition (E-7) can be simplified by setting $dL_{Ci} = d\delta_{ij} = d\varphi_{ij} = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and $dW_j = 0$. Then, we can use condition (B-29) to get:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = I_{\tau Ii}d\tau_{Ii} + I_{\tau Xi}\ d\tau_{Xi},$$

where $I_{\tau Ii} = \frac{\varepsilon\tau^{\varepsilon}}{\tau+(2\varepsilon-1)\tau^{\varepsilon}} > 0$ and $I_{\tau Ii} = \frac{\varepsilon\tau^{\varepsilon}}{\tau+(2\varepsilon-1)\tau^{\varepsilon}} > 0$.

(d) This follows from the previous points. ■

E.5 Proof of Lemma 5

Proof We prove Lemma 5 point by point.

(a) Using conditions (B-11), (B-17) and (B-19) we can rewrite the production-efficiency effect in (33) as

$$\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)dL_{Ci} = E_{\tau Ii}d\tau_{Ii} + E_{\tau Xi}d\tau_{Xi} + E_{\tau Li}d\tau_{Li},$$

where $E_{\tau Ii} \equiv \frac{L_{Ci}(1-\delta_{ii})\delta_{ii}}{(1-2\delta_{ii})^2(\varepsilon-1)^2}\left[(\varepsilon-1)((1-\alpha)(1-2\delta_{ii})+\varepsilon)+\varepsilon\Phi_i\right]$, $E_{\tau Xi} \equiv -\frac{L_{Ci}(1-\delta_{ii})}{(1-2\delta_{ii})^2(\varepsilon-1)^2}[(\varepsilon-1)(1+\delta_{ii}(\alpha+2\delta_{ii}(1-\alpha)+\varepsilon-3))+\delta_{ii}\varepsilon\Phi_i]$ and $E_{\tau Li} \equiv L_{Ci}\frac{(\varepsilon-1)[2\delta_{ii}^2(\varepsilon+\alpha-2)-1-\delta_{ii}(2\varepsilon+\alpha-4)]-2(1-\delta_{ii})\delta_{ii}\varepsilon\Phi_i}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$. To see why $E_{\tau Ii} > 0$ it is sufficient to notice that $(1-\alpha)(1-2\delta_{ii})+\varepsilon = (1-\alpha)(1-\delta_{ii})+\varepsilon-(1-\alpha)\delta_{ii}$. What remains to show is that: (i) $E_{\tau Xi} < 0$ and (ii) $E_{\tau Li} < 0$.

(i) A sufficient condition for $E_{\tau Xi} < 0$ is $\overline{E}_{\tau Xi}(\delta_{ii}) \equiv 1+\delta_{ii}(\alpha+\varepsilon-3)+2\delta_{ii}^2(1-\alpha) > 0$ for all $0 \leq \delta_{ii} \leq 1$. In what follows we show that this is the case.

First, consider that $\overline{E}_{\tau Xi}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{E}''_{\tau Xi}(\delta_{ii}) = 4(1-\alpha) > 0$ (i.e., the function has a minimum) and the minimum is equal to $\min \overline{E}_{\tau Xi}(\delta_{ii}) \equiv \overline{E}^M_{\tau Xi}(\varepsilon,\alpha) = -\frac{(1+\alpha)^2-2(3-\alpha)\varepsilon+\varepsilon^2}{8(1-\alpha)}$. Second, note that $\overline{E}_{\tau Xi}(\delta_{ii}) > 0$ for both $\delta_{ii} = 0$ and $\delta_{ii} = 1$ since $\overline{E}_{\tau Xi}(0) = 1$ and $\overline{E}_{\tau Xi}(1) = \varepsilon-\alpha > 0$. This implies that if $\overline{E}'_{\tau Xi}(0) \geq 0$ ($\overline{E}'_{\tau Xi}(1) \leq 0$), then $\overline{E}_{\tau Xi}(\delta_{ii})$ is monotonically increasing (decreasing) and always positive for $0 \leq \delta_{ii} \leq 1$. Therefore, the two necessary conditions for $\overline{E}_{\tau Xi}(\delta_{ii}) < 0$ for $0 < \delta_{ii} < 1$ are $\overline{E}'_{\tau Xi}(0) = \varepsilon+\alpha-3 < 0$ and $\overline{E}'_{\tau Xi}(1) = \varepsilon+1-3\alpha > 0$, i.e., $\max\{1, 3\alpha-1\} < \varepsilon < 3-\alpha$. Hence, the last step to demonstrate that $\overline{E}_{\tau Xi}(\delta_{ii}) > 0$ for all $0 \leq \delta_{ii} \leq 1$ is to show that $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha) > 0$ always when $\max\{1, 3\alpha-1\} < \varepsilon < 3-\alpha$. Let's call $\overline{E}^{M\prime}_{\tau Xi}(\varepsilon)$ the partial derivative of $\overline{E}^M_{\tau Xi}(\varepsilon)$ w.r.t. ε. Note that $\overline{E}^{M\prime}_{\tau Xi}(\varepsilon) = \frac{3-\varepsilon-\alpha}{4(1-\alpha)}$ decreases in ε and is greater than zero as long as $\varepsilon < 3-\alpha$. This implies that $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)$ increases in ε in the admissible parameter range. What remains to do is then to evaluate the sign of $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)$ at the minimum admissible range for ε. There are two cases. If $\alpha > \frac{2}{3}$, then $\varepsilon = \max\{1, 3\alpha - 1\} = 3\alpha - 1$. Instead if $\alpha < \frac{2}{3}$, then $\varepsilon = \max\{1, 3\alpha - 1\} = 1$. In the first case, $\overline{E}^M_{\tau Xi}(3\alpha - 1, \alpha) = 2\alpha - 1 > 0$ always for $\alpha > \frac{2}{3}$. In the second case, $\overline{E}^M_{\tau Xi}(1, \alpha) = \frac{4-\alpha(4+\alpha)}{8(1-\alpha)}$. Note that $\overline{E}^M_{\tau Xi}(1, \alpha) > 0$ for $\alpha^1 < \alpha < \alpha^2$ where $\alpha^1 = -2 - \sqrt{2} < 0$ and $\alpha^2 = -2 + 2\sqrt{2} > \frac{2}{3}$. As a consequence, $\overline{E}^M_{\tau Xi}(1, \alpha) > 0$ in the relevant parameter range $0 < \alpha < \frac{2}{3}$. We can thus conclude that if $\overline{E}_{\tau Xi}(\delta_{ii})$ has a minimum for $0 < \delta_{ii} < 1$, such a minimum is always positive.

(ii) A sufficient condition for $E_{\tau Li} < 0$ is $\overline{E}_{\tau Li}(\delta_{ii}) \equiv -1 - \delta_{ii}(2\varepsilon + \alpha - 4) + 2\delta_{ii}^2(\varepsilon + \alpha - 2) < 0$ for all $0 \leq \delta_{ii} \leq 1$. In what follows we show that this is always the case.

First, note that $\overline{E}_{\tau Li}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{E}''_{\tau Li}(\delta_{ii}) = 4(\varepsilon - 2 + \alpha)$ and its critical point is equal to $\overline{E}^M_{\tau Li}(\varepsilon, \alpha) = -\frac{\varepsilon}{2} - \frac{\alpha^2}{8(-2+\alpha+\varepsilon)}$. Second, observe that $\overline{E}_{\tau Li}(0) = -1 < \overline{E}_{\tau Li}(1) = -(1 - \alpha) < 0$. As a consequence, if $\overline{E}''_{\tau Li}(\delta_{ii}) > 0$, $\overline{E}_{\tau Li}(\delta_{ii})$ has a minimum for $0 \leq \delta_{ii} < 1$ and it is always negative in this range. Thus, what remains to show is that $\overline{E}_{\tau Li}(\delta_{ii}) < 0$ even when $\overline{E}''_{\tau Li}(\delta_{ii}) < 0$ i.e., when $\varepsilon < 2 - \alpha$ and $\overline{E}_{\tau Li}(\delta_{ii})$ has a maximum. Two scenarios are possible. If $\varepsilon \geq 2 - \frac{3}{2}\alpha$, then $\overline{E}'_{\tau Li}(1) = -4 + 3\alpha + 2\varepsilon \geq 0$. As a result, $\overline{E}_{\tau Li}(\delta_{ii})$ is monotonically increasing and thus always negative for $0 < \delta_{ii} \leq 1$. Instead, when $1 < \varepsilon < 2 - \frac{3}{2}\alpha$, $\overline{E}_{\tau Li}(\delta_{ii})$ has a maximum for $0 < \delta_{ii} < 1$. Hence, the last step is to show that such a maximum is always negative. Notice that $\overline{E}^M_{\tau Li}(\varepsilon, \alpha) = 0$ for $\varepsilon^1 = 1 - \sqrt{1 - \alpha} - \frac{\alpha}{2}$ and $\varepsilon^2 = 1 + \sqrt{1 - \alpha} - \frac{\alpha}{2}$. It is easy to see that $\varepsilon^1 < 1$ and that $\varepsilon^2 > 2 - \frac{3}{2}\alpha$, i.e., $\overline{E}^M_{\tau Li}(\varepsilon, \alpha)$ never changes sign in $1 < \varepsilon < 2 - \frac{3}{2}\alpha$ and $0 < \alpha < 1$. To complete the proof it is then enough to show that $\overline{E}^M_{\tau Li}(\varepsilon, \alpha) < 0$ at one point in our interval. For example, if $\alpha = 0.5$, $\varepsilon = 1.2 < 2 - \frac{3}{2}\alpha$ and $\overline{E}^M_{\tau Li}(1.2, 0.5) = -0.49 < 0$.

(b) It is easy to see that the consumption-efficiency effect in (33) is zero for all policy instruments when $\tau_{Ii} = \tau_{Xi} = 1$.

(c) At the laissez-faire allocation we can use conditions (E-4), (E-5) and (E-6) and impose $dW_i = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ to rewrite terms of trade effects in (E-7) as:

$$C_{ij}[d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})] = \Sigma_{\tau Ii} d\tau_{Ii} + \Sigma_{\tau Xi} d\tau_{Xi} + \Sigma_{\tau Li} d\tau_{Li}, \tag{E-9}$$

where $\Sigma_{\tau Ii} \equiv -\frac{L_{Ci}(1-\delta_{ii})[(1-\delta_{ii})(\varepsilon-1)(1-\alpha+(\varepsilon-1+\alpha)2\delta_{ii})+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$, $\Sigma_{\tau Li} \equiv \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-\alpha(2\delta_{ii}-1))(\varepsilon-1)+\varepsilon 2\delta_{ii}\Phi_{ii}]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$ and $\Sigma_{\tau Xi} \equiv \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-1)(\delta_{ii}+2\delta_{ii}^2(\varepsilon-1)+(\varepsilon+\alpha(1-\delta_{ii}))(1-2\delta_{ii}))+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$. If is easy to show that $\Sigma_{\tau Ii} < 0$ in the relevant parameter range. To see why $\Sigma_{\tau Li} > 0$ it is sufficient to observe that $\varepsilon - \alpha(2\delta_{ii} - 1) > 0$ for all $0 < \delta_{ii} < 1$. Therefore, what remains to demonstrate is that $\Sigma_{\tau Xi} > 0$.

A sufficient condition for $\Sigma_{\tau Xi} > 0$ is $\overline{\Sigma}_{\tau Xi}(\delta_{ii}) \equiv \delta_{ii} + 2\delta_{ii}^2(\varepsilon - 1) + (\varepsilon + \alpha(1 - \delta_{ii}))(1 - 2\delta_{ii}) > 0$ for $0 \leq \delta_{ii} \leq 1$. First, consider that $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{\Sigma}''_{\tau Xi}(\delta_{ii}) = 4(\varepsilon - 1 + \alpha) > 0$ i.e., the function has a minimum and this minimum is equal to $\min \overline{\Sigma}_{\tau Xi}(\delta_{ii}) \equiv \overline{\Sigma}^M_{\tau Xi}(\delta_{ii}) = \frac{4\varepsilon(\alpha+\varepsilon-1)-(1+\alpha)^2}{8(\varepsilon-1+\alpha)}$. Second, observe that $\overline{\Sigma}_{\tau Xi}(0) = \varepsilon + \alpha > \varepsilon - 1 = \overline{\Sigma}_{\tau Xi}(1) > 0$ i.e., $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is positive at both ends of the relevant interval. Then, there are two cases. If $\varepsilon \leq \frac{3-\alpha}{2}$, $\overline{\Sigma}'_{\tau Xi}(1) = 2\varepsilon + \alpha - 3 < 0$ implying $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is monotonically decreasing and always positive for $0 \leq \delta_{ii} \leq 1$. By contrast, if $\varepsilon > \frac{3-\alpha}{2}$, then $\overline{\Sigma}'_{\tau Xi}(1) > 0$ implying $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ reaches a minimum for $0 \leq \delta_{ii} \leq 1$. However, when $\varepsilon > \frac{3-\alpha}{2}$ then $\overline{\Sigma}^M_{\tau Xi}(\delta_{ii}) > 0$. Indeed, in this case $4\varepsilon(\alpha + \varepsilon - 1) - (1 + \alpha)^2 > 4\frac{3-\alpha}{2}(\alpha + \frac{3-\alpha}{2} - 1) - (1 + \alpha)^2 = 2(1 - \alpha^2) > 0$.

(d) Combining the effects found at point (a), (b) and (c) we find that (33) can be rewritten as:

$$\begin{aligned} dU_i =& \frac{1}{I_i}\left[\left(\frac{\varepsilon}{\varepsilon - 1} - 1\right) dL_{Ci} + C_{ji}\left(d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})\right)\right] \\ =& \frac{1}{I_i}\left[E_{\tau Ii} d\tau_{Ii} + E_{\tau Xi} d\tau_{Xi} + E_{\tau Li} d\tau_{Li} + \Sigma_{\tau Ii} d\tau_{Ii} + \Sigma_{\tau Xi} d\tau_{Xi} + \Sigma_{\tau Li} d\tau_{Li}\right] \\ =& \frac{1}{I_i}\left[\Omega_{\tau Ii} d\tau_{Ii} + \Omega_{\tau Xi} d\tau_{Xi} + \Omega_{\tau Li} d\tau_{Li}\right], \end{aligned} \tag{E-10}$$

where $\Omega_{\tau Ii} \equiv \frac{L_{Ci}(1-\delta_{ii})[\delta_{ii}\varepsilon-(2\delta_{ii}-1)(1-\alpha)]}{(2\delta_{ii}-1)(\varepsilon-1)} d\tau_{Ii} > 0$, $\Omega_{\tau Xi} \equiv \frac{L_{Ci}(1-\delta_{ii})[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} < 0$ and $\Omega_{\tau Li} \equiv \frac{L_{Ci}[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} < 0$ iff $\delta_{ii} > 1/2$. To see why this is the case first note that the denominators of all these coefficients are positive iff $\delta_{ii} > 1/2$. Moreover, the numerator of $\Omega_{\tau Ii}$ is always positive since $\delta_{ii} > 2\delta_{ii} - 1$ for $\delta_{ii} < 1$, while the numerators of $\Omega_{\tau Xi}$ and $\Omega_{\tau Li}$ are always negative since $1 - 2\delta_{ii} < 1 - \delta_{ii}$ and $1 - \alpha < \varepsilon$ and as a consequence $(1 - 2\delta_{ii})(1 - \alpha) - (1 - \delta_{ii})\varepsilon < 0$. ■

F The Design of Trade Agreements in the Presence of Domestic Policies

In this section we prove Propositions 3, 4 and 7, which state the main results on strategic policies when all policy instruments (Proposition 3) or only production taxes (Propositions 4 and 7) are available. In both cases, we solve the Nash problems using the total-differential approach described in Appendix B. We focus on symmetric Nash equilibria in the two-sector model for which $\alpha < 1$ and $W_i = W_j = 1$ for $i, j = H, F$. We also prove Lemma 6, which concerns unilateral deviations from the first-best allocation.

F.1 Proof of Proposition 3

Proof We prove Proposition 3 point by point.

(a) First, we write the differential of the terms-of trade effect in (33) in terms of dL_{Ci}, dC_{ii}, dC_{ij}. For this purpose, we use the differentials of the equilibrium conditions derived in Appendix B.2.2 – imposing symmetry and the restrictions $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ – to evaluate each component of the terms-of-trade effects as decomposed in (E-7). In particular, we use: conditions (B-15) and (B-16) for term (ii) (differential of the amount of labor in both countries allocated to the differentiated sectors); conditions (B-7) and (B-14) jointly with the fact that $d\delta_{ji} = -d\delta_{ii}$ for term (iii) (differential of the average-profit shares in the export markets) and conditions (B-3), (B-6) and (B-10) for term (iv) (differentials of the export productivity cut-offs). Finally, we employ (B-11), (B-17) and (B-19) to substitute out $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ to obtain:

$$C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij}) = \Sigma_{Cii} dC_{ii} + \Sigma_{Cij} dC_{ij} + \Sigma_{LCi} dL_{Ci} \tag{F-1}$$

where:

$$\Sigma_{Cii} = -\frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{Li}\tau_{Xi}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}} \delta_{ii}(\varepsilon-1)^2} \frac{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \alpha\delta_{ii}(\varepsilon-1) + \alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}] + \delta_{ii}\varepsilon[\alpha+(1-\alpha)\tau_{Li}]\Phi_i}{\delta_{ii}[\alpha+(1-\alpha)\delta_{ii}\tau_{Li}] - (1-\delta_{ii})\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}] - \frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}\delta_{ii}[\alpha+(1-\alpha)\tau_{Li}]}$$

$$\Sigma_{Cij} = \frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}} \varphi_{ij}} \frac{[(\varepsilon-1+\delta_{ii})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})(\alpha\varepsilon+(1-\alpha)(1-\delta_{ii})\tau_{Ii})\tau_{Li}\tau_{Xi} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\varepsilon-1}((1-\alpha)\tau_{Li}+\alpha)]}{(\delta_{ii}H-\Pi)(\varepsilon-1) - \delta_{ii}\varepsilon[(1-\alpha)\tau_{Li}+\alpha](\varepsilon-1+\Phi_i)}$$

$$\Sigma_{LCi} = \frac{\tau_{Li}\tau_{Xi}\left[(\varepsilon-\delta_{ii})\frac{1-\alpha}{\varepsilon-1}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \alpha\delta_{ii} + \alpha\frac{\varepsilon}{\varepsilon-1}(1-\delta_{ii})\tau_{Li}\tau_{Xi} + \delta_{ii}\frac{\varepsilon}{(\varepsilon-1)^2}(\alpha+(1-\alpha)\tau_{Li})\Phi_i\right]}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where Σ_{Cii}, Σ_{Cij}, and Σ_{LCi} have been simplified using equations (7)-(13). Moreover, $\Pi = (1-\delta_{ii})(\alpha + (1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}$ and $H = \alpha + (1-\alpha)\tau_{Li}[\delta_{ii} + (1-\delta_{ii})\tau_{Ii}\tau_{Xi}]$. Condition (F-1) allows us to write (33) as follows:

$$\begin{aligned} dV_i =& (1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) \\ =& E_{Cii}dC_{ii} + E_{Cij}dC_{ij} + E_{LCi}dLC_i + \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \\ =& \Omega_{Cii}dC_{ii} + \Omega_{Cij}dC_{ij} + \Omega_{LCi}dL_{Ci} \end{aligned} \tag{F-2}$$

where $E_{Cii} \equiv (1-\tau_{Xi})P_{ii}$, $E_{Cij} \equiv (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}$, $E_{LCi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1$, $\Omega_{Cii} \equiv E_{Cii} + \Sigma_{Cii}$, $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$, and $\Omega_{LCi} \equiv E_{LCi} + \Sigma_{LCi}$. Condition (F-2) corresponds to condition (35) in the main text.

(b) In appendix B.1.2 we explained how to apply the total differential approach to solve a constrained optimization problem in n variables with m constraints. In this case we have 25 variables (22 endogenous variables plus 3 policy instruments) and 22 constraints i.e., exactly 3 degrees of freedom to choose the policy instruments so has to maximize world welfare.[63]. In point (a) we show how to rewrite the total differential of (35) as function of 3 total differentials ($dC_{ii}, dC_{ij}, dL_{Ci}$ with $i = H, F$ and $i \neq j$). As explained in B.1.2, at the optimum the wedges multiplying each differential needs to be individually equal to zero, i.e., $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$. This gives a set of 3 additional equations which can be used to solve for the optimal policy instruments. Once we have the solution for the instruments we can use the 22 constraints to determine the solution of the remaining 22 variables.

Before moving to point (c) we simplify each of these wedges to make them tractable.

First, consider $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$. Using (11) and imposing symmetry, the consumption-efficiency wedge E_{Cij} in (F-2) can be written as:

$$E_{Cij} = \frac{(\tau_{Ii}-1)(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ij}}$$

Then, recalling condition (F-1) we obtain

$$\Omega_{Cij} = \frac{\overline{\Omega}_{Cij}\tau_{ij}\tau_{Li}\tau_{Xi}(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}}{\varphi_{ij}(\varepsilon-1)(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\left[(\delta_{ii}H - \Pi)(\varepsilon-1) - \delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)\right]},$$

where

$$\overline{\Omega}_{Cij} = (\varepsilon-1)((\varepsilon-1)(1-\delta_{ii})H + \varepsilon\tau_{Ii}(\delta_{ii}H - \Pi)) - \delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)((1-\alpha)\tau_{Li}+\alpha)(\varepsilon\tau_{Ii} - \varepsilon + 1). \tag{F-3}$$

Second, consider $\Omega_{Cii} \equiv E_{Cii} + \Sigma_{Cii}$. Again using (11), the consumption-efficiency wedge E_{Cii} in (F-2) can be simplified as:

$$E_{Cii} = \frac{(\tau_{Xi}-1)(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{Li}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ii}}$$

Therefore, by (F-1)

$$\Omega_{Cii} = \frac{\overline{\Omega}_{Cii}(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\tau_{Li}(L_{Ci}\delta_{ii})^{-\frac{1}{\varepsilon-1}}(\varepsilon-1)^{-2}\varphi_{ii}^{-1}}{\delta_{ii}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha + (1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}(\alpha + (1-\alpha)\tau_{Li})},$$

[63] When $\alpha = 1$ we have 24 variables and 21 constraints while for the model with homogeneous firms we have 13 variables and 10 constraints.

where

$$\begin{aligned}\overline{\Omega}_{Cii} \equiv\ & (1-\tau_{Xi})[\varepsilon(\varepsilon-1)(\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}))\\ & -(\varepsilon-1+\Phi_i)\varepsilon^2\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})]\\ & -\tau_{Xi}[(\varepsilon-1)(\varepsilon(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})-(1-\alpha)\delta_{ii}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})\\ & +\alpha\delta_{ii}(\varepsilon-1)+\alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi})+\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i] \end{aligned} \tag{F-4}$$

Finally, consider $\Omega_{LCi} \equiv E_{LCi} + \Sigma_{LCi}$. Combining the production-efficiency wedge in (F-2) and condition (F-1) we obtain:

$$\Omega_{LCi} = \frac{\overline{\Omega}_{LCi}(\varepsilon-1)^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where

$$\begin{aligned}\overline{\Omega}_{LCi} \equiv\ & \delta_{ii}(\varepsilon-1)\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})-\varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ & -(\varepsilon-1)[\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ & -(\tau_{Li}\tau_{Xi}-1)\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i \end{aligned} \tag{F-5}$$

Notice that from (F-3), (F-4) and (F-5) we can conclude that $\Omega_{LCi} = \Omega_{Cii} = \Omega_{Cij} = 0$ iff $\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = \overline{\Omega}_{Cij} = 0$.

(c) First recall that from point (b) in the Nash equilibrium

$$\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = \overline{\Omega}_{Cij} = 0, \tag{F-6}$$

where $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ are defined in (F-3), (F-4), and (F-5). These wedges are functions of 8 variables only: τ_{Li}, τ_{Ii}, τ_{Xi}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}. Observe that once we impose symmetry and we take into account that $\delta_{ji} = 1-\delta_{ii}$ also conditions (6) -(9) are functions of these variables only. Therefore, we can fully characterize the symmetric Nash equilibrium using the 3 conditions in (F-6) jointly with the 5 equilibrium equations (6)-(9). In what follows we use the superscript N to indicate that a variable is evaluated at the Nash equilibrium.

To prove point (c), we proceed in 3 steps. First, we show that in the Nash equilibrium it must be the case that $\tau_L^N = \frac{\varepsilon-1}{\varepsilon}$. Second, we show that $\overline{\Omega}_{LCi} > 0$ always when $\tau_X < 1$ and $\tau_L = \tau_L^N$. Therefore, when a Nash equilibrium exists it must be such that $\tau_X^N > 1$. Finally, we show that $\overline{\Omega}_{Cij} < 0$ always when $\tau_I > 1$, $\tau_X > 1$ and $\tau_L = \tau_L^N$. Hence, when a Nash equilibrium exists it must be such that $\tau_I^N < 1$.

(1) We use $\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = 0$ to solve for τ_L and τ_I and we obtain two sets of solutions, (τ_L^1, τ_I^1) and (τ_L^2, τ_I^2):

$$\begin{aligned}\tau_L^1 &= \frac{\varepsilon-1}{\varepsilon}\\ \tau_I^1 &= \frac{(1-\alpha)\delta_{ii}^2(\varepsilon(1-\tau_X)+\tau_X)-\alpha\varepsilon\tau_X+\delta_{ii}\varepsilon((\varepsilon-1+\alpha)\tau_X-\varepsilon)}{(1-\alpha)(1-\delta_{ii})\tau_X[\varepsilon(1-\delta_{ii})+\delta_{ii}\tau_X(\varepsilon-1)]}\\ &\quad+\frac{\delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon(\tau_X-1)-\tau_X)\Phi_i}{(1-\alpha)(1-\delta_{ii})(\varepsilon-1)^2\tau_X[\varepsilon(1-\delta_{ii})+\delta_{ii}\tau_X(\varepsilon-1)]}\\ \tau_L^2 &= -\alpha\frac{1+\varepsilon(\varepsilon-2+\Phi_i)}{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})+\alpha(1-\delta_{ii})\tau_X]+(1-\alpha)\varepsilon\Phi_i}\\ \tau_I^2 &= -\frac{\alpha}{1-\alpha}\end{aligned}$$

Note that $\tau_I^2 < 0$, which is outside the admissible range for τ_I. Thus, the only possible solution is (τ_L^1, τ_I^1), implying that when a Nash equilibrium exists, it must be that $\tau_L^N = \frac{\varepsilon-1}{\varepsilon}$. We can thus substitute τ_L^N into $\overline{\Omega}_{LCi}$,

$\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ (labeling these expressions $\overline{\Omega}^N_{LCi}$, $\overline{\Omega}^N_{Cii}$, and $\overline{\Omega}^N_{Cij}$ respectively) to obtain:

$$\overline{\Omega}^N_{LCi} = \overline{\Omega}^{\mathcal{N}}_{LCi} + \overline{\Omega}^{\Phi}_{LCi}$$
$$\overline{\Omega}^N_{Cii} = -\frac{\overline{\Omega}^N_{LCi}}{\varepsilon}$$
$$\overline{\Omega}^N_{Cij} = \overline{\Omega}^{\mathcal{N}}_{Cij} + \overline{\Omega}^{\Phi}_{Cij}$$

where

$$\overline{\Omega}^{\mathcal{N}}_{LCi} \equiv (\varepsilon-1)^2[\delta_{ii}(\varepsilon-(\varepsilon-1)\tau_X)(\varepsilon-(1-\alpha)\delta_{ii}) + \delta_{ii}(\varepsilon-1)\tau_X((1-\alpha)(1-\delta_{ii})\tau_I\tau_X) + \varepsilon((1-\delta_{ii})(\alpha+(1-\alpha)(1-\delta_{ii})\tau_I)\tau_X)]$$
$$\overline{\Omega}^{\Phi}_{LCi} \equiv \delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon-(\varepsilon-1)\tau_X)\Phi_i$$
$$\overline{\Omega}^{\mathcal{N}}_{Cij} \equiv (\varepsilon-1)\left[\delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1) + \delta_{ii}\tau_I(\alpha\varepsilon+\delta_{ii}(\varepsilon-1)(1-\alpha)) + (1-\delta_{ii})(\varepsilon-1)\left(\alpha+\varepsilon^{-1}\delta_{ii}(1-\alpha)(\varepsilon-1)\right) + (1-\delta_{ii})(\varepsilon-1)\tau_I\tau_X\left(\varepsilon^{-1}(1-\alpha)(\varepsilon-1)(1-\delta_{ii})-\alpha-(1-\alpha)(1-\delta_{ii})\tau_I\right)\right]$$
$$\overline{\Omega}^{\Phi}_{Cij} \equiv \delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1)\Phi_i$$

Note that $\overline{\Omega}^N_{Cii}$ and $\overline{\Omega}^N_{LCi}$ are collinear. In the next steps we thus use only $\overline{\Omega}^N_{LCi}$ and $\overline{\Omega}^N_{Cij}$ to characterize the Nash equilibrium for the remaining two instruments, τ^N_X and τ^N_I.

(2) First, observe that $\varepsilon-(\varepsilon-1)\tau_X > 0$ iff $\tau_X < \frac{\varepsilon}{\varepsilon-1}$. This implies that when $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then both $\overline{\Omega}^{\mathcal{N}}_{LCi} > 0$ and $\overline{\Omega}^{\Phi}_{LCi} > 0$. Therefore, $\overline{\Omega}^N_{LCi} > 0$ for all $\tau_X < \frac{\varepsilon}{\varepsilon-1}$, implying that there cannot be a Nash equilibrium in this region as it will never be the case that $\overline{\Omega}^N_{LCi} = 0$. Thus, in the Nash equilibrium it must be the case that $\tau^N_X > \frac{\varepsilon}{\varepsilon-1} > 1$.

(3) What remains to show is that $\tau^N_I < 1$. We prove this by contradiction. Assume $\tau^N_I > 1$. In the previous point, we already showed that $\tau^N_X > 1$, thus if $\tau^N_I > 1$ also $\tau^N_I\tau^N_X > 1$. First, consider that $\overline{\Omega}^{\Phi}_{Cij} < 0$ when $\tau^N_I > 1$. As a consequence, a necessary condition for the Nash equilibrium to exist in the region $\tau_I > 1$ is that there exist a $\tau_I > 1$ such that $\overline{\Omega}^{\mathcal{N}}_{Cij} > 0$. To see whether this is the case, observe that $\overline{\Omega}^{\mathcal{N}}_{Cij}$ is linear in α since δ_{ii} (as implicitly determined by conditions (6)-(9)) is independent of α. Moreover, when $\alpha = 0$ $\overline{\Omega}^{\mathcal{N}}_{Cij} = (\varepsilon-1)^2\left[-\delta_{ii}(1-\delta_{ii}+\varepsilon(\tau_I-1)(\varepsilon-\delta_{ii})) - (1-\delta_{ii})^2(1+\varepsilon(\tau_I-1))\tau_I\tau_X\right] < 0$ while when $\alpha = 1$, $\overline{\Omega}^{\mathcal{N}}_{Cij} = -(\varepsilon-1)^2[(\tau_I\tau_X-1)(1-\delta_{ii}) + \delta_{ii}\varepsilon(\tau_I-1)] < 0$. This implies that $\overline{\Omega}^{\mathcal{N}}_{Cij} < 0$ for all $\tau_I > 1$. Therefore, $\overline{\Omega}^N_{Cij} < 0$ for all $\tau_I > 1$ which contradicts our original hypothesis of a Nash equilibrium with $\tau^N_I > 1$. Thus, if a Nash equilibrium exists it must be such that $\tau^N_I < 1$. ■

F.2 Proof of Lemma 6

Proof We prove Lemma 6 point by point.

(a) First note that, when $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$, both production efficiency and consumption efficiency effects are zero so that condition (33) simplifies to:

$$dV_i = \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \tag{F-7}$$

where we made use of (F-1) to write the terms-of-trade effect as function of dL_{Ci}, dC_{ii}, and dC_{ij}.

As explained in section B.2.2, conditions (B-11), (B-17), and (B-19) can be used to find an explicit solution for dL_{Ci}, dC_{ii} and dC_{ij} as linear functions of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ for $i = H, F$. Imposing symmetry of the initial conditions, $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$, as well as $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$, we can rewrite (F-7) as function only of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$, and evaluate the welfare effects of a unilateral marginal change in each of the policy instruments.

When $d\tau_{Li} = d\tau_{Xi} = 0$ then $dV_i = \frac{L_{Ci}(\delta_{ii}-1)((1-\delta_{ii})(\varepsilon-1)^2(\varepsilon 2\delta_{ii}-(1-\alpha)(2\delta_{ii}-1))+\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)\varepsilon} d\tau_{Ii}$. Note that $\varepsilon > 1-\alpha$ and $2\delta_{ii} > 2\delta_{ii} - 1$. Therefore, $\varepsilon 2\delta_{ii} - (1-\alpha)(2\delta_{ii} - 1) > 0$ implying that the numerator is always negative. The sign of the denominator depends on AB where $A \equiv 2\delta_{ii} - 1$ and $B \equiv \alpha + (2\delta_{ii} - 1)(\varepsilon - 1)$. Note that $A > 0$ if and only if $\delta_{ii} > \frac{1}{2}$ and $B > 0$ if and only if $\delta_{ii} > \frac{1}{2} - \frac{\alpha}{2(\varepsilon-1)}$. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Ii} < 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

When $d\tau_{Li} = d\tau_{Ii} = 0$ then $dV_i = \frac{L_{Ci}(1-\delta_{ii})((\varepsilon-1)^2(\varepsilon(1-\delta_{ii})-\delta_{ii}(\varepsilon-1+\alpha)(1-2\delta_{ii}))+\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)\varepsilon} d\tau_{Xi}$. Note that $\varepsilon > \varepsilon - 1 + \alpha$ and $1-\delta_{ii} > 1-2\delta_{ii}$ thus, $\varepsilon(1-\delta_{ii}) > \delta_{ii}(\varepsilon-1+\alpha)(1-2\delta_{ii})$ and the numerator is always positive. The denominator is the same as in the previous point. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Xi} > 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $dV_i = \frac{L_{Ci}(1-\delta_{ii})\varepsilon((\varepsilon-1)^2+2\delta_{ii}(\varepsilon-1+\alpha)\Phi_i)}{(2\delta_{ii}-1)(\alpha+(2\delta_{ii}-1)(\varepsilon-1))(\varepsilon-1)^2} d\tau_{Li}$. Note that the numerator is always positive. The sign of the denominator depends on AB where A and B have been defined above. Therefore, the denominator is positive and thus $dV_i > 0$ when $d\tau_{Li} > 0$ if and only if either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$.

(b) We now compute the imports from the differentiated sector in the 3 scenarios.

When $d\tau_{Li} = d\tau_{Xi} = 0$ then

$$dC_{ij} = -C_{ij}\varepsilon \frac{A_{\tau_{Ii}} + \Phi_i B_{\tau_{Ii}}}{C_{\tau_{Ii}}} d\tau_{Ii}$$

where $A_{\tau_{Ii}} \equiv (\varepsilon-1)^2((\varepsilon-1)(2\alpha\delta_{ii}(\delta_{ii}^2+1-\delta_{ii})+((1-\delta_{ii})^2+\delta_{ii}^2)((1-\alpha)(1-\delta_{ii})+\delta_{ii}(\varepsilon-1)))+\alpha\delta_{ii}(1-\delta_{ii})+\alpha^2\delta_{ii}^2)$, $B_{\tau_{Ii}} \equiv \alpha(\varepsilon-1)(2\delta_{ii}^2+1-\delta_{ii})+\alpha^2\delta_{ii}+((1-\delta_{ii})^2+\delta_{ii}^2)(\varepsilon-1)^2$, and $C_{\tau_{Ii}} \equiv -(1-2\delta_{ii})2(\varepsilon-1)\left(\delta_{ii} - \frac{1}{2}\left(1 - \frac{\alpha}{\varepsilon-1}\right)\right)(\varepsilon-1)^2(\varepsilon-1+\alpha)$

When $d\tau_{Li} = d\tau_{Ii} = 0$ then

$$dC_{ij} = C_{ij}(1-\delta_{ii})\varepsilon \frac{A_{\tau_{Xi}} + \Phi_i B_{\tau_{Xi}}}{C_{\tau_{Xi}}} d\tau_{Xi}$$

where $A_{\tau_{Xi}} \equiv 2(\varepsilon-1)(1-\alpha)\delta_{ii}(1-\delta_{ii})+\alpha(\varepsilon-(1-\alpha)\delta_{ii})+2\delta_{ii}^2(\varepsilon-1)(\varepsilon-1+\alpha)$, $B_{\tau_{Xi}} \equiv \delta_{ii}(\alpha+2\delta_{ii}(\varepsilon-1))\varepsilon(\alpha+\varepsilon-1)$, and $C_{\tau_{Xi}} = C_{\tau_{Ii}}$

When $d\tau_{Ii} = d\tau_{Xi} = 0$ then

$$dC_{ij} = C_{ij}\varepsilon^2 \frac{A_{\tau_{Li}} + \Phi_i B_{\tau_{Li}}}{C_{\tau_{Li}}} d\tau_{Li}$$

where $A_{\tau_{Li}} \equiv 2\delta_{ii}(\varepsilon-1)(1-\delta_{ii})+\delta_{ii}(\varepsilon-1)^2+\alpha(\varepsilon-1)(1-\delta_{ii}+2\delta_{ii}^2)+\alpha(1-\delta_{ii})+\delta_{ii}\alpha^2$, $B_{\tau_{Li}} \equiv \delta_{ii}\varepsilon(\alpha+\varepsilon-1)^2$, and $C_{\tau_{Li}} = C_{\tau_{Ii}(\varepsilon-1)}$

First note that $A_{\tau_{Ii}}$, $A_{\tau_{Xi}}$, $A_{\tau_{Li}}$, $B_{\tau_{Ii}}$, $B_{\tau_{Xi}}$, and $B_{\tau_{Li}}$ are always positive. Note that $C_{\tau_{Ii}} > 0$ (and therefore also $C_{\tau_{Xi}} > 0$ and $C_{\tau_{Li}} > 0$) when either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$. It then follows that $dC_{ij} > 0$ when either $0 < \delta_{ii} < \frac{1}{2} - \frac{\alpha}{2(\epsilon-1)}$ or $\delta_{ii} > \frac{1}{2}$ and $d\tau_{Ii} < 0$, or $d\tau_{Xi} > 0$, or $d\tau_{Li} > 0$. ■

F.3 Proof of Proposition 4

Proof We prove Proposiiton 4 point by point.

(a) When only production taxes are available $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Therefore, the consumption-efficiency wedges in (33) are absent. Hence, to prove this point it is sufficient to rewrite the term-of-trade effect as a function of dL_{Ci} only, and then add it to the production-efficiency term.

For this purpose, we follow the same approach used in point (a) of Proof F.1. We use the differentials of the equilibrium conditions derived in Appendix B.2.2 to evaluate each component of the terms-of-trade effects as decomposed in (E-7) with the difference that in this case we do not only impose symmetry and $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ but also the restrictions $d\tau_{Ii} = d\tau_{Xi} = 0$. Moreover, given the system of 3 equations ((B-11), (B-17), and (B-19)) in 6 variables ($d\tau_{Li}$, $d\tau_{Ii}$, $d\tau_{Xi}$, dL_{Ci}, dC_{ii}, dC_{ij}) and given that here we are imposing $d\tau_{Ii} = d\tau_{Xi} = 0$, we are able to express $d\tau_{Li}$, dC_{ii}, dC_{ij} as a function of dL_{Ci} only. This allows us to obtain:

$$C_{ji}dP_{ji} - C_{ij}dP_{ij} = \Sigma_i dL_{Ci}$$

with:

$$\Sigma_i \equiv \frac{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})[(\alpha(2\delta_{ii}-1)(1+\varepsilon(\tau_{Li}-1)) - \varepsilon\tau_{Li}) - 2\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i]}{(\varepsilon-1)\Sigma_{di}} \tag{F-8}$$

$$\begin{aligned}\Sigma_{di} \equiv\, &(\varepsilon-1)\left[(1-\delta_{ii})(1+2\delta_{ii}(\varepsilon-1))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li})\right]\\ &+ 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i\end{aligned} \tag{F-9}$$

Then, in this case condition (33) can be simplified as:

$$\begin{aligned}dV_i =&\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1\right)dL_{Ci} + C_{ji}dP_{ji} - C_{ij}dP_{ij}\\ =&E_i dL_{Ci} + \Sigma_i dL_{Ci}\\ =&\Omega_i dL_{Ci}\end{aligned} \tag{F-10}$$

where $\Omega_i \equiv E_i + \Sigma_i$ and $E_i \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1$. Condition (F-10) corresponds to condition (36) in the main text.

(b) Characterizing the Nash problem when only production taxes are available means solving the constrained problem in (32) imposing $\tau_{Ii} = \tau_{Xi} = 1$. We follow the same steps explained in general terms in Appendix B.1.2. The problem can be reduced to a maximization problem in 23 variables (22 endogenous variables plus 1 policy instrument) subject to the equilibrium conditions (6)-(13). In the previous point we showed how to rewrite the total differential of (32) as in (36) namely as a function of one total differential only, dL_{Ci}. The number of policy instruments available to the individual-country policy maker is also one. This implies that at the optimum condition (36) must be equal to zero, i.e., $\Omega_i = 0$. Note how we can rewrite Ω_i as:

$$\Omega_i = \frac{\overline{\Omega}_i}{(\varepsilon-1)\Sigma_{di}}$$

where

$$\begin{aligned}\overline{\Omega}_i \equiv\, &(\varepsilon-1)\left[(1+\varepsilon(\tau_{Li}-1))((1-\delta_{ii})(1-\alpha+2\delta_{ii}(\varepsilon-(1-\alpha)))(\alpha+(1-\alpha)\tau_{Li})\right.\\ &\left.+(1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li})) - (1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})\varepsilon\tau_{Li}\right]\\ &+ 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-(1-\alpha))(\tau_{Li}-1)\Phi_i\end{aligned} \tag{F-11}$$

Given this last condition we can conclude that $\Omega_i = 0$ iff $\overline{\Omega}_i = 0$.

(c) First, note that $\overline{\Omega}_i$ is a function of 6 variables: τ_{Li}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}. Second, under symmetry and when $\tau_{Ii} = \tau_{Xi} = 1$, the equilibrium equations (6)-(9) give us 5 conditions, which provide a solution for φ_{ii}, φ_{ji}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ji}$, and δ_{ii} independently from τ_{Li}. Hence, condition

$$\overline{\Omega}_i = 0 \tag{F-12}$$

jointly with conditions (6)-(9) allows us to fully characterize the Nash equilibrium when only the production tax is available.

For what follows, note that $\overline{\Omega}_i$ can be conceived as a quadratic polynomial in τ_{Li} (called $\overline{\Omega}_i(\tau_{Li})$). Differently from the Nash problem with all instruments, the symmetric Nash-equilibrium policy will not affect the profit-share from sales in the domestic market and thus δ_{ii} can be determined independently of τ_{Li}. Moreover, $\overline{\Omega}_i(0) < 0$ for $0 < \delta_{ii} \leq 1$ and $\overline{\Omega}_i(0) = 0$ when $\delta_{ii} = 0$ since $\overline{\Omega}_i(0) = -(\varepsilon-1)^2\alpha\left[(1-\delta_{ii})(1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1)) -(1-2\delta_{ii})(1-\alpha)\right] - 2\alpha(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i$ and both $1-\delta_{ii} > 1-2\delta_{ii}$ and $1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1) > 1-\alpha$. In addition, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = -(1-\delta_{ii})(\alpha+\varepsilon-1)\left[(\varepsilon-1)^2 + 2\delta_{ii}(\alpha+\varepsilon-1)\Phi_i\right]\varepsilon^{-1}$. Hence, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$ for $0 \leq \delta_{ii} < 1$

and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = 0$ when $\delta_{ii} = 1$. Moreover, observe that $\overline{\Omega}_i(1) = (2\delta_{ii}-1)(\varepsilon-1)\left[(1-\delta_{ii})(\varepsilon-1+\alpha)+\delta_{ii}(1-\alpha)\right]$. As a consequence, $\overline{\Omega}_i(1) \geq 0$ iff $\delta_{ii} \geq \frac{1}{2}$. Finally, take into account that $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon[(\varepsilon-1)\varpi_i(\delta_{ii}) +2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$ is linear in δ_{ii} and can be characterized as follows: $\varpi_i(0) = 2\varepsilon+\alpha-3 \geq 0$ iff $\varepsilon \geq \frac{3-\alpha}{2}$, $\varpi_i(1) = 1-\alpha > 0$ and $\varpi_i(\delta_{ii}) \geq 0$ iff $\delta_{ii} \geq \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}$. Now, we are ready to prove points (i) and (ii) point by point.

(i) Consider the case $\delta_{ii} \geq \frac{1}{2}$. This implies that $\overline{\Omega}_i(1) \geq 0$. Recall that $\overline{\Omega}_i(\tau_{Li})$ is quadratic, implying that it has at most two zeros. Note that $\overline{\Omega}_i(0) < 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$. If $\overline{\Omega}_i''(\tau_{Li}) \geq 0$ then $\overline{\Omega}_i(\tau_{Li})$ is convex, and the zeros must be such that $\tau_L^1 < 0$ and $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^2 \leq 1$. However, $\tau_{Li} \geq 0$ by assumption. Hence, as long as $\delta_{ii} \geq \frac{1}{2}$ and $\overline{\Omega}_i''(\tau_{Li}) \geq 0$, there exist a unique symmetric Nash equilibrium, namely $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N = \tau_L^2 \leq 1$. Therefore, what remains to show in order to prove point (c) (i) is that $\overline{\Omega}_i''(\tau_{Li}) \leq 0$ when $\delta_{ii} \geq \frac{1}{2}$. The second derivative is given by $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon\left[(\varepsilon-1)\varpi_i(\delta_{ii})+2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i\right]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$.

Note that if $\varepsilon \geq \frac{3-\alpha}{2}$, then by linearity $\varpi_i(\delta_{ii}) \geq 0$ for all $0 \leq \delta_{ii} \leq 1$. Instead, if $\varepsilon < \frac{3-\alpha}{2}$, then $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} \leq 1$. However, we can show that $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ when $\varepsilon < \frac{3-\alpha}{2}$. Indeed, $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $\frac{2\varepsilon+\alpha-3}{\varepsilon+\alpha-2} < 1$ and $\varepsilon+\alpha-2 < 0$ when $\varepsilon < \frac{3-\alpha}{2}$. Therefore, in this case $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $2\varepsilon+\alpha-3 > \varepsilon+\alpha-2$. This inequality holds since $\varepsilon > 1$. As a consequence, $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{1}{2} \leq \delta_{ii} \leq 1$, which implies that $\overline{\Omega}_i(\tau_{Li})$ is convex in this parameter range.

(ii) Now consider the case $\delta_{ii} < \frac{1}{2}$. In this case $\overline{\Omega}_i(1) < 0$. In the previous point we have already argued that $\overline{\Omega}_i(\tau_{Li})$ is convex when either $\varepsilon \geq \frac{3-\alpha}{2}$ or when $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ and $\varepsilon < \frac{3-\alpha}{2}$. Since $\overline{\Omega}_i(\tau_{Li})$ is quadratic $\overline{\Omega}_i(0) \leq 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$, there exist two zeros of $\overline{\Omega}_i(\tau_{Li})$ such that $\tau_L^1 \leq 0$ and $\tau_L^2 > 1$. Again, we can exclude $\tau_L^1 \leq 0$ since $\tau_{Li} > 0$ by assumption. As a consequence, there exists a unique symmetric Nash equilibrium with $\tau_L^N = \tau_L^2 \geq 1$.

■

F.4 Proof of Proposition 7

Proof I We prove Proposition 7 point by point.

(a) First, consider the case of heterogeneous firms. According to Proposition 4, when $\delta_{ii} \geq \frac{1}{2}$ and only domestic policies are available any symmetric Nash equilibrium is such that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$. Hence, a sufficient condition for the Nash allocation to entail higher welfare than the free-trade allocation is that in a symmetric equilibrium individual-country welfare is monotonically decreasing in τ_{Li}. In other words, we need to demonstrate that in a symmetric equilibrium $\frac{dU_i}{d\tau_{Li}} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. To show this result, first observe that $\frac{dU_i}{d\tau_{Li}} = \frac{dU_i}{dL_{Ci}}\frac{dL_{Ci}}{d\tau_{Li}}$. Second, consider that the total differential of the utility in (3) can be written as in condition (D-2). Then, if we combine this total differential with the total differential of (12) and (13) departing from a symmetric allocation we get:

$$dU_i = -\frac{P_{ii}}{I_i}dC_{ii} - \frac{P_{ij}}{I_i}dC_{ij} + -\frac{1}{I_i}dL_{Ci}$$

Moreover, it can be shown[64] that under symmetry $dC_{ij} = \frac{C_{ij}}{L_{Ci}}\frac{\varepsilon}{\varepsilon-1}dL_{Ci}$ for $i, j = H, F$. By substituting these conditions into the differential above and taking into account conditions (10) and (11) we obtain:

$$dU_i = \frac{1}{I_i}\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)dL_{Ci} \tag{F-13}$$

This last result follows directly from the fact that symmetric deviations of the production subsidy from a symmetric allocation do not have an impact on the cut offs φ_{ij} and on the market shares δ_{ij}, implying that terms-of-trade effects are zero. Moreover, consumption-efficiency wedges are also zero since import tariffs and export taxes are absent. Hence, changes in welfare in condition (33) are equal to the production-efficiency effects only. Finally, it can be shown that:

$$\frac{dL_{Ci}}{d\tau_{Li}} = -\frac{(1-\alpha)L_{Ci}}{\alpha+\tau_{Li}(1-\alpha)} < 0$$

[64] The proof is available on request.

This allows us to conclude that $\frac{dU_i}{d\tau_{Li}} = -\frac{L_{Ci}}{I_i}\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)\frac{1-\alpha}{\alpha+\tau_{Li}(1-\alpha)} \leq 0$ if and only if $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. Moving to the homogeneous-firm set up, it is easy to show that when starting from a symmetric allocation condition (F-13) still holds. Moreover, Campolmi et al. (2014) have already proved that also in this case $\frac{dL_{Ci}}{d\tau_{Li}} = \frac{dN_i}{d\tau_{Li}} < 0$. Therefore with both homogeneous and heterogeneous firms, $\frac{dU_i}{d\tau_{Li}} \leq 0 \iff \tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$ and independently of the value of δ_{ii}. We know from Proposition 4 that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$ when $\delta_{ii} \geq \frac{1}{2}$. As a consequence, when $\delta_{ii} < \frac{1}{2}$ the symmetric Nash equilibrium is welfare dominated by the free-trade allocation.

(b) By taking the the differential of conditions (6), (7) and (8) with respect to f_{ij} and τ_{ij}, it can be shown that:

$$d\delta_{ii} = \frac{(\varepsilon - 1 + \Phi_i)\delta_{ii}(1-\delta_{ii})}{\tau_{ij}} d\tau_{ij} + \frac{\Phi_i \delta_{ii}(1-\delta_{ii})\tilde{\varphi}_{ij}^{1-\varepsilon}\varphi_{ij}^{\varepsilon-1}}{(\varepsilon-1)f_{ij}} df_{ij},$$

which confirms that δ_{ii} is monotonically increasing in both τ_{ij} and f_{ij}. ■