Supplementary Appendix for "Productivity Differences in an Interdependent World"

Harald Fadinger*

University of Vienna

August 2009

*Correspondence: Department of Economics, University of Vienna, Hohenstaufengasse 9, A-1010 Vienna, Austria. e-mail: harald.fadinger@univie.ac.at

1 The Productivity Calibration Problem (PCP)

Definition 2: A **Productivity Calibration Problem** (**PCP**) is a collection of goods prices $\{\tilde{p}_i\}$, productivity-equivalent wages $\{\hat{w}_d\}$, productivity-equivalent rental rates $\{\hat{r}_d\}$, numbers of sectoral varieties $\{N_{id}\}$ and factor productivities $\{A_{Hc}\}$, $\{A_{Kc}\}$ such that given a cross section of human capital endowments $\{H_c\}$, physical capital endowments $\{K_c\}$, wages $\{w_c\}$, rentals $\{r_c\}$ and parameters $\{\alpha_i\}$, $\{\beta_i\}$ and ϵ the following system of equations holds for all $d \in D$:

$$[\alpha_i^{\epsilon}\hat{r}_d^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\hat{w}_d^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \geq \tilde{p}_i \tag{1}$$

with

$$\{\tilde{p}_i - [\alpha_i^{\epsilon}\hat{r}_d^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\hat{w}_d^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\}N_{id} = 0 \tag{2}$$

$$\sum_{i\in I}[\alpha_i^{\epsilon}\hat{r}_d^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\hat{w}_d^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}(1-\alpha_i)^{\epsilon}\hat{w}_d^{-\epsilon}N_{id} = \sum_{c\in d} A_{Hc}H_c \tag{3}$$

$$\sum_{i\in I}[\alpha_i^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}(\alpha_i)^{\epsilon}\hat{r}_d^{-\epsilon}N_{id} = \sum_{c\in d} A_{Kc}K_c \tag{4}$$

$$\tilde{p}_i \sum_{d\in D} N_{id} = \beta_i \sum_{c\in C} Y_c \quad i = 1, ..., I-1 \tag{5}$$

$$\prod_{i=1}^{I}\left(\frac{P_i}{\beta_i}\right)^{\beta_i} = 1; \tag{6}$$

$$A_{Hc} = \frac{w_c}{\hat{w}_d} \tag{7}$$

$$A_{Kc} = \frac{r_c}{\hat{r}_d} \tag{8}$$

1.1 Relation between Equilibrium and PCP

Regarding the connection between the **PCP** and an **Equilibrium**, one can establish the following relationships.

Lemma 1: If given $\{H_c\}$, $\{K_c\}$, $\{w_c\}$, $\{r_c\}$, parameters $\{\alpha_i\}$, $\{\beta_i\}$, and ϵ we have that $\{\tilde{p}_i\}$, $\{\hat{w}_d\}$, $\{\hat{r}_d\}$, $\{N_{id}\}$, $\{A_{Hc}\}$, $\{A_{Kc}\}$ are a solution to the **PCP** then $\{\tilde{p}_i\}$, $\{\hat{w}_d\}$, $\{\hat{r}_d\}$, $\{N_{id}\}$ are also an **Equilibrium** given $\{A_{Hc}H_c\} = \{\hat{H}_c\}$, $\{A_{Kc}K_c\} = \{\hat{K}_c\}$.

Proof: Follows from inspecting the equations of **PCP**.

Lemma 2: If given $\{H_c\}$, $\{K_c\}$, $\{\alpha_i\}$, $\{\beta_i\}$ and ϵ, we have that $\{\tilde{p}_i\}$, $\{w_d\}$, $\{r_d\}$, $\{N_{id}\}$ are an **Equilibrium** then they also solve the **PCP** given $\{H_c\}$, $\{K_c\}$, $\{w_d\}$ and $\{r_d\}$ with $\{A_{Hc}\} = \{A_{Kc}\} = \{1\}$.

Proof: Follows from inspecting the equations of **PCP**.

1.2 Uniqueness of Solution to PCP

Simsek, Ozdaglar and Acemoglu (2006) have derived a sufficient condition for the uniqueness of the solution to nonlinear complementarity problems. If this condition is met it guarantees uniqueness of the solution to **PCP** for the particular parameter values considered.

Let $F : R^n \to R^n$ be a function. The nonlinear complementarity problem is to find a vector x that satisfies the following

$$x \geq 0, F(x) \geq 0 \tag{9}$$

$$x^T F(x) = 0 \tag{10}$$

Note that **PCP** has the structure of a nonlinear complementarity problem.

Denote the set of solutions to (9), (10) by NCP(F). Define the index sets

$$I^{NB}(x) = \{i \in \{1, ..., n\} | x_i > 0\} \tag{11}$$

$$I^F(x) = \{i \in \{1, ..., n\} | F_i(x) = 0\} \tag{12}$$

A.A1: There exists a compact set $C \subset R^n_+$ such that for all x $\in R^n_+ - C$ there exists some $y \in C$ and $i \in 1, ..., n$ such that $(y_i - x_i)F_i(x) < 0$.

A.A2: Let $U^n_+ \subset R^n$ be an open set containing R^n_+ and $F : U^n_+ \to R^n$ be a continuous function that is continuously differentiable at every $x \in NCP(F)$. We have $det(\bigtriangledown F(x)|_J) > 0$ for every $x \in$ NCP(F) and for every index set J such that $I^{NB}(x) \subseteq J \subseteq I^F(x)$.

Theorem (Simsek, Ozdaglar, Acemoglu): Let $U^n_+ \subset R^n$ be an open set containing R^n_+ and $F : U^n_+ \to R^n$ be a continuous function which is continuously differentiable at every x $\in NCP(F)$. Let Assumptions **A.A1** and **A.A2** hold. Then NCP(F) has a unique element.

As noted above the **PCP** is a nonlinear complementarity problem. Continuity can be checked by inspection. In addition, the **PCP** satisfies **A.A1**.

Proof: Let $p^{max} \in R_+ \equiv \beta_i \sum_{c \in C} Y_c$. Let C be the rectangle $(0, (p^{max}, ..., p^{max}))$. Let $F_i(x) = \tilde{p}_i \sum_{d \in D} N_{id} - \beta_i \sum_{c \in C} Y_c$. Then for all $x \in R^n - C$ we have that $F_i(x) > 0$. Hence, it follows that by choosing $y = 0 \in C$ the condition $(y_i - x_i)F_i(x) < 0$ is satisfied.

The condition that the determinant of the Jacobian must be positive in NCFP(x), can be checked numerically. It is satisfied for all the examples with $\epsilon > 1$. When $\epsilon < 1$ multiple solutions may exist. However, the solution is unique provided that the following additional assumptions are made: 1) Countries are ranked by K_c/H_c and 2) Factor price equalization holds, when a set of countries can have equalized efficient factor prices.

2 Development Accounting

In this section I perform the typical development accounting exercise which is to ask why some countries are so much richer than others. The first question I pose is: What would the world income distribution look like if all countries had the same per capita factor endowments given their factor productivities? The experiment is to endow each country at a time with the per worker endowments of human and physical capital of the US and to compute its counterfactual income per worker for given productivities A_{Hc} and A_{Kc}.[1]

In the one-sector case, the counterfactual income per worker is given by

$$\tilde{y_c} = [\alpha(A_{Kc}k_{US})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{Hc}h_{US})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}. \quad (13)$$

The upper left panel of Figure 1 plots predicted income per worker given US per capita endowments against income per worker in the one-sector case.[2] The ratio of income per worker of the 90th to the 10th percentile is reduced from 25 to 4.5. The lower left panel plots the output gain (the ratio of predicted to actual income per worker) for this case. Obviously, income gains are largest for poor countries.

Alternatively, I ask the question what the world income distribution would look like if all countries had the US factor productivities but their own factor endowments. In this case, counterfactual income is the following,

$$\tilde{y_c} = [\alpha(A_{KUS}k_c)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{HUS}h_c)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}. \quad (14)$$

The upper right panel plots predicted against actual income per worker, while the lower right panel plots the predicted output gain. The ratio of income per worker of the 90th to the 10th percentile is reduced from 25 to 6.96, which is a smaller reduction in inequality than in the first case, where all countries have the same endowments per worker. This can also be seen from output gains which are smaller for most

[1] This experiment differs somewhat from the one performed by Caselli (2005), who asks the question: How much dispersion of the income distribution could we observe if all countries had the same A_K and A_H? He defines 100% success as a model that can generate the actual dispersion of the cross country income distribution without productivity differences. However, with factor augmenting productivities, this statistic is not very meaningful because the effect of productivities and endowments on the variance of income cannot be separated. I ask the question how compressed the income distribution would be if countries had their own productivities but the same endowments, which seems more natural to me because it addresses the question which policy would help to increase the income of poor countries.

[2] For $\epsilon = 0.836$.

countries in the second case.[3] The reason is that the US has some of the largest per capita human capital and physical capital endowments and a very high human capital productivity, whereas its physical capital productivity is rather low, so that in efficiency units poor countries have higher and more balanced per capita endowment levels when endowed with the US per capita endowments than when given the US factor productivities.

3 An Alternative Method for Estimating Factor Productivities

In the main text I have estimated productivities from **PCP** and plugged these productivities into the factor content prediction to estimate parameters. Here, I show that an alternative approach gives similar results. In this section I proceed exactly in the opposite way as in the main text - I first use the factor content prediction to obtain a set of productivity estimates and then plug these productivity estimates in the equilibrium conditions of the model to obtain parameter estimates.

In order to to so, it is convenient to consider the factor content prediction in physical units. The per unit factor use of human capital in physical units in the model is $\bar{d}_{Hic} = \left(\frac{\sigma}{\sigma-1}\right)^{-\epsilon} p_i^{\epsilon} w_c^{-\epsilon}(1-\alpha_i)^{\epsilon} A_{Hc}^{\epsilon-1}$. Hence, we can write the factor use vector of factor f in country c as $\bar{D}_{fc} = \left(\frac{\Pi_{US}}{\Pi_c}\right)^{\epsilon}\left(\frac{A_{fc}}{A_{fUS}}\right)^{\epsilon-1}\bar{D}_{fUS}$

Using the definition of the factor content of trade from Trefler and Zhu (2005), the factor content prediction (in physical units) can be written as

$$\left[\left(\frac{\Pi_{US}}{\Pi_1}\right)^{\epsilon}\left(\frac{A_{f1}}{A_{fUS}}\right)^{\epsilon-1}\bar{D}_{fUS}, ..., \left(\frac{\Pi_{US}}{\Pi_C}\right)^{\epsilon}\left(\frac{A_{fC}}{A_{fUS}}\right)^{\epsilon-1}\bar{D}_{fUS}\right](I-B)^{-1}T_c = V_{fc} - s_c\sum_{c\in C} V_{fc}. \qquad (15)$$

For a given value of ϵ, this is a system of $C-1$ linear equations in $\left(\frac{A_{fc}}{A_{USc}}\right)^{\epsilon-1}$ given data on $\bar{D}_{fUS}$, $\left(\frac{\Pi_c}{\Pi_{US}}\right)$, B and T_c. This provides me with a set of estimates $\left\{\frac{A_{fc}}{A_{USc}}\right\}$.

[3] Caselli (2005) notes that the one-sector model is able to replicate the cross-country variance in income per worker even if all countries have the same productivities (those of the US) when ϵ is sufficiently low (around 0.5). In that sense the whole cross country variation in income per worker is "explained" by factor endowments. However, this does not imply that factor accumulation would help much in reducing cross-country income differences given that we know that productivities differ. In fact, the lower the elasticity of substitution, the less powerful is factor accumulation in reducing income differences.

In a second step I use these estimates to generate productivity-equivalent endowments and I use the equilibrium conditions of the one-sector model or the ones of the two-sector muti-cone model to generate predicted income, Y_c, wages, w_c, and rentals, r_c. I choose the set of parameters ϵ (for the one-sector model) or $\alpha_h, \alpha_k, \beta_h, \epsilon$ (for the two-sector multi-cone model) to match the mean and the variance of Y_c, w_c, r_c between the model and the data (6 conditions). Note that one problem with this approach is that the one-sector and the two- sector multi-cone model have a different number of parameters and - differently from the case where I use the factor content prediction to compare across models - there is no clear criterion to discriminate between the two models.

Table 1 provides estimates of ϵ for the one-sector and the two-sector multi-cone model.[4] In both cases ϵ is estimated to be smaller than one. When using all the moment conditions in the one-sector model, the over-identification test is not passed. Therefore, I also estimate the model fitting only data on income. This gives a somewhat lower but less precise estimate of ϵ. Figure 4 provides plots of the estimates of factor productivities against income per worker, as well as plots of fitted wages, rentals and income against the data for the one sector model for $\epsilon = 0.909$.

Factor prices in the one-sector model are $w_c = \left(\frac{Y_c}{H_c}\right)^{\frac{1}{\epsilon}} A_{Hc}^{\frac{\epsilon-1}{\epsilon}}$ and $r_c = \left(\frac{Y_c}{K_c}\right)^{\frac{1}{\epsilon}} A_{Kc}^{\frac{\epsilon-1}{\epsilon}}$. With $\epsilon < 1$ factor productivities are inversely related to factor prices and therefore dampen differences in output-factor ratios. Since some poor countries have very low estimated capital productivities a lower estimate of ϵ would exacerbate rental rates for these countries. An estimate of ϵ larger than one, on the other hand, would predict too large differences in wages, since $\frac{Y_c}{H_c}$ and A_{Hc} are positively correlated.

Note that both the sets of factor productivities and estimates of ϵ in the one sector model derived using the PCP (fitting first factor prices and income data) and the ones that come out of this exercise are similar. In both cases there is a positive relation between the productivity of human capital and income per worker and no clear relation between the productivity of physical capital and income per worker.

[4] The other parameters estimates in the two-sector case are $\alpha_H = 0.2, \alpha_K = 0.7, \beta_H = 0.5$. Again, the problem is not continuously differentiable in the parameter vector because specialization patterns change discontinuously with changes in parameters and therefore standard errors of the coefficients are hard to obtain

4 Non-tradables

One concern is that a substantial fraction of goods, such as services, is not traded in international markets, and that the part of factor endowments that is used in non-tradables should be substracted from countries' endowments in the factor content predictions. In this section I show that introducing a non-tradable sector does not affect the main results on factor productivities and parameter estimates much.

Davis and Weinstein (2001) provide a definition of the factor content prediction in the presence of non-tradable goods. One disadvantage of their approach is that it does not allow for trade in intermediate goods, which is a large part of trade. Another necessary assumption is that preferences are Cobb-Douglas between tradables and non-tradables, so that the expenditure share on non-tradables is fixed.

With non-tradables the definition of the factor content of trade prediction in productivitity-equivalent units is

$$F_{fc}^{*NT} = \tilde{E}_{fcT} X_c - \sum_{c' \neq c} \tilde{E}_{fc'T} M_{cc'} = \hat{V}_{fcT} - s_c \sum_{c \in C} \hat{V}_{fcT}. \tag{16}$$

Here, $\tilde{E}_{fcT}$ is the submatrix of tradables in the total factor use matrix in productivity-equivalent units $\tilde{E}_{cT} = [\tilde{E}_{fcT}, \tilde{E}_{fcNT}]$, where $\tilde{E}_{fc} = D_{fc}(I - B_c)^{-1}$ converts direct productivity-equivalent factor use into total productivity-equivalent factor use. The matrix of direct factor use can be split into a part that corresponds to tradables and one that corresponds to non-tradables $D_{fc} = [D_{cT}, D_{cNT}]$. According to the model this can be written as $[\mathrm{D}_{UST} \left(\frac{A_{fc}}{A_{fUS}}\right)^{\epsilon} \left(\frac{\Pi_c}{\Pi_{US}}\right)^{-\epsilon}, D_{USNT} \left(\frac{A_{fc}}{A_{fUS}}\right)^{\epsilon} \left(\frac{\Pi_c}{\Pi_{US}}\right)^{-\epsilon} \left(\frac{p_{NTc}}{p_{NTUS}}\right)^{\epsilon}]$, where $\left(\frac{p_{NTc}}{p_{NTUS}}\right)$ is the relative price of non-tradables in country c. Factor endowments in productivity-equivalent units available in the tradable sector can be written as total productivity-equivalent endowments minus productivity-equivalent factor use in the non-tradable sectors, $\hat{V}_{cT} = \hat{V}_c - \hat{V}_{cNT}$. Productivity-equivalent factor use in the non-tradable sectors is computed as $\hat{V}_{cNT} = \tilde{E}_{fcNT} Q_{cNT}$, where Q_{cNT} is the vector of net output in the non-tradable sectors. Countries that have lower factor prices in productivity-equivalent units use a larger fraction of their endowments in the non-tradable sector, which tends to reduce differences in productivity-equivalent factor endowments across countries. Note that in the single-cone case, non-tradables can be ignored because when productivity-adjusted factor prices are equalized we have that

$\hat{V}_{fcNT} = s_c \sum_{c \in C} \hat{V}_{fcNT}$.

The equations of **PCP** still hold with a non-tradable sector but one needs to add C market clearing conditions for the non-tradable sector of the type $\tilde{p}_{NT} N_{cNT} = \beta_{NT} Y_c$ and country-level price indices are no longer equal because of potential differences in the price of non-tradables $P_c = \left(\frac{P_{NT}}{\beta_{NT}}\right)^{\beta_{NT}} \prod_{i=1}^{I_T} \left(\frac{P_i}{\beta_i}\right)^{\beta_i}$. Apart from these changes, the same method to estimate productivities and parameters θ as described in the main text can be applied. Since market clearing in the non-tradable sector adds one equation per country to **PCP** computations become so complex that I am not able to obtain reliable estimates for the parameters in the two-sector multi-cone case.[5]

In the empirical implimentation I assume that all service sectors are non-traded.

Table 4 reports the estimates for ϵ for the one-sector case. Again, for similar reasons as in the main text, ϵ is estimated to be significantly smaller than one. In the one-sector model non-tradables help to improve the factor content prediction to some extent because differences in factor-endowments are reduced, as rich countries use more productivity-equivalent capital in the non-tradable sector. Productivity-equivalent wages are only slightly smaller in poor countries than in rich ones, because rich countries have much higher human capital productivities. This reduces poor countries' human-capital abundance somewhat because they use more of this factor in the non-tradable sector than rich countries. Again, both predicted and measured factor content of human capital are small. Since productivity-equivalent rentals are smaller in rich countries, they use more of their capital in in the non-tradable sector than poor countries and this reduces differences in productivity equivalent endowments. At the same time also the measured factor content becomes smaller since there is no trade in intermediates. In the single-cone equilibrium, as noted above, nontradables would not affect the predicted factor content of trade. Since the definition of the measured factor content used in this section does not allow for trade in intermediates, the measured factor content is even lower than with the definition used in the main text. As discussed there, the predicted factor content of trade for physical capital is much larger than the measured one, because differences in

[5]In the multiple-cone case, I would need to determine also two additional parameters: I calibrate $\beta_{NT} = 0.6$, which is the average share of value added in services in total value added in the sample of 53 countries and I would also need a calibration for α_{NT}.

productivity-equivalent factor endowments across countries are large. Therefore, the single-cone model does not perform well.

Case	Moment Cond.	ϵ	Std(ϵ)	t-statistic	J-statistic	P-value
One-sector	all	0.909	0.017	-5.24	64.73	0
	income	0.635	0.187	-1.65	0.25	0.62
Multi Cone	all	0.72	-	-	-	-

Table 1: Robustness: Model fit alternative procedure - Parameters. GMM estimates of the elasticity of substitution between human and physical capital when productivities are estimated from the factor content prediction. Model is estimated by fitting factor price and income data. The last column is the P-value for the validity of the overidentifying restrictions.

Case	Moment Cond.	ϵ	Std(ϵ)	t-statistic	J-statistic	P-value
One-sector	all	0.833	0.095	-1.75	0.72	0.87

Table 2: Robustness: Model fit non-tradables - Parameters. GMM estimates of the elasticity of substitution between human and physical capital in model with non-tradable sector. The last column is the P-value for the validity of the overidentifying restrictions.

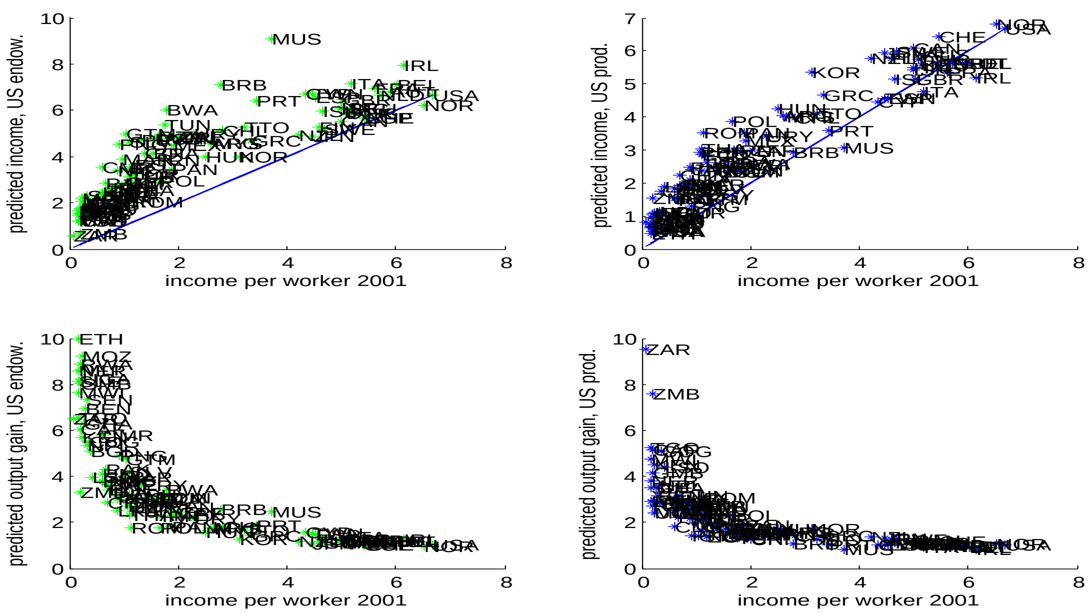

Figure 1: Development Accounting. The upper panels plot the predicted income per worker (vertical axis) against actual income per worker (horizontal axis) for the case in which all countries have the US endowments of factors per worker (left panel) or the US factor productivities (right panel) for the factor deepening world with $\epsilon = 0.836$. The lower panels plot the output gain (vertical axis) against income per worker (horizontal axis) for the same experiments.

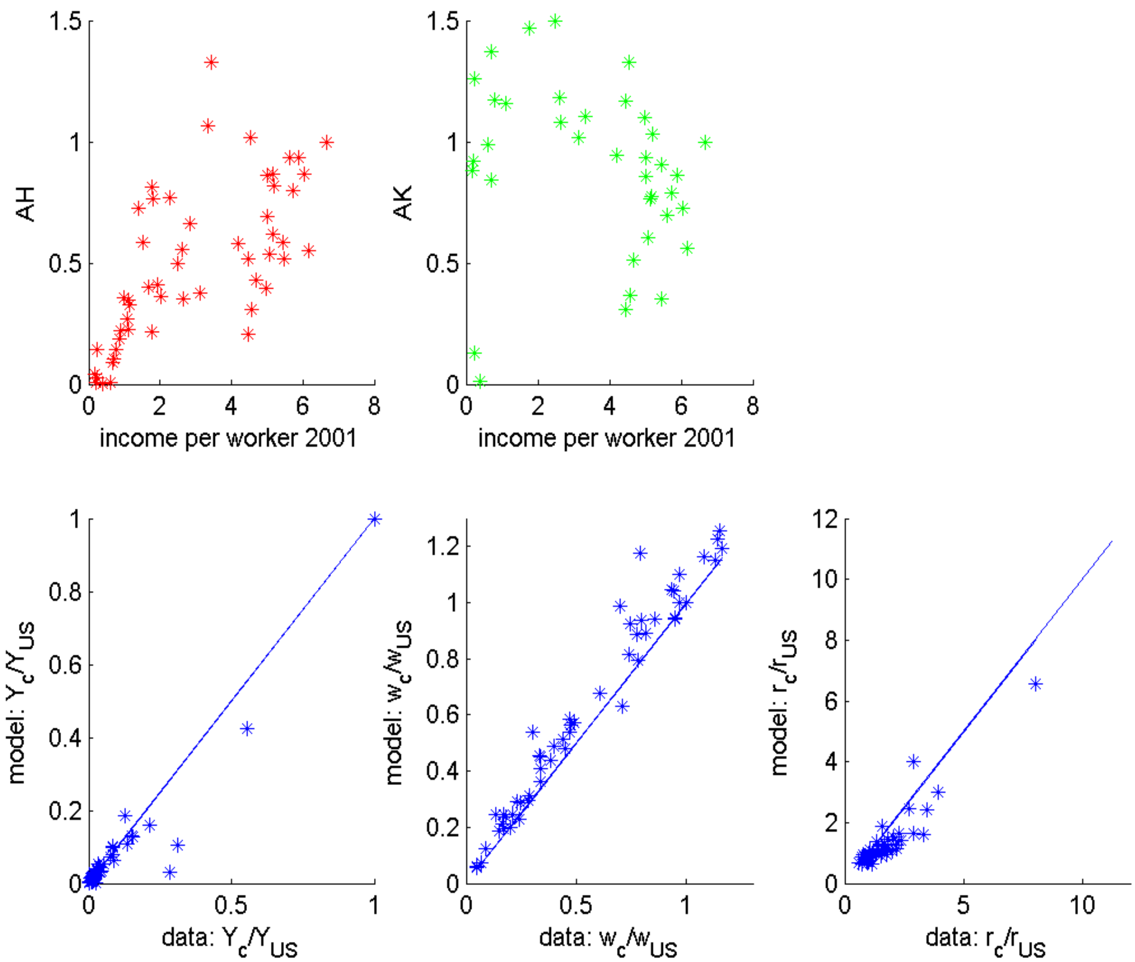

Figure 2: Robustness 1. The upper panels plot human capital productivity (left panel) and physical capital productivity (right panel) estimated from the factor content equations against income per worker. Parameter estimates are for the one-sector economy with $\epsilon = 0.909$. The lower panels plot income and factor prices predicted by the model against the data.