Income Differences and Input-Output Structure *

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Abstract

We consider a multi-sector general equilibrium model with input-output (IO) linkages and sector-specific productivities to investigate how the IO structure interacts with sectoral productivities in determining cross-country differences in aggregate income per worker. Using tools from network theory, we show that aggregate income can be approximated as a simple function of the first and second moments of the joint distribution of the IO multipliers and sectoral productivities. We then estimate the parameters of the model to fit their joint empirical distribution. Poor countries have few high-multiplier sectors, while most sectors have very low multipliers; by contrast, rich countries have more sectors with intermediate multipliers. Moreover, the correlations of sectoral IO multipliers with productivities are positive in poor countries, while being negative in rich ones. The estimated model predicts cross-country income differences extremely well and significantly better than a multi-sector model without IO linkages. Finally, we perform a number of counterfactuals and compute optimal tax rates.

KEY WORDS: input-output structure, networks, productivity, cross-country income differences, development accounting

JEL CLASSIFICATION: O11, O14, O47, C67, D85

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1 Introduction

One of the fundamental debates in economics is about how important differences in factor endowments – such as physical or human capital stocks – are relative to aggregate productivity differences in terms of explaining cross-country differences in income per capita. The standard approach to address this question is to specify an aggregate production function for value added (see, e.g., Hsieh and Klenow, 2010). Given data on aggregate income and factor endowments and the imposed mapping between them, one can back out productivity differences as a residual that explains differences between predicted and actual income. However, this standard approach ignores that GDP aggregates value added of many economic activities which are connected to each other through input-output (IO) linkages.[1] By contrast, a literature in development economics initiated by Hirschman (1958) has long emphasized that economic structure is of first-order importance to understand cross-country income differences. More recent contributions highlighting the role of IO linkages for aggregate income are Ciccone (2002) and Jones (2011 a,b).

Consider, for example, a productivity increase in the Transport sector. This reduces the price of transport services and thereby increases productivity in sectors that use transport services as an input (e.g., Mining). Increased productivity in Mining in turn increases productivity of the Steel sector by reducing the price of iron ore, which in turn increases the productivity of the Transport Equipment sector. In a second-round effect, the productivity increase in Transport Equipment improves productivity of the Transport sector and so on. Thus, IO linkages between sectors lead to multiplier effects. The IO multiplier of a given sector summarizes all these intermediate effects and measures by how much aggregate income will change if productivity of this sector changes by one percent. The size of the sectoral multiplier effect depends to a large extent on the number of sectors to which a given sector supplies and the intensity with which its output is used as an input by the other sectors.[2] We document that there are large differences in IO multipliers across sectors – e.g., most infrastructure sectors, such as Transport and Energy, have high multipliers because they are used as inputs by many other sectors,[3] while a sector such as Textiles – which does not provide inputs to many sectors – has a low multiplier. As a consequence, low productivity levels in different sectors will have very distinct effects on aggregate income, depending on the size of the sectoral IO multiplier.

[1] An important exception that highlights sectoral TFP differences is the recent work on dual economies (e.g., Restuccia, Yang, and Zhu, 2008). This literature finds that productivity gaps between rich and poor countries are much more pronounced in agriculture than in manufacturing or service sectors and this fact together with the much larger value added or employment share of agriculture in poor countries can explain an important fraction of cross-country income differences.

[2] The intensity of input use is measured by the IO coefficient, which states the cents spent on that input per dollar of output produced. There are also higher-order effects, which depend on the number and the IO coefficients of the sectors to which the sectors that use the initial sector's output as an input supply.

[3] The view that infrastructure sectors are of crucial importance for aggregate outcomes has also been endorsed by the World Bank. In 2010, the World Bank positioned support for infrastructure as a strategic priority in creating growth opportunities and targeting the poor and vulnerable. Infrastructure projects have become the single largest business line for the World Bank Group, with $26 billion in commitments and investments in 2011 (World Bank Group Infrastructure Update FY 2012-2015).

In this paper, we address the question how differences in economic structure across countries – as captured by IO linkages between sectors – affect cross-country differences in aggregate per capita income. To this end, we combine data from the World Input-Output Database (Timmer, 2012) and the Global Trade Analysis project (GTAP Version 6), in order to construct a unique dataset of IO tables and sectoral total factor productivities for a large cross section of countries in the year 2005.[4] With this data in hand, we investigate how the IO structure interacts with sectoral TFP differences to determine aggregate per capita income. First, we document that in all countries there is a relatively small set of sectors which have very large IO multipliers and whose performance thus crucially affects aggregate outcomes. Moreover, despite this regularity, we also find that there do exist substantial differences in the network characteristics of IO linkages between poor and rich economies. In particular, low-income countries typically have a large number of sectors with very low multipliers and only relatively few sectors with intermediate multipliers, while high-income countries have a more dense input-output network. To visualize these differences, in Figure 1 we plot a graphical representation of the IO matrices of two countries: Uganda (a very poor country with a per capita GDP of 964 PPP dollars in 2005) and the U.S. (a major industrialized economy with a per capita GDP of around 42,500 PPP dollars in 2005). The columns of the IO matrix are the producing sectors, while the rows are the sectors whose output is used as an input. Thus, a dot in the matrix indicates that the column sector uses some of the row sector's output as an input and a blank space indicates that there is no significant connection between the two sectors.[5]

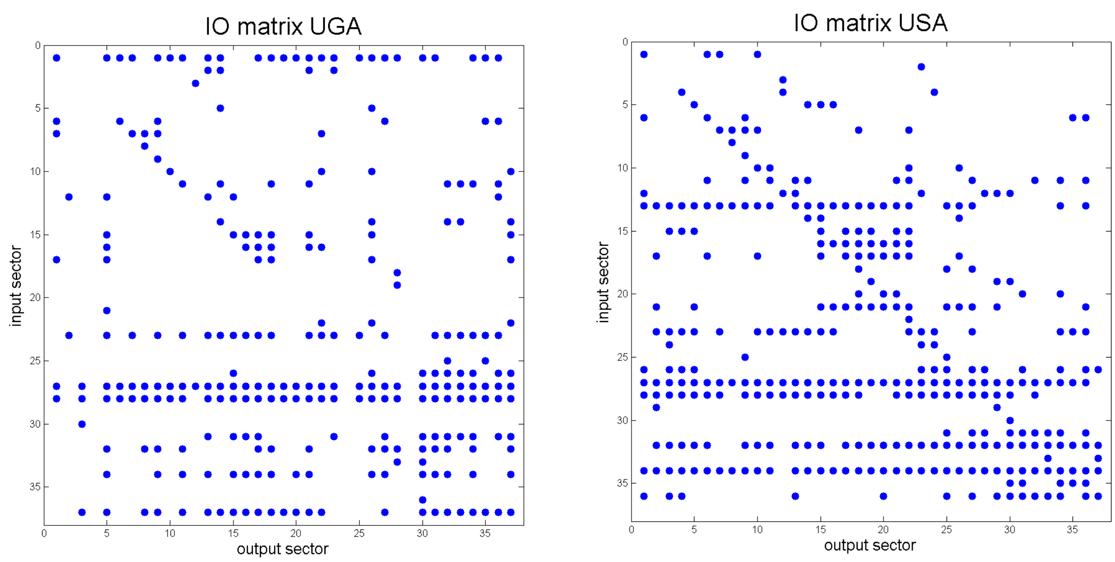


Figure 1: IO-matrices by country: Uganda (left), USA (right)

[4]Data on sectoral TFPs are available for 36 countries and data on IO tables for 65 countries.

[5]Data are from GTAP version 6, see the data appendix for details. The figure plots IO coefficients defined as cents of industry j output (row j) used per dollar of output of industry i (column i). To make the figure better interpretable, we only plot linkages with at least 2 cents per dollar of output.

By comparing the matrices it is apparent that in Uganda there are only four sectors which supply to most other sectors.[6] These are Agriculture (row 1), Electricity (row 23), Wholesale and Retail Trade (row 27), and Transport (row 28). These sectors are the high-IO-multiplier sectors, where a change in sectoral productivity has a relatively large effect on aggregate output. Most other sectors are quite isolated in Uganda, in the sense that their output is not used as an input by many sectors. By contrast, the U.S. has a much larger number of sectors that supply to many others: Chemicals (row 13), Electricity (row 23), Construction (row 26), (Wholesale and Retail) Trade (row 27), Transport (row 28), Financial Services (row 32), and Business Services (row 34), among others. This difference in IO structure between rich and poor countries has important implications for aggregate income differences: in Uganda changes in the productivity of a few crucial sectors can have large effects on aggregate income, while productivity in most sectors does not matter much for aggregate outcomes, because these sectors are isolated. By contrast, in the U.S. productivity levels of many more sectors have a significant impact on GDP because the IO network is much denser. To some extent this is good news for low-income countries: in those countries policies that focus on a few crucial sectors can have a large effect on aggregate income, while this is not true for middle-income and rich countries.

Having described the salient features of cross-country differences in IO structure, we model IO structures using tools from network theory. We analytically solve a multi-sector general equilibrium model with IO linkages and sector-specific productivities. We then estimate this model using a statistical approach that employs the moments of the distributions instead of actual values. The crucial advantage of this strategy is that it allows us to derive a simple closed-form expression for aggregate per capita income that conveniently summarizes the interactions between IO structure and sectoral productivities, without having to deal with the complicated input-output matrices directly: aggregate income is a simple function of the first and second moments of the distribution of IO multipliers and sectoral productivities.

Higher average IO multipliers and average sectoral productivity levels have a positive effect on income per capita. Moreover, a positive correlation between sectoral IO multipliers and productivities increases income. This is intuitive: high sectoral productivities have a larger positive impact if they occur in high-multiplier sectors. We estimate the parameters of the model to fit the joint empirical distribution of IO multipliers and productivities for the countries in our sample, allowing them to vary across countries in order to account for cross-country differences in these characteristics. We find that low-income countries have more extreme distributions of IO multipliers: while most sectors have very low multipliers, there are a few very high-multiplier sectors. In contrast, rich countries have relatively more sectors with intermediate multipliers. Moreover, while sectoral IO multipliers and productivities are positively correlated in low-income countries, they are negatively correlated in high-income ones.

[6] See Table A-3 in the Supplementary Appendix for the complete list of sectors.

With the parameter estimates in hand, we use our closed-form expression for income per capita as a function of the moments of the joint distribution of IO multipliers and productivities to predict income differences across countries. In contrast to standard development accounting, where the model is exactly identified, this provides an over-identification test because parameter estimates have been obtained using data on IO multipliers and productivities only. We find that our model predicts cross-country income differences extremely well both within the sample of countries that we have used to estimate the parameter values and also out of sample, i.e., in the full Penn World Tables sample (155 countries). Our model correctly predicts up to 98% of the cross-country variation in relative income per capita, which is extremely large compared to standard development accounting exercises. Moreover, our model with IO linkages does significantly better in terms of predicting income differences than a model that just averages estimated sectoral productivities and ignores IO structure (the model with IO structure explains up to 8 percentage points more of income variation).[7] In fact, the model without IO structure predicts too large cross-country income differences. The reason is that the large sectoral TFP differences that we observe in the data are to some extent mitigated by countries' IO structures, since low-productivity sectors tend to be isolated in low- and middle-income countries and the same is true for high-productivity sectors in rich countries. Thus, if we measured aggregate productivity levels by just averaging sectoral productivities, income levels of middle- and low-income countries would be significantly lower than they actually are, whereas income levels of rich countries would be even higher.

We show that our results are robust to various alternative specifications. In our baseline model, differences in IO coefficients across countries are exogenously given. However, one may be concerned that observed IO coefficients are affected by tax wedges. We thus extend our baseline model for sector-country-specific wedges on gross output, which we identify as deviations of sectoral intermediate input shares from their cross-country average value: a below-average intermediate input share in a given sector identifies a positive implicit tax wedge. We show that wedges correlate positively with IO multipliers in poor countries and negatively in rich ones, while the shape of the distribution of multipliers and the correlation between multipliers and productivities is not significantly affected by allowing for tax wedges. Moreover, introducing wedges does not improve the model's explanatory power in terms of predicting cross-country income levels.

Alternatively, when relaxing the assumption of a unit elasticity of substitution between intermediate

[7] In the light of Hulten's (1978) results, one may ask whether using a structural general equilibrium model and modeling the statistical features of the IO matrix adds much compared to computing aggregate TFP as a weighted average (where the adequate 'Domar' weights correspond to the shares of sectoral gross output in GDP) of sectoral productivities. Absent distortions, Domar weights equal sectoral IO multipliers and summarize the direct and indirect effect of IO linkages. However, such a reduced-form approach does not allow to assess which features of the IO structure matter for aggregate outcomes or to compute counterfactual outcomes due to changes in IO structure, or productivities, as we do. Finally, as Basu and Fernald (2002) show, in the presence of sector-specific distortions (as we consider in an extension) the simple reduced-form connection between sectoral productivities and aggregate TFP breaks down.

inputs, IO coefficients may be endogenous to prices. We thus extend the model to allow for a constant elasticity of substitution between intermediates different from unity. We show that the CES model is hard to reconcile with the data because – depending on whether intermediates are substitutes or complements – it predicts that IO multipliers and productivities should either be positively or negatively correlated in *all* countries. Instead, we observe a positive correlation between these variables in poor economies and a negative one in rich countries.

Moreover, we extend our baseline model and incorporate cross-country differences in final demand structure and imported intermediate inputs; we also differentiate between skilled and unskilled labor inputs. We find that our results are robust to all of these extensions.

We also perform a number of counterfactuals. First, we impose the IO structure of the U.S. on all countries, which forces them to use the relatively dense U.S. IO network. We find that this would significantly reduce income of low- and middle-income countries. For a country at 40% of the U.S. income level (e.g., Mexico) per capita income would decline by around 20% and income reductions would amount to up to 60% for the world's poorest economies (e.g., Congo). The intuition for this result is that poor countries tend to have higher-than-average relative productivity levels (relative to those of the U.S. in the same sector) in precisely those sectors that have higher IO multipliers,[8] while their typical sector is quite isolated from the rest of the economy. This implies that they do relatively well given their really low productivity levels in many sectors. Consequently, if we impose the much denser IO structure of the U.S. on poor countries – which would make their typical sector much more connected to the rest of the economy – they would be significantly poorer.

Second, we impose that sectoral IO multipliers and productivities are uncorrelated. This scenario would again hurt low-income countries, which would lose up to 10% of their per capita income, because they have above-average productivity levels in high-multiplier sectors. By contrast, high-income countries would gain up to 40% in terms of income per capita, since they tend to have below-average productivity levels in high-multiplier sectors.

Third, removing the correlation beween wedges and multipliers would have more modest effects. If low-income countries did not have above-average tax rates in high-multiplier sectors, they would gain up to 10% of per capita income.[9]

Finally, we study optimal taxation and the welfare gains from moving from the current tax rates to an optimal tax system that keeps tax revenue constant. Our results suggest that when the government is concerned with maximizing GDP per capita subject to a given level of tax revenue, the actual distribution

[8]An important exception is agriculture, which, in low-income countries, has a high IO multiplier and a below-average productivity level.

[9]These gains from reducing sector-specific wedges are more modest than those of removing plant-specific distortions for manufacturing plants in China and India, (Hsieh and Klenow, 2009).

of tax rates in rich countries is close to optimum. By contrast, in poor countries, the mean of the distribution is too low and the variance is too high relative to the optimal values. Furthermore, for a given value of tax variance, a negative correlation of taxes with IO multipliers is optimal. The poorest countries in the world could gain up to 10 % in terms of income per capita by moving to an optimal tax system.

1.1 Literature

We now turn to a discussion of the related literature.

Our work is related to the literature on development accounting (level accounting), which aims at quantifying the importance of cross-country variation in factor endowments – such as physical, human or natural capital – relative to aggregate productivity differences in explaining disparities in income per capita across countries. This literature typically finds that both are roughly equally important in accounting for cross-country income differences (see, e.g., Klenow and Rodriguez-Clare, 1997; Hall and Jones, 1999; Caselli, 2005, Hsieh and Klenow, 2010). The approach of development accounting is to specify an aggregate production function for value added (typically Cobb-Douglas) and to back out productivity differences as residual variation that reconciles the observed income differences with those predicted by the model given observed variation in factor endowments. Thus, this approach naturally abstracts from any cross-country differences in the underlying economic structure across countries. We contribute to this literature by showing how aggregate value added production functions can be derived in the presence of IO linkages that differ across countries. Moreover, we show that incorporating cross-country variation in IO structure is of first-order importance in explaining cross-country income differences.

The importance of intermediate linkages and IO multipliers for aggregate income differences has been highlighted by Fleming (1955), Hirschmann (1958), and, more recently, by Ciccone (2002) and Jones (2011 a,b). The last two authors emphasize that if the intermediate share in gross output is sizable, there exist large multiplier effects: small firm (or industry-level) productivity differences or distortions that lead to misallocation of resources across sectors or plants can add up to large aggregate effects. These authors make this point in a purely theoretical context. While our setup in principle allows for a mechanism whereby intermediate linkages amplify small sectoral productivity differences, we find that there is little empirical evidence for this channel: cross-country sectoral productivity differences estimated from the data are even larger than aggregate ones, and the sparse IO structure of low-income countries actually helps to mitigate the impact of very low productivity levels in some sectors on aggregate outcomes.

Our finding that sectoral productivity differences between rich and poor countries are larger than aggregate ones is instead similar to those of the literature on dual economies and sectoral productivity gaps in agriculture (Caselli, 2005; Chanda and Dalgaard, 2008; Restuccia, Yang, and Zhu, 2008; Vollrath, 2009; Gollin et al., 2014). Also closely related to our work – which focuses on changes in the IO structure as countries' income level increases – is a literature on structural transformation. It emphasizes sectoral productivity gaps and transitions from agriculture to manufacturing and services as a reason for cross-country income differences (see, e.g., Duarte and Restuccia, 2010 for a recent contribution). However, this literature abstracts from intermediate linkages between industries.

In terms of modeling approach, our paper adopts the framework of the multi-sector real business cycle model with IO linkages of Long and Plosser (1983); in addition we model the input-output structure as a network, quite similarly to the setup of Acemoglu et. al. (2012).[10] In contrast to these studies, which deal with the relationship between sectoral productivity shocks and economic fluctuations, we are interested in the question how sectoral productivity *levels* interact with the IO structure to determine aggregate income *levels* and provide corresponding structural estimation results.

Other recent related contributions are Oberfield (2013) and Carvalho and Voigtländer (2014), who develop an abstract theory of endogenous input-output network formation, and Boehm (2015), who focuses on the role of contract enforcement on aggregate productivity differences in a quantitative structural model with IO linkages. Differently from these papers, we do not try to model the IO structure as arising endogenously and we take sectoral productivity differences as exogenous. Instead, we aim at understanding how given differences in IO structure and sectoral productivities translate into aggregate income differences.

The number of empirical studies investigating cross-country differences in IO structure is quite limited. In the most comprehensive study up to that date, Chenery, Robinson, and Syrquin (1986) find that the intermediate input share of manufacturing increases with industrialization and – consistent with our evidence – that input-output matrices become less sparse as countries industrialize. Most closely related to our paper is the contemporaneous work by Bartelme and Gorodnichenko (2015). They also collect data on IO tables for many countries and investigate the relationship between IO linkages and aggregate income.[11] In reduced-form regressions of *aggregate* IO multipliers on income per worker, they find a positive correlation between the two variables. Moreover, they investigate how distortions affect IO linkages and income levels. Differently from the present paper, they neither use data on sectoral productivities nor network theory to represent IO tables. As a consequence, they do not investigate how differences in the distribution of sectoral multipliers and their correlations with productivities impact on aggregate income, which is the focus of our work. Furthermore, they do not address the question of

[10] Barrot and Sauvagnat (2016) provide reduced-form evidence for the short-run propagation of firm-specific shocks in the production network of U.S. firms.

[11] Grobovsek (2015) performs a development accounting exercise in a more aggregate structural model with two final and two intermediate sectors.

optimal taxation given the IO structure, while we do.

The outline of the paper is as follows. In the next section we describe our dataset and present some descriptive statistics. In the following section, we lay out our theoretical model and derive an expression for aggregate GDP in terms of the IO structure and sectoral productivities. Subsequently, we turn to the estimation and model fit. We then present a number of robustness checks and the counterfactual results, followed by the results on optimal taxation. The final section presents our conclusions.

2 Dataset and descriptive analysis

2.1 Data

IO tables measure the flow of intermediate products between different plants, both within and between sectors. The ji'th entry of the IO table is the value of output from establishments in industry j that is purchased by different establishments in industry i for use in production.[12] Dividing the flow of industry j to industry i by gross output of industry i, one obtains the IO coefficient γ_{ji}, which states the cents of industry j's output used in the production of each dollar of industry i's output.

To construct a dataset of IO tables for a range of high- and low-income countries and to compute sectoral total factor productivities and countries' aggregate income and factor endowments, we combine information from three datasets: the World Input-Output Database (WIOD, Timmer, 2012), the Global Trade Analysis Project (GTAP version 6, Dimaranan, 2006), and the Penn World Tables, Version 7.1 (PWT, Heston et al., 2012).

The first dataset, WIOD, contains IO data for 36 countries classified into 35 sectors in the year 2005. The list of countries and sectors is provided in the Supplementary Appendix Tables A-1 to A-3. WIOD IO tables are available in current national currency at basic prices.[13] In our main specification, IO coefficients are defined as the value of domestically produced plus imported intermediates divided by the value of gross output at basic prices.[14] As explained in more detail below, WIOD data also allows us to compute sectoral total factor productivities.

The second dataset, GTAP version 6, contains data for 65 countries and 37 sectors in the year 2004. We use GTAP data to obtain more information about IO tables of low-income countries. We construct IO coefficients for all 65 countries.[15]

[12]While IO Tables in principle record flows independently of whether they occur within the boundaries of the firm or between plants owned by different companies, intermediate output must usually be traded between establishments in order to be recorded in the IO tables. Flows that occur within a given plant are not measured.

[13]Basic prices exclude taxes and transport margins.

[14]In a robustness check, we separate domestically produced from imported intermediates and define domestic IO coefficients as the value of domestically produced intermediates divided by the value of gross output, while IO coefficients for imported intermediates are defined as the value of imported intermediates divided by the value of gross output. We show in the robustness section that this choice does not affect our results.

[15]Compared to the original GTAP classification, we aggregate all agricultural commodities in the GTAP data into a

Finally, the third dataset, PWT, includes data on income per capita in PPP, aggregate physical capital stocks (constructed from investment data with the perpetual inventory method) and labor endowments for 155 countries in the year 2005. In our analysis, PWT data is mainly used to make out-of-sample predictions with our model.

2.2 IO structure

To begin with, we provide some descriptive analysis of the input-output structure of the set of countries in our data. To this end, we consider the sample of countries from the GTAP database. First, we sum IO multipliers of all sectors to compute the aggregate IO multiplier. While a sectoral multiplier indicates the change in aggregate income caused by a one-percent change in productivity of one specific sector, the aggregate IO multiplier tells us by how much aggregate income changes due to a one-percent change in productivity of all sectors.[16] Figure 2 (left panel) plots aggregate IO multipliers for each country against GDP per capita (relative to the U.S.).

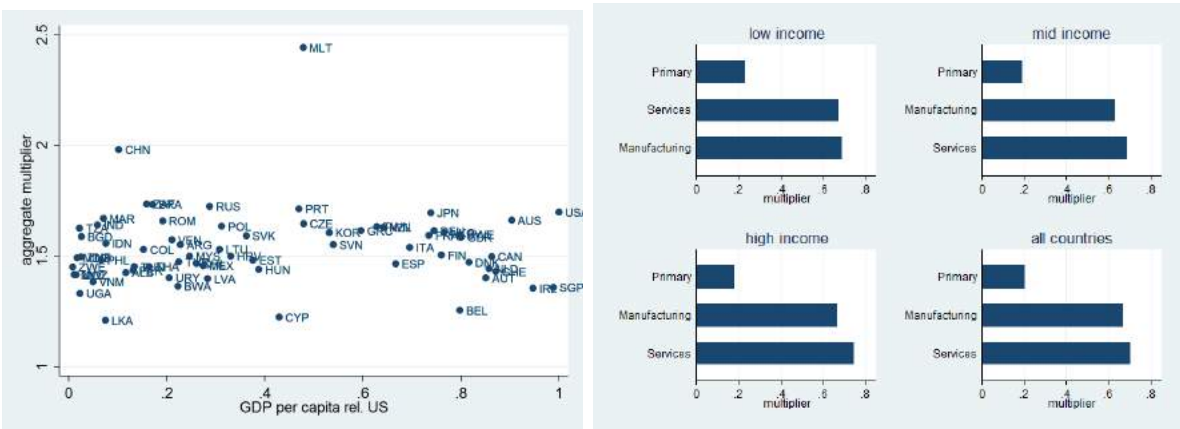


Figure 2: Aggregate IO-multipliers by country (left), sectoral IO-multipliers by income level (right)

We observe that aggregate multipliers for the GTAP sample average around 1.6 and are uncorrelated with the level of income. This implies that a one-percent increase in productivity of all sectors raises per-capita income by around 1.6 percent on average.[17]

Next, we separately compute the aggregate IO multipliers for the three major sector categories: primary sectors (which include Agriculture, Coal, Oil and Gas Extraction and Mining), manufacturing and services. Figure 2 (right panel) plots these multipliers by income level. Here, we divide countries into low income (less than 10,000 PPP Dollars of per capita income), middle income (10,000-20,000 PPP

single sector. IO coefficients are computed as payments to intermediates (domestic and foreign) divided by gross output at purchasers' prices. Purchasers' prices include transport costs and net taxes on output (but exclude deductible taxes, such as VAT).

[16] We provide a formal definition of IO multipliers in section 3.3.

[17] Aggregate multipliers for the WIOD sample are somewhat larger (with a mean of around 1.8) and also uncorrelated with the level of per-capita income. A simple regression of the aggregate multipliers from the GTAP sample on those from the WIOD data gives a slope coefficient of around 0.8 and the relationship is strongly statistically significant.

Dollars of per capita income) and high income (more than 20,000 PPP Dollars of per capita income).

We find that multipliers are largest in services (around 0.65 on average), slightly lower in manufacturing (around 0.62) and smallest in the primary sector (around 0.2). As before, the level of income does not play an important role in this result: the comparison is similar for countries at all levels of income per capita.[18] We conclude that at the aggregate-economy level or for major sectoral aggregates there are no systematic differences in IO structure across countries.

Let us now look at differences in IO structure at a more disaggregate level. To this end, we compute sectoral IO multipliers separately for each sector and country. Figure 3 presents kernel density plots of the distribution of (log) sectoral multipliers for different levels of income per capita. The left panel presents the distributions of multipliers for the GTAP sample (37 sectors) and the right panel the one for the WIOD sample (35 sectors).

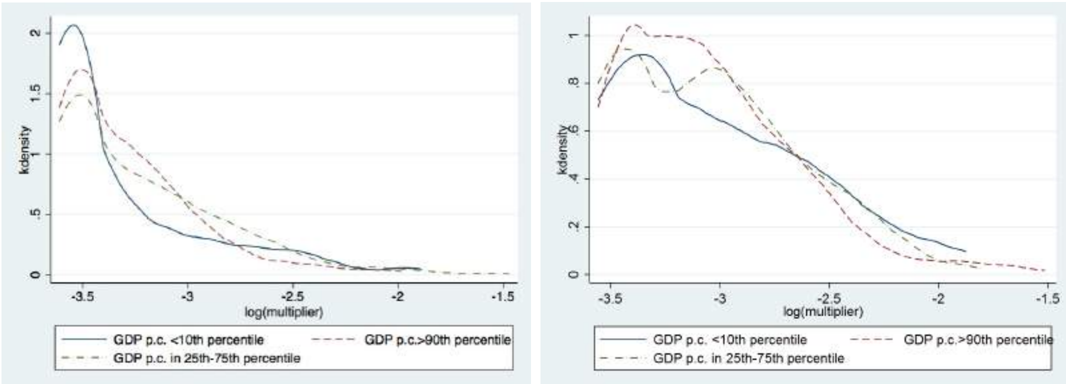


Figure 3: Distribution of sectoral log multipliers. GTAP sample (left panel); WIOD sample (right panel)

The following two facts stand out. First, for any given country the distribution of sectoral multipliers is *highly skewed*: while most sectors have low multipliers, a few sectors have multipliers way above the average. A typical low-multiplier sector (at the 10th percentile of the distribution of multipliers) has a multiplier of around 0.02 and the median sector has a multiplier of around 0.03. By contrast, a typical high-multiplier sector (at the 90th percentile of the distribution of multipliers) has a multiplier of around 0.065, while a sector at the 99th percentile has a multiplier of around 0.134.[19]

Second, the distribution of multipliers in low-income countries is *more skewed towards the extremes* than it is in high-income countries. In poor countries, almost all sectors have very low multipliers and a few sectors have very high multipliers. Differently, in rich countries the distribution of sectoral multipliers has significantly more mass in the center.

Finally, we investigate which sectors tend to have the largest multipliers. We thus rank sectors according to the size of their multiplier for each country. The upper panels of Figure 4 plots sectoral

[18] Very similar results are obtained for the WIOD sample. The only difference is that primary sectors are somewhat more important in low-income countries compared to others.

[19] These numbers correspond to the GTAP sample.

multipliers for a few selected countries, which are representative for the whole sample: a very poor African economy (Uganda (UGA)), a large emerging economy (India (IND)) and a large high-income economy (United States (USA)). It is apparent that the distribution of multipliers in Uganda is such that the bulk of sectors have low multipliers, with the exception of Agriculture, Electricity, Trade and Inland Transport. By contrast, a typical sector in the U.S. has a larger multiplier, while the distribution of multipliers in India lies between the one of Uganda and the one of the U.S.[20]

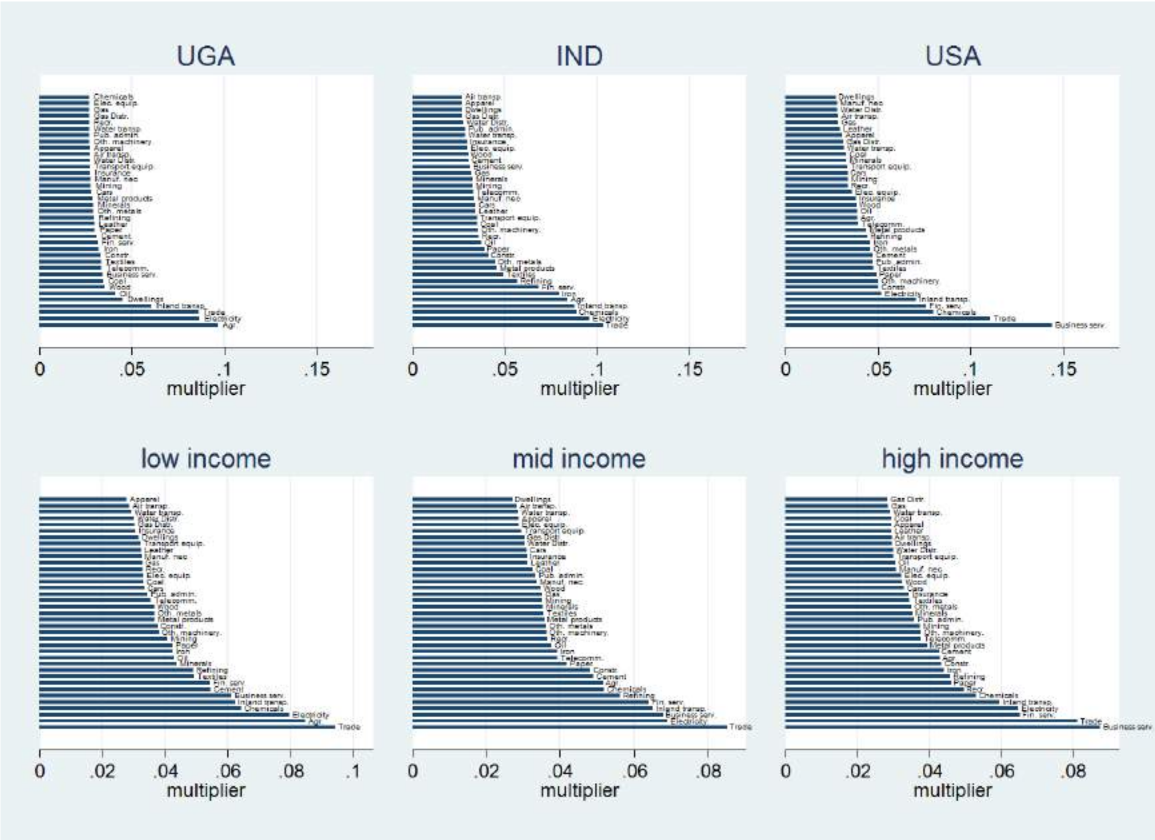


Figure 4: Sectoral IO-multipliers by country (top panel)/ income level (bottom panel)

In the lower panels of the same figure we plot sectoral multipliers averaged across countries by income level. Note that while the distributions of multipliers now look quite similar for different levels of income, this is an aggregation bias, which averages out much of the heterogeneity at the country level. From this figure we see that, in low-income countries, the sectors with the highest multipliers are Trade, Electricity, Agriculture, Chemicals, and Inland Transport, while in the set of middle- and high-income countries, the most important sectors in terms of multipliers are Trade, Electricity, Business Services, Inland Transport and Financial Services.

[20]One might be concerned that the IO structure in poor countries is mismeasured due to the importance of the informal sector in these countries and that the size of linkages is thus understated (manufacturing census and survey data used to construct IO tables do not include the informal sector). However, the fact that estimated average multipliers do not differ with GDP per capita and that agriculture has strong IO linkages in developing countries, even though most agricultural establishments are in the informal sector, mitigates this concern. In addition, the largest firms in a sector (which operate in the formal economy) typically account for the bulk of sectoral output and inputs and even more so in developing countries (Alfaro et al., 2008), so that the mismeasurement in terms of aggregate output and intermediate input demand is probably small.

Thus, overall the sectors with the highest multipliers are mostly service sectors. Agriculture is one notable exception for countries with an income level below 10,000 PPP dollars, where agricultural products are an input to many sectors. Moreover, in low-income countries the sector Chemicals and Petroleum Refining tends to have a large multiplier, too. In general though, typical manufacturing sectors have intermediate multipliers (around 0.04). Finally, the sectors with the lowest multipliers are also mostly services: Apparel, Air Transport, Water Transport, Gas Distribution and Dwellings (Owner-occupied houses). Given the large number of sectors with low multipliers, the specific sectors differ more across income groups. The figures for individual countries confirm the overall picture.

2.3 Productivities

We now explain the construction of a sectoral total factor productivity (TFP) relative to the U.S. and provide some descriptive evidence on sectoral TFPs as well as their correlation with sectoral multipliers. Here, we use the countries in the WIOD sample, because this information is available only for this dataset.

In particular, WIOD contains all the necessary information to compute gross-output-based sectoral total factor productivity: nominal gross output and material use, sectoral capital and labor inputs, sectoral factor payments to labor, capital and inputs for 35 sectors. Crucially, WIOD also provides purchasing power parity (PPP)-deflators (in purchasers' prices) for sector-level gross output that we use to convert nominal values into PPP units and which thus allow us to compute real TFPs at the sector level.[21] These deflators have been constructed by Inklaar and Timmer (2014) and are consistent in methodology and outcome with the latest version of the PWT. They combine expenditure prices and levels collected as part of the International Comparison Program (ICP) with data on industry output, exports and imports and relative prices of exports and imports from Feenstra and Romalis (2014). The authors use export and import values and prices to correct for the problem that the prices of goods consumed or invested domestically do not take into account the prices of exported products, while the prices of imported goods are included. To our knowledge, WIOD combined with these PPP deflators is the best available cross-country dataset for computing sector-level productivities using production data.

Given that we only have information on inputs and outputs in PPPs for a single year, we follow the development/growth accounting literature (e.g. Caselli, 2005; Jorgenson and Stiroh, 2000) and calibrate sector-level production functions. We compute TFP at the sector level relative to the U.S. (measured in constant 2005 PPPs) assuming constant-returns-to-scale Cobb-Douglas sectoral technologies for gross

[21] WIOD data comprises socio-economic accounts that are defined consistently with the IO tables. We use sector-level data on gross output, physical capital stocks in constant 1995 prices, the price series for investment, and labor inputs in hours. Using the sector-level PPPs for gross output, we convert nominal gross output and inputs into constant 2005 PPP prices. Furthermore, using price series for investment from WIOD and the PPP price index for investment from PWT 7.1, we convert sector-level capital stocks from WIOD into constant 2005 PPP prices.

output with *country-sector-specific* input shares:

$$\Lambda_{ic}^{rel} \equiv \frac{\Lambda_{ic}}{\Lambda_{iUS}} = \frac{q_{ic}}{q_{iUS}} \frac{\left(k_{iUS}^{\alpha_{iUS}} l_{iUS}^{1-\alpha_{iUS}}\right)^{1-\gamma_{iUS}} d_{1iUS}^{\gamma_{1iUS}} d_{2iUS}^{\gamma_{2iUS}} \cdot \ldots \cdot d_{niUS}^{\gamma_{niUS}}}{\left(k_{ic}^{\alpha_{ic}} l_{ic}^{1-\alpha_{ic}}\right)^{1-\gamma_{ic}} d_{1ic}^{\gamma_{1ic}} d_{2ic}^{\gamma_{2ic}} \cdot \ldots \cdot d_{nic}^{\gamma_{nic}}}, \tag{1}$$

where i is the sector index and c is the country index. The notation uses Λ_{ic}^{rel} for TFP of sector i normalized relative to the U.S., q_{ic} for the gross output of sector i, k_{ic} and l_{ic} for the quantities of capital and labor inputs and d_{ji} for the quantity of intermediate good j used in the production of sector i; α_{ic}, $1-\alpha_{ic}$ are the empirical factor income shares in GDP, $\gamma_{jic} \in [0,1)$ are the intermediate input shares in gross output from the WIOD IO tables and $\gamma_{ic} = \sum_{j=1}^{n} \gamma_{jic}$.[22] The specification thus allows for cross-country differences in technology for a given sector, by allowing the output elasticity of any given input j in the production of sector i to vary by country c.

In Table 1 we report means and standard deviations of relative productivities by income level, as well as the correlation between sectoral multipliers and productivities. To compute the standard deviations and correlations, we consider deviations from country means, so they are to be interpreted as within-country variation.

Sample	N	avg. TFP	std. TFP (within)	corr. TFP, mult. (within)
low income	236	0.445	0.950	0.251
mid income	340	0.619	0.667	0.06
high income	745	1.109	0.475	-0.156
all	1,321	0.891	0.646	-0.101

Table 1: Descriptive statistics for sectoral TFPs and multipliers

The following empirical regularities arise. First, average sectoral productivities are highly positively correlated with income per capita. Second, the within-country standard deviation is highest for poor countries and lowest for rich countries, as is also apparent from Figure 5, which plots histograms of log relative productivities by income level. Thus, low-income countries have much more dispersion in relative productivities across sectors than rich ones. Third, in low-income countries, productivity levels of high-multiplier sectors are above their average productivity level relative to the U.S., while in richer countries productivities in these sectors tend to be below average. This is demonstrated by the examples in Figure 6. For instance, India has productivity levels above its average in the high-multiplier sectors Chemicals, Inland Transport and Refining and Electricity, while its productivity levels in the low-multiplier sectors

[22] Applying more sophisticated parametric estimation methods developed for plant-level data to obtain consistent estimates of output elasticities (e.g., Olley and Pakes, 1996) is not feasible, since it requires many observations for a given sector. These methods solve the simultaneity bias that arises when estimating the output elasticities of inputs with regression techniques by taking logs of (1), since unobserved TFP is correlated with input choice. Note, however, that using the empirical intermediate input shares γ_{jic} solves this simultaneity problem when the production function is Cobb-Douglas and intermediate inputs are freely adjustable. Under these assumptions the first-order conditions for profit maximization imply that intermediate input shares are independent of (unobserved) TFP.

such as Car Retailing, Telecommunications and Business Services are below average. An exception is India's high-multiplier sector Agriculture, where the productivity level is very low. This confirms the general view that poor countries tend to have particularly low productivity levels in this sector. In contrast, rich European economies, such as Germany, tend to have below-average productivity levels in high-multiplier sectors such as Financial Services, Business Services and Transport.

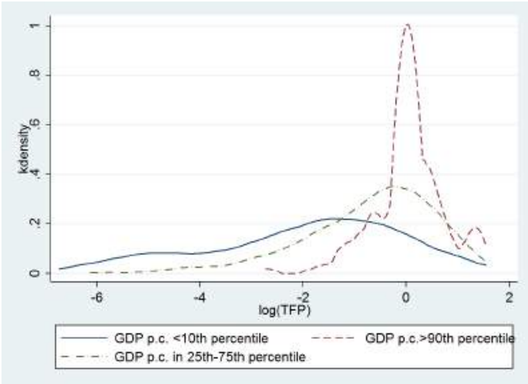


Figure 5: Distribution of sectoral log(TFP) relative to the U.S.

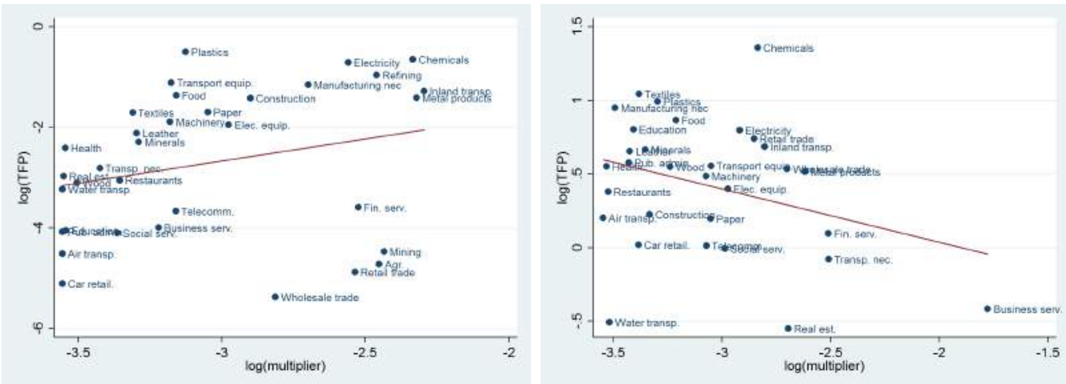


Figure 6: Correlation between IO-multipliers and productivities: India (left) and Germany (right)

3 Theoretical framework

3.1 Model

In this section we present our theoretical framework, which will be used in the remainder of our analysis. Consider a static multi-sector economy. n competitive sectors each produce a distinct good that can be used either for final consumption or as an input for production. The technology of sector $i \in 1:n$ is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}, \tag{2}$$

where Λ_i is the exogenous total factor productivity of sector i, k_i and l_i are the quantities of capital and labor used by sector i and d_{ji} is the quantity of good j used in production of good i (intermediate goods produced by sector j).[23] The exponent $\gamma_{ji} \in [0, 1)$ represents the share of good j in the production technology of firms in sector i, and $\gamma_i = \sum_{j=1}^{n} \gamma_{ji} \in (0, 1)$ is the total share of intermediate goods in gross output of sector i. Parameters α, $1-\alpha \in (0, 1)$ are the shares of capital and labor in the remainder of the inputs (value added).

Given the Cobb-Douglas technology in (2) and competitive factor markets, γ_{ji}'s also correspond to the entries of the IO matrix, measuring the value of spending on input j per dollar of production of good i. We denote this IO matrix by $\mathbf{\Gamma}$. Then the entries of the j'th row of matrix $\mathbf{\Gamma}$ represent the values of spending on a given input j per dollar of production of each sector in the economy. On the other hand, the elements of the i'th column of matrix $\mathbf{\Gamma}$ are the values of spending on inputs from each sector in the economy per dollar of production of a given good i.[24]

Output of sector i can be used either for final consumption, y_i, or as an intermediate good:

$$y_i + \sum_{j=1}^{n} d_{ij} = q_i \qquad i = 1 : n \tag{3}$$

Final consumption goods are aggregated into a single final good through another Cobb-Douglas production function:

$$Y = y_1^{\frac{1}{n}} \cdot ... \cdot y_n^{\frac{1}{n}}. \tag{4}$$

This aggregate final good is used as households' consumption, C, so that $Y = C$. Note that the symmetry in exponents of the final good production function implies symmetry in consumption demand for all goods. This assumption is useful as it allows us to focus on the effects of the IO structure and the interaction between the structure and sectors' productivities in an otherwise symmetric framework. It is, however, straightforward to introduce asymmetry in consumption demand by defining the vector of demand shares $\boldsymbol{\beta} = (\beta_1, .., \beta_n)$, where $\beta_i \neq \beta_j$ for $i \neq j$ and $\sum_{i=1}^{n} \beta_i = 1$. The corresponding final good production function is then $Y = y_1^{\beta_1} \cdot ... \cdot y_n^{\beta_n}$. This more general framework is analyzed in section 5, where we consider extensions of our benchmark model.

Finally, the total supply of capital and labor in this economy are assumed to be exogenous and fixed

[23] In section 5 and in the Supplementary Appendix we consider the case of an open economy, where sectors' production technology employs both domestic and imported intermediate goods.

[24] According to our notation, the sum of elements in the i'th column of matrix $\mathbf{\Gamma}$ is equal to γ_i, the total intermediate share of sector i.

at the levels of K and 1, respectively:

$$\sum_{i=1}^{n} k_i = K, \tag{5}$$

$$\sum_{i=1}^{n} l_i = 1. \tag{6}$$

To complete the description of the model, we provide a formal definition of a competitive equilibrium.

Definition A competitive equilibrium is a collection of quantities q_i, k_i, l_i, y_i, d_{ij}, Y, C, G and prices p_i, p, w, and r for $i \in 1:n$ such that

1. y_i solves the profit maximization problem of a representative firm in a perfectly competitive final good's market:

$$\max_{\{y_i\}} p y_1^{\frac{1}{n}} \cdot \ldots \cdot y_n^{\frac{1}{n}} - \sum_{i=1}^{n} p_i y_i,$$

taking $\{p_i\}, p$ as given.

2. $\{d_{ij}\}$, k_i, l_i solve the profit maximization problem of a representative firm in the perfectly competitive sector i for $i \in 1:n$:

$$\max_{\{d_{ji}\}, k_i, l_i} p_i \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}} - \sum_{j=1}^{n} p_j d_{ji} - r k_i - w l_i,$$

taking $\{p_i\}$ as given (Λ_i is exogenous).

3. Households' budget constraint determines C: $C = w + rK$.

4. Markets clear:

   (a) r clears the capital market: $\sum_{i=1}^{n} k_i = K$,

   (b) w clears the labor market: $\sum_{i=1}^{n} l_i = 1$,

   (c) p_i clears the sector i's market: $y_i + \sum_{j=1}^{n} d_{ij} = q_i$,

   (d) p clears the final good's market: $Y = C$.

5. Production function for q_i is $q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}$.

6. Production function for Y is $Y = y_1^{\frac{1}{n}} \cdot \ldots \cdot y_n^{\frac{1}{n}}$.

Note that households' consumption is simply determined by the budget constraints, so that there is no decision for the households to make. Moreover, total production of the aggregate final good, Y, which is equal to $\sum_{i=1}^{n} p_i y_i$, represents real GDP (total value added) per capita.

3.2 Equilibrium

The following proposition characterizes the equilibrium value of the logarithm of GDP per capita, which we later refer to equivalently as aggregate output or aggregate income or value added of the economy.

Proposition 1. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of GDP per capita, $y = \log(Y)$, is given by*

$$y = \sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \sum_{j \ s.t. \ \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) - \log n + \alpha \log K, \tag{7}$$

where

$$\boldsymbol{\mu} = \{\mu_i\}_i = \frac{1}{n}[\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\mathbf{1}, \qquad n \times 1 \textit{ vector of multipliers}$$

$$\boldsymbol{\lambda} = \{\lambda_i\}_i = \{\log \Lambda_i\}_i, \qquad n \times 1 \textit{ vector of sectoral log-productivity coefficients}$$

Proof. The proof of Proposition 1 is provided in the Appendix.

Thus, due to the Cobb-Douglas structure of our economy, aggregate per capita GDP can be represented as a log-linear function of (i) terms representing aggregate productivity and summarizing the aggregate impact of sectoral productivities via the IO structure; and (ii) the capital stock per worker weighted by the capital share in GDP, α.

The proposition highlights two important facts. First, aggregate output is an increasing function of sectoral productivity levels. Second, and more importantly, the impact of each sector's productivity on aggregate output is proportional to the value of the sectoral IO multiplier μ_i, and hence, the larger the multiplier, the stronger the effect. This means that the positive effect of higher sectoral productivity on aggregate output is stronger in sectors with larger multipliers.[25]

The vector of sectoral multipliers, in turn, is determined by the features of the IO matrix through the Leontief inverse, $[\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}$.[26] The interpretation and properties of this matrix as well as a simpler representation of the vector of multipliers are discussed in the next section.

3.3 Intersectoral network. Multipliers as sectors' centrality

The input-output matrix $\boldsymbol{\Gamma}$, where a typical element γ_{ji} captures the value of spending on input j per dollar of production of good i, can be equivalently represented by a directed weighted network on n nodes. Nodes of this network are sectors and directed links indicate the flow of intermediate goods between sectors. Specifically, the link from sector j to sector i with weight γ_{ji} is present if sector j is an input supplier to sector i.

[25] The value of sectoral multipliers is positive due to a simple approximation result (9) in the next section.
[26] See Burress (1994).

For each sector in the network we define the *weighted in-* and *out-degree.* The weighted in-degree of a sector is the share of intermediate inputs in its production. It is equal to the sum of elements in the corresponding *column* of matrix $\mathbf{\Gamma}$; that is, $d_i^{in} = \gamma_i = \sum_{j=1}^{n} \gamma_{ji}$. The weighted out-degree of a sector is the share of its output in the input supply of the entire economy. It is equal to the sum of elements in the corresponding *row* of matrix $\mathbf{\Gamma}$; that is, $d_j^{out} = \sum_{i=1}^{n} \gamma_{ji}$. Note that if the weights of all links that are present in the network are identical, the in-degree of a given sector is proportional to the number of sectors that supply to it and its out-degree is proportional to the number of sectors to which it is a supplier.

The interdependence of sectors' production technologies through the network of intersectoral trade, helps to obtain some insights into the meaning of the Leontief inverse matrix $[\boldsymbol{I}-\mathbf{\Gamma}]^{-1}$ and the vector of sectoral multipliers $\boldsymbol{\mu}$.[27] A typical element l_{ji} of the Leontief inverse can be interpreted as the percentage increase in the output of sector i following a one-percent increase in productivity of sector j. This result takes into account all – direct and indirect – effects at work, such as for example, the effect of raising productivity in sector A that makes sector B more efficient and via this raises the output in sector C. Then multiplying the Leontief inverse matrix by the vector of weights $\frac{1}{n}\mathbf{1}$ adds up the effects of sector j on all the other sectors in the economy, weighting each by its share $\frac{1}{n}$ in GDP. Thus, a typical element of the resulting vector of IO multipliers reveals how a one-percent increase in productivity of sector j affects the overall value added in the economy.

In particular, for a simple one-sector economy, the multiplier is given by $\frac{1}{1-\gamma}$, where γ is a share of the intermediate input in the production of that sector. Moreover, $\frac{1}{1-\gamma}$ is also the value of the *aggregate* multiplier in an n-sector economy where only one sector's output is used (in the proportion γ) as an input in the production of all other sectors.[28] Thus, if the share of intermediate inputs in gross output of each sector is, for example, $\frac{1}{2}$ ($\gamma = \frac{1}{2}$), then a one-percent increase in TFP of each sector increases aggregate value added by $\frac{1}{1-\gamma} = 2$ percent. In more extreme cases, the aggregate multiplier – and hence, the effect of sectoral productivity increases on aggregate value added – becomes infinitely large when $\gamma \to 1$ and it is close to 1 when $\gamma \to 0$. This is consistent with the intuition in Jones (2011b).

One important observation is that the vector of multipliers is closely related to the *Bonacich centrality* vector corresponding to the intersectoral network of the economy.[29] This means that sectors that are more "central" in the network of intersectoral trade have larger multipliers and hence, play a more

[27]Observe that in this model the Leontief inverse matrix is well-defined since CRS technology of each sector implies that $\gamma_i < 1$ for any $i \in 1:n$. According to the Frobenius theory of non-negative matrices, this means that the maximal eigenvalue of $\mathbf{\Gamma}$ is bounded above by 1, and this, in turn, implies the existence of $[\boldsymbol{I}-\mathbf{\Gamma}]^{-1}$.

[28]Recall that aggregate multiplier is equal to the sum of all sectoral multipliers and represents the effect on aggregate income of a one-percent increase in the productivity of each sector.

[29]An analogous observation is made in Acemoglu et al. (2012), with respect to the *influence vector*. For the definition and other applications of the Bonacich centrality notion in economics see Bonacich, 1987; Jackson, 2008; and Ballester et al., 2006.

important role in determining aggregate output.

To see what centrality means in terms of simple network characteristics, such as sectors' out-degree, consider the following useful approximation for the vector of multipliers. Since none of $\mathbf{\Gamma}$'s eigenvalues lie outside the unit circle (cf. footnote 27), the Leontief inverse and hence the vector of multipliers can be expressed in terms of a convergent power series:

$$\boldsymbol{\mu} = \frac{1}{n}[\boldsymbol{I} - \mathbf{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\left(\sum_{k=0}^{+\infty}\mathbf{\Gamma}^k\right)\mathbf{1}.$$

As long as the elements of $\mathbf{\Gamma}$ are sufficiently small, this power series is well approximated by the sum of the first terms. Namely, consider the norm of $\mathbf{\Gamma}$, $\|\mathbf{\Gamma}\|_\infty = \max_{i,j\in 1:n}\gamma_{ji}$, and assume that it is sufficiently small. Then

$$\frac{1}{n}\left(\sum_{k=0}^{+\infty}\mathbf{\Gamma}^k\right)\mathbf{1} \approx \frac{1}{n}(\boldsymbol{I} + \mathbf{\Gamma})\mathbf{1} = \frac{1}{n}\mathbf{1} + \frac{1}{n}\mathbf{\Gamma}\mathbf{1}.$$

Consider that $\mathbf{\Gamma}\mathbf{1} = \boldsymbol{d^{out}}$, where $\boldsymbol{d^{out}}$ is the vector of sectors' out-degrees, $\boldsymbol{d^{out}} = \left(d_1^{out}, .., d_n^{out}\right)'$. This leads to the following simple representation of the vector of multipliers:

$$\boldsymbol{\mu} \approx \frac{1}{n}\mathbf{1} + \frac{1}{n}\boldsymbol{d^{out}}, \tag{8}$$

so that for any sector i,

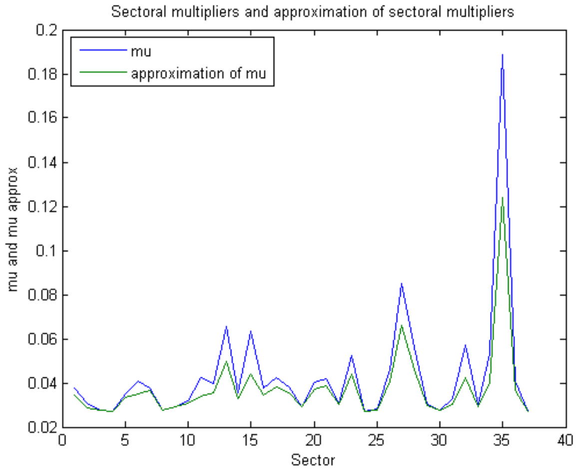
$$\mu_i \approx \frac{1}{n} + \frac{1}{n}d_i^{out}, \qquad i = 1:n. \tag{9}$$

Thus, larger multipliers correspond to sectors with larger out-degree, the simplest measure of sector's centrality in the network. In view of the statement in the previous section, this implies that sectors with the largest out-degree have the most pronounced impact on aggregate value added of the economy: the changes in productivity of such "central" sectors affect aggregate output most.

For the sample of countries in our data, both rich and poor, the approximation of sectoral multipliers by sectors' out-degree (times and plus $1/n$) turns out to be quite good, as demonstrated by Figure 7.

In what follows, we will consider that the in-degree (intermediate input share) of all sectors is the same, $\gamma_i = \gamma$ for all i. While clearly a simplification, this assumption turns out to be broadly consistent with the empirical distribution of sectoral in-degrees of the countries in our sample. In fact, the distribution of in-degrees in all countries is strongly peaked around the mean value, which suggests that on the demand side sectors are rather homogeneous, i.e., they use intermediate goods in approximately equal proportions.[30] This is in sharp contrast with the observed distribution of sectoral out-degrees that puts most weight on small values of out-degrees but also assigns a non-negligible weight to the out-degrees

[30]Note that essentially the same assumption of constant in-degree ($\gamma_i = 1$) is employed in Acemoglu et al., 2012, and in Carvalho et al., 2010.



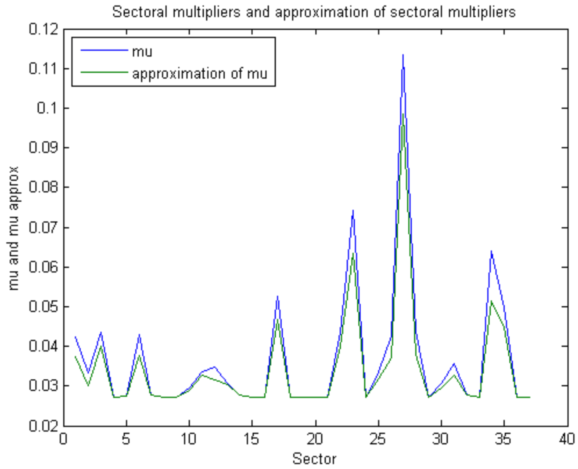


Figure 7: Sectoral multipliers in Germany (left) and Botswana (right). GTAP sample.

that are way above the average, displaying a fat tail. That is, on the supply side sectors are rather heterogenous: relatively few sectors supply their product to a large number of sectors in the economy, while many sectors supply to just a few. The upper left panel of Figure 10 in section 5 presents the empirical distributions of in-degrees for different levels of per capita income for the countries in the WIOD sample and Figure A-1 in the Supplementary Appendix plots the distributions of in- and out-degrees for the countries in the GTAP sample.

Note that the fat-tail nature of out-degree distribution carries over to the distribution of sectoral multipliers (see Figure 3 in section 2). Moreover, according to both distributions, the proportion of sectors with extreme out-degrees and multipliers is larger in low-income countries. This similarity between the distributions of sectoral out-degrees and multipliers is consistent with the derived relationship (9) between d_i^{out} and μ_i for each sector.

3.4 Expected aggregate output

To estimate our baseline model we use a statistical approach that allows us to represent aggregate income as a simple function of the first and second moments of the distribution of the IO multipliers and sectoral productivities. The distribution of multipliers, or sectors' centralities, captures the properties of the intersectoral network in each country, while the correlation between the distribution of multipliers and productivities captures the interaction of the input-output structure with sectoral productivities.

Figures 3 and 5 in section 2 suggest that the *joint* distribution of sectoral multipliers and productivities (relative to the U.S.) (μ_i, Λ_i^{rel}) is close to *log-Normal*, so that the joint distribution of log's of the corresponding variables, $(\log(\mu_i), \log(\Lambda_i^{rel}))$ is *Normal*.[31] Here i refers to the sector and $\Lambda_i^{rel} = \frac{\Lambda_i}{\Lambda_i^{US}}$. In

[31] To be precise, the distribution of $(\log(\mu_i), \log(\Lambda_i^{rel}))$ is a *truncated* bivariate Normal, where $\log(\mu_i)$ is censored from below at a certain $a > 0$. This is taken into account in our empirical analysis. However, the difference from a usual, non-truncated Normal distribution turns out to be inessential. Therefore, for simplicity of exposition, in this section we

particular, the fact that the distribution of μ_i is log-Normal means that while the largest probability is assigned to relatively low values of a multiplier, a non-negligible weight is assigned to high values, too. That is, the distribution is positively skewed, or possesses a fat right tail. Empirically, we find that the tail is fatter in countries with lower income.[32]

Given the log-Normal distribution of (μ_i, Λ_i^{rel}), the expected value of the aggregate output in each country can be evaluated using the expression for y in (7). This requires a few simplifying assumptions on our theoretical model. First, we consider that for each sector i the couple (μ_i, Λ_i^{rel}) is drawn from the *same* bivariate log-Normal distribution, that is, it is independent of the sector (but obviously country specific). Second, we assume that all variables on the right-hand side of (7), apart from μ_i and Λ_i^{rel}, are not random. Moreover, as already mentioned in the previous section, we assume that the in-degree γ_i is independent of the sector, $\gamma_i = \gamma$ for all i, and we adopt a coarse approximation that all non-zero elements of the input-output matrix $\mathbf{\Gamma}$ are the same, that is, $\gamma_{ji} = \widehat{\gamma}$ for any i and j whenever $\gamma_{ji} > 0$. In the robustness checks we show that the empirical predictions of our baseline model for cross-country differences remain practically unchanged when the latter assumption is relaxed and γ_{ji} are considered as random draws from a log-Normal distribution (see section 5 and the Supplementary Appendix).[33] Finally, in order to express sectoral log-productivity coefficients λ_i in terms of the relative productivity Λ_i^{rel}, we use the approximation $\lambda_i = \log(\Lambda_i) \approx \Lambda_i^{rel} + \big(\log(\Lambda_i^{US}) - 1\big)$, which, strictly speaking, is only good when Λ_i is sufficiently close to Λ_i^{US}.

Under these assumptions, the expression for the aggregate output y in (7) simplifies and can be written as:

$$y = \sum_{i=1}^{n} \mu_i \Lambda_i^{rel} + \sum_{i=1}^{n} \mu_i \gamma \log(\widehat{\gamma}) + \log(1-\gamma) - \log n + \alpha \log(K) - (1+\gamma) + \sum_{i=1}^{n} \mu_i \log(\Lambda_i^{US}). \qquad (10)$$

The expected aggregate output, $E(y)$, is then equal to :

$$\begin{aligned} E(y) \quad = \quad & n\Big(E(\mu)E(\Lambda^{rel}) + cov(\mu, \Lambda^{rel})\Big) + (1+\gamma)(\gamma\log(\widehat{\gamma}) - 1) + \\ & + \log(1-\gamma) - \log n + \alpha\log(K) + E(\mu)\sum_{i=1}^{n} \log\left(\Lambda_i^{US}\right). \end{aligned} \qquad (11)$$

From this expression, we see that higher expected multipliers $E(\mu)$ lead to larger expected income $E(y)$ for the same fixed levels of $E(\Lambda^{rel})$ and covariance $cov(\mu, \Lambda^{rel})$. Moreover, since aggregate value

refer to the distribution of $(\log(\mu_i), \log(\Lambda_i^{rel}))$ as Normal and to the distribution of (μ_i, Λ_i^{rel}) as log-Normal.

[32]See the distribution parameter estimates in the next section.

[33]Note that our conditions on γ_{ji} and γ allow us to express $\sum_{j \text{ s.t. } \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji}$ as $\mu_i \gamma \log(\widehat{\gamma})$ since the number of non-zero elements in each column of $\mathbf{\Gamma}$ is equal to $\frac{\gamma}{\widehat{\gamma}}$, and $\sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) = log(1-\gamma)$ since $\sum_{i=1}^{n} \mu_i(1-\gamma_i) = \mathbf{1}'[\boldsymbol{I} - \mathbf{\Gamma}] \cdot \frac{1}{n}[\boldsymbol{I} - \mathbf{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\mathbf{1}'\mathbf{1} = 1$. Moreover, $\sum_{i=1}^{n} \mu_i \approx 1+\gamma$ because from (9) it follows that $\sum_{i=1}^{n} \mu_i \approx 1 + \frac{\sum_{i=1}^{n} d_i^{out}}{n} = 1 + \frac{\sum_{i=1}^{n} d_i^{in}}{n}$ and $d_i^{in} = \gamma_i = \gamma$ for all i.

added depends positively on the covariance term $cov(\mu, \Lambda^{rel})$, higher relative productivities have a larger impact if they occur in sectors with higher multipliers.

The expression for expected aggregate income in (11) can be written in terms of the parameters of the normally distributed $(\log(\mu), \log(\Lambda^{rel}))$, by means of the relationships between Normal and log-Normal distributions:[34]

$$\begin{aligned} E(y) &= n\left(e^{m_\mu + m_\Lambda + 1/2(\sigma_\mu^2 + \sigma_\Lambda^2) + \sigma_{\mu,\Lambda}}\right) + (1+\gamma)(\gamma \log(\widehat{\gamma}) - 1) + \\ &+ \log(1-\gamma) - \log n + \alpha \log(K) + e^{m_\mu + 1/2\sigma_\mu^2} \sum_{i=1}^{n} \log\left(\Lambda_i^{US}\right), \end{aligned} \tag{13}$$

where m_μ, m_Λ are the means and σ_μ^2, σ_Λ^2 and $\sigma_{\mu,\Lambda}$ are the elements of the variance-covariance matrix of the Normal distribution. This is the ultimate expression that we use in the empirical analysis of the benchmark model in the next section.

4 Empirical analysis

In this section we estimate the parameters of the Normal distribution of $(\log(\mu), \log(\Lambda^{rel}))$ for the sample of countries for which we have data. We allow parameter estimates to vary across countries in order to model the systematic underlying differences in IO structure and productivity that we have discussed in section 2. With the parameter estimates in hand we then use equation (13) to evaluate the predicted aggregate income in these countries (relative to the one of the U.S.)[35] and compare our baseline model with three simple alternatives which abstract from some of the elements present in our model: (i) sectoral productivity differences; (ii) IO linkages; (iii) country-specific IO structure. We show that all these elements are important for understanding cross-country income differences.

4.1 Structural estimation

We assume that the vector of log multipliers and log relative productivities $\mathbf{Z} \equiv (\log(\mu), \log(\Lambda^{rel}))$ is drawn from a (truncated) bivariate Normal distribution with country-specific parameters $\mathbf{\Theta} = (\mathbf{m}, \mathbf{\Sigma})$, where $\mathbf{m}$ is the vector of means and $\mathbf{\Sigma}$ denotes the variance-covariance matrix. In order to allow the distributions of log multipliers and productivities to differ across countries, we first estimate the

[34]These relationships are:

$$E(\mu) = e^{m_\mu + 1/2\sigma_\mu^2}, \qquad E(\Lambda^{rel}) = e^{m_\Lambda + 1/2\sigma_\Lambda^2}, \qquad cov(\mu, \Lambda) = e^{m_\mu + m_\Lambda + 1/2(\sigma_\mu^2 + \sigma_\Lambda^2)} \cdot (e^{\sigma_{\mu,\Lambda}} - 1)\,. \tag{12}$$

[35]In order to predict relative rather than absolute output, we use equation (13) differenced with the value of predicted aggregate income for the U.S.

parameters separately for each country using Maximum Likelihood.[36] Observe that in the estimation we do not impose any structure on the data except for assuming joint log Normality. In a second step, we then regress the estimated country-specific parameters $\hat{\mathbf{\Theta}}$ on (log) per capita income in order to test if the parameters indeed systematically vary with countries' income level, as suggested by the evidence presented in section 2.[37]

We estimate the statistical model using the empirical data for log multipliers and log TFPs constructed from the WIOD dataset (35 sectors, 36 countries). In the panels of Figure 8 we plot the country-specific estimates of all parameters against log(GDP per capita) and in Table 2 we report the corresponding results of regressing each parameter on log(GDP per capita). Because the coefficients are maximum-likelihood estimates, we report bootstrapped standard errors.

We find that m_μ does not vary systematically with the income level (column (1)). Instead, σ_μ decreases significantly in (log) per capita GDP with a slope of -0.076 (column (2)). Thus, in the WIOD sample, poor countries have a distribution of log multipliers with the same average but with more dispersion than rich countries. Average log productivity, m_Λ, strongly increases in log GDP per capita (with a slope of around 1.4, see column (3)), while the standard deviation of log productivity, σ_Λ, is a decreasing function of the same variable (column (4)). This implies that rich countries have much higher average productivity levels and less dispersion in relative productivities across sectors than poor economies. Finally, note that the covariance between log multipliers and log productivity, $\sigma_{\mu,\Lambda}$, has a positive intercept and is a decreasing function of log(GDP per capita) (column (5)). Hence, poor countries have above-average productivity levels in sectors with higher multipliers, while rich countries have productivities which are lower than their average level in these sectors. We label predicted values from these regressions $\tilde{\mathbf{\Theta}}$.

To obtain more information on the IO structure of low-income countries, we now redo the estimation using data for the GTAP sample (37 sectors, 65 countries). For this sample, we only have information on IO multipliers but not on productivity levels available. Therefore, we estimate a univariate (truncated) Normal distribution for m_μ and σ_μ for each country. The results of regressing the country-specific parameter estimates on (log) per capita GDP are reported in columns (6) and (7) of Table 2. The results are quite similar to those for the WIOD sample: m_μ does not vary significantly with the income

[36]The formula for the truncated bivariate Normal, where $\log(\mu)$ is censored from below at a is given by $f(\mathbf{Z}|log(\mu) \geq a) = \frac{1}{\sqrt{(2\Pi)^2|\mathbf{\Sigma}|}} exp[-1/2(\mathbf{Z}-\mathbf{m})'\mathbf{\Sigma}^{-1}(\mathbf{Z}-\mathbf{m})]/(1-F(a))$, where $F(a) = \int_{-\infty}^{a} \frac{1}{\sigma_\mu\sqrt{(2\Pi)}} exp[-1/2(\log(\mu)-m_\mu)^2/\sigma_\mu^2] d\log(\mu)$ is the cumulative marginal distribution of $\log(\mu)$ and where

$$\mathbf{m} = \begin{pmatrix} m_\mu \\ m_\Lambda \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} \sigma_\mu^2 & \sigma_{\mu,\Lambda} \\ \sigma_{\mu,\Lambda} & \sigma_\Lambda^2 \end{pmatrix}. \tag{14}$$

[37]We obtain very similar results if we, alternatively, pool observations across countries and model coefficients as linear functions of (log) per capita income. Such a one-step procedure is statistically more efficient than our two-step procedure, but it also imposes more structure on the data ex ante, which we would like to avoid.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	WIOD sample					GTAP sample	
	m_μ	σ_μ	m_Λ	σ_Λ	$\sigma_{\mu,\Lambda}$	m_μ	σ_μ
Constant	-5.462***	1.461***	-14.216***	3.606***	2.320***	-8.749***	1.868***
	(1.125)	(0.392)	(2.119)	(0.619)	(0.478)	(2.959)	(0.443)
log(GDP p.c.)	0.168	-0.076*	1.396***	-0.303***	-0.234***	0.368	-0.100**
	(0.112)	(0.039)	(0.209)	(0.061)	(0.047)	(0.300)	(0.046)
R-squared	0.002	0.046	0.590	0.557	0.343	0.012	0.057
Observations	36	36	36	36	36	65	65

Table 2: Regression of estimated country-specific parameters on log(GDP p.c.).
Bootstrapped standard errors significant at 1% (***), 5% (**), 10% (*) significance level in parenthesis.

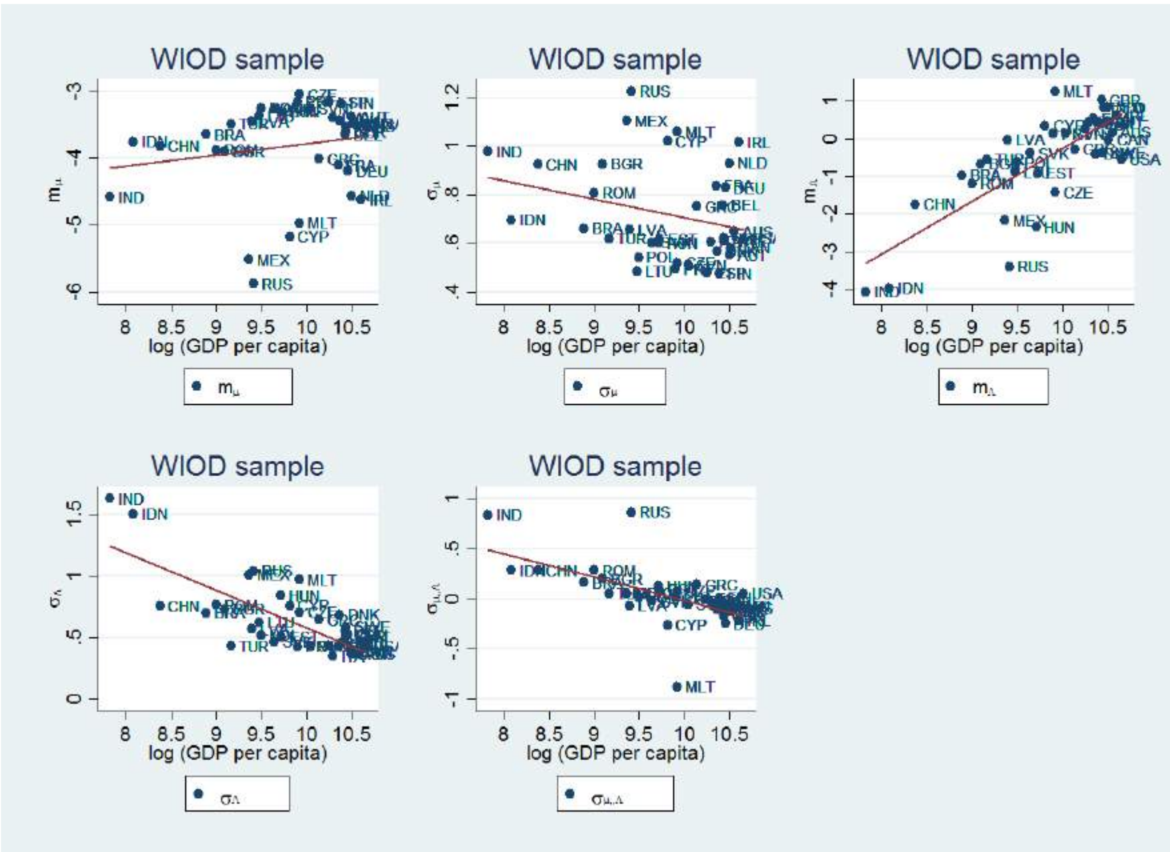


Figure 8: Correlation of country-specific coefficient estimates with log per-capita GDP.

level (column (6)), while the standard deviation of log multipliers, σ_μ, is a decreasing function of (log) per capita income with a slope of -0.1 (column (7)). Again, this implies that in poor countries the average sector has the same log multiplier but there is more mass at the extremes of the distribution than in rich countries. We summarize these empirical findings below.

Summary of estimation results:

1. *The estimated distribution of log IO multipliers has a larger variance with more mass at the extremes in poor countries compared to rich ones.*

2. *The estimated distribution of log productivities has a lower mean and a larger variance in poor countries compared to rich ones.*

3. *Log IO multipliers and productivities correlate positively in poor countries and negatively in rich ones.*

4.2 Predicting cross-country income differences

We now plug the predicted values from the regressions of coefficient estimates on (log) income per capita, $\tilde{\mathbf{\Theta}}$, into the expression for the expected income per capita derived from the baseline model (13) (differenced relative to the U.S.) to forecast per capita income levels relative to the U.S.[38]

We compare our baseline model, which features country-specific IO linkages and sectoral productivity differences, with three simple alternatives. The first one, which we label the 'naive model', has no IO structure and no productivity differences, so that $y = E(y) = \alpha log(K)$. The second model, by contrast, features sectoral productivity differences but no IO linkages. It is easy to show that under the assumption that sectoral productivities follow a log-Normal distribution, predicted log income in this model is given by $E(y) = e^{m_\Lambda + 1/2\sigma^2_\Lambda} + \alpha \log(K) + \frac{1}{n}\sum_{i=1}^{n}(\log(\Lambda_i^{US})) - 1$.[39] The third alternative model features sectoral productivity differences and IO linkages but keeps the IO structure constant across countries (by restricting the mean and the variance of the distribution of log multipliers and its covariance with log productivity to be constant across countries). In addition to the estimated parameter values $\tilde{\mathbf{\Theta}}$ we also need to calibrate a few other parameters. As standard, we set $(1-\alpha)$, the labor income share in GDP, equal to 2/3. Finally, we set n equal to 35, which corresponds to the number of sectors in the WIOD dataset.

To evaluate model fit, we provide the following tests: first, we regress model-predicted income per capita relative to the U.S. on actual data for income per capita relative to the U.S. If the model fits

[38] The expression for $E(y)$ for the truncated distribution of (μ_i, Λ_i^{rel}) is somewhat more complicated and less intuitive. However, the results for aggregate income using a truncated normal distribution for μ are very similar to the estimation of (13) and we therefore use the formulas for the non-truncated distribution. The details can be provided by the authors.

[39] $Y = \prod_{i=1}^{n} \Lambda_i^{1/n}(K)^\alpha$, hence $y = \frac{1}{n}\sum_{i=1}^{n}\lambda_i + \alpha\log(K)$. Using our approximation for productivity relative to the U.S., taking expectations and assuming that Λ_i follows a log-Normal distribution, we obtain the above formula.

relative per capita income levels perfectly, the estimate for the intercept should be zero and the regression slope and the R-squared should equal unity.[40] Second, as a graphical measure for the goodness of fit, we also plot predicted income per capita relative to the U.S. against actual relative income. Note that these tests provide over-identification restrictions for our model, since there is no intrinsic reason for the model to fit data on relative per capita income well: we have not matched income data in order to estimate the parameters of the distribution of log IO multipliers and productivities. Instead, we have just allowed their joint distribution to vary with the level of per capita income in the estimation procedure.

We first predict income differences for the sample of WIOD countries (36 countries), then for the GTAP sample (65 countries) and finally for the Penn World Table sample (155 countries). Starting with the WIOD sample, the results of the first test are reported in Table 3. In column (1), we report statistics for the 'naive' model. In column (2), we report results for the model with productivity differences but no IO structure. In column (3) we report results for our baseline model (13), where we take the parameter estimates obtained from the WIOD data (using predicted values of the parameters from Table 2, columns (1)-(4)). In column (4), we force the distribution of multipliers to be the same across countries by restricting both m_{μ}, σ_{μ}^2 and $\sigma_{\mu,\Lambda}$ to be constant. Finally, in column (5) we report results for the baseline model when the distribution of multipliers is estimated using the GTAP dataset (using predicted values for the distribution of log multipliers from Table 2, columns (5) and (6)).

We now present the results of this exercise. The 'naive' model fails to predict relative income levels (see column (1) of Table 3 and green squares in the upper left panel of Figure 9). As is well known, a model without productivity differences predicts too little variation in income per capita across countries and over-predicts income levels for poor countries. Still, in the WIOD sample, which consists mostly of medium- and high-income countries, it does relatively well: the intercept is 0.371, the slope coefficient is 0.832 and the R-squared is 0.710. The simple model with productivity differences but no IO linkages (column (2)) performs much better but it makes many countries significantly poorer than they are in the data (red triangles in the upper left panel of Figure 9), implying that productivity differences estimated from sectoral data are larger than those necessary to generate the observed income differences: the intercept is -0.141, the slope coefficient is 0.967 and the R-squared is 0.927. We now move to the first specification with IO structure. In column (3) we report results for our baseline model with varying IO structure, estimated from WIOD data. This model indeed performs better than the one without IO structure in terms of predicting relative income levels: the intercept is no longer statistically different from zero, the slope coefficient equals 1.000 and the R-squared is 0.939. A visual comparison of actual vs. predicted relative income in the upper left panel of Figure 9 confirms the substantially better fit of

[40] Denoting model-predicted per-capita income by $\hat{y}$ and actual per capita income by $GDP\ p.c.$, the R^2 is given by $\frac{\sum(intercept+slope*GDP\ p.c.)^2}{\sum \hat{y}^2}$, which equals unity when $intercept = 0$, $slope = 1$ and $Var(\hat{y}) = Var(GDP\ p.c)$.

the model with IO linkages (blue circles) compared to the one without IO structure, which underpredicts relative income levels of most countries and the naive model, which overpredicts relative income levels for virtually all countries.

Next, we test if the inclusion of an IO structure per se or rather cross-country differences in IO structure account for improved model fit. In column (4) we thus restrict the parameters m_μ, σ^2_μ and $\sigma_{\mu,\Lambda}$ to be the same for all countries. We find that this model fits the data significantly worse than the one with income-varying IO structure and roughly similarly as the model without IO structure: the intercept is -0.145, the slope coefficient drops to 0.963 and the R-squared to 0.925. This implies that cross-country variation in IO structure is important for predicting differences in income across countries. Finally, in column (5) we plug the IO structure estimated from the GTAP sample in our baseline IO model. The GTAP data is more informative about cross-country differences in IO linkages than the WIOD data because it includes a much larger sample of low- and middle-income countries, which allows estimating differences in structure across countries more precisely. In particular, the estimates from the GTAP data indicate that poorer countries have a distribution of log multipliers with a significantly larger variance compared to rich countries. Using these estimates, we find that the intercept is not statistically different from zero, while the slope coefficient is equal to 1.004 and the R-squared is 0.940. Thus, this specification performs comparably to the one where the IO structure is estimated from the WIOD data.

Observe that there are two main factors that determine the improved fit of the baseline model with IO structure compared to the model without IO structure or with constant structure. First, differences in IO structure between high and low-income countries: poor countries have only few highly connected sectors and many sectors that are relatively isolated, while rich countries have more intermediately connected sectors; second, the fact that – in contrast to rich countries – poor economies have above-average productivity levels in high-multiplier sectors. We will investigate the impact of each of these factors in the section on counterfactuals.

	(1)	(2)	(3)	(4)	(5)
	naive model	no IO structure	WIOD IO structure	constant IO structure	GTAP IO structure
intercept	0.371***	-0.141***	-0.033	-0.145***	-0.023
	(0.060)	(0.029)	(0.023)	(0.029)	(0.021)
slope	0.832***	0.967***	1.000***	0.963***	1.040***
	(0.101)	(0.051)	(0.039)	(0.052)	(0.039)
R-squared	0.710	0.927	0.939	0.925	0.940
Observations	36	36	36	36	36

Table 3: Model fit: WIOD sample. Standard errors significant at 1% (***), 5% (**), 10% (*) significance level in parenthesis.

Next, we turn to testing model fit in the sample of GTAP countries and the sample of countries in

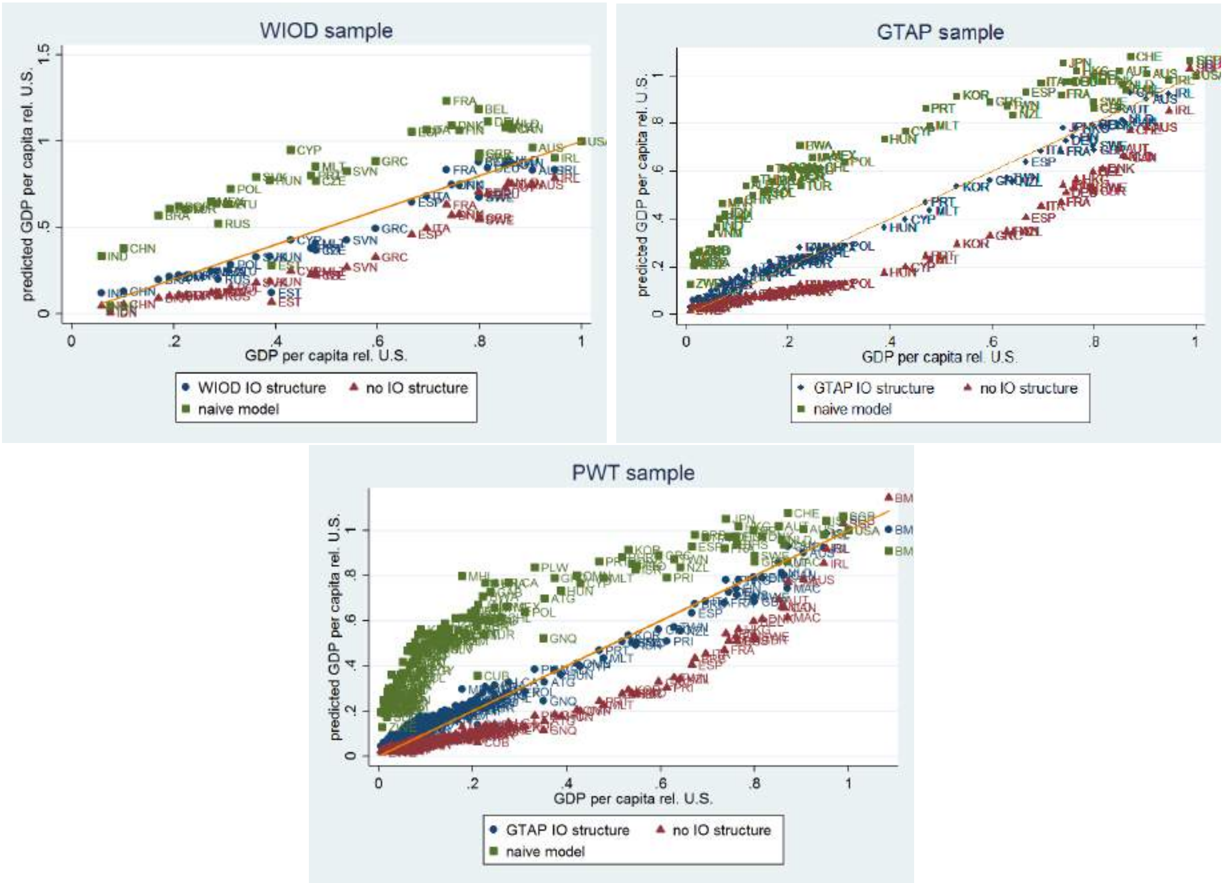


Figure 9: Predicted income per capita: model fit for different samples.

the Penn World Tables for which we have the necessary information on capital stocks. The latter sample is usually employed for development accounting exercises. In Table 4, columns (1)-(4), we present results for the GTAP sample. In column (1) we report results for the 'naive' model, which does relatively poorly in predicting relative income for this sample: the intercept is 0.365, the slope coefficient is 0.779 and the R-squared is 0.887. In column (2) we present results for the model with productivity differences but no IO structure. As before, this model performs much better than the 'naive' one: the intercept drops to -0.044, the slope coefficient rises to 0.804 and the R-squared improves to 0.916. Next, turning to the baseline model with IO structure, in column (3) we report the results using parameter estimates from the GTAP sample. This model performs significantly better than the 'naive' model and the model without IO structure in terms of fitting the regression of predicted on actual income: the intercept is 0.054, the slope coefficient is 0.839 and the R-squared is 0.968. The increased goodness of fit can also be seen from Figure 9, upper right panel, where we plot predicted income against actual income for the baseline model (blue circles), the model without IO structure (red triangles) and the 'naive' model (green squares). While the naive model considerably over-predicts and the model without IO structure under-predicts relative income levels for most countries, the model with IO structure is extremely close to the 45-degree line. Only for the poorest countries it slightly over-predicts their relative income levels.

In column (4) we report results for the baseline model with parameter estimates from the WIOD sample: we now get an intercept of 0.042, a slope coefficient of 0.918 and an R-squared of 0.987. Thus, this model performs even better than the one with the GTAP IO structure.

Finally, we discuss the results for predicting relative income levels in the full PWT sample (see columns (5)-(8)), which requires to predict the distributions of IO structure and productivities out of sample. Here, the performance of the 'naive' model is again quite poor, as it strongly over-predicts income for poor countries (green squares in the lower panel of Figure 9), indicating that productivity differences matter for explaining aggregate income differences. In column (5) the intercept is 0.342 and the slope coefficient is 0.823 with an R-squared of 0.830. In column (6) we report results for the model with productivity differences but without IO structure, which has a negative intercept (-0.018), a slope coefficient of 0.763 and an R-squared of 0.910 and thus under-predicts income levels for many countries (red triangles in the lower panel of Figure 9). This model is again significantly outperformed by our baseline model with the GTAP IO structure (column (7)): the slope coefficient is 0.802 and the R-squared increases substantially to 0.965, implying a 5.5-percentage-point gain in the model-explained variation in relative income from introducing the IO structure. Thus, the model performs very well in predicting relative income levels across countries, even in a sample that is much larger than the one from which we have estimated the parameters of the model. The good fit can also be seen clearly from the lower panel of Figure 9 (blue circles), where most data points are extremely close to the 45-degree line. Finally, in column (8), we report results for the baseline model when estimating the IO structure from the WIOD sample. This model does even better than the previous one: the slope coefficient is 0.897, and the R-squared is 0.984, a 8-percentage-point increase compared to the model without IO structure. We conclude that including an IO structure into the model helps to significantly improve model fit. To wrap up, we now present a summary of our findings.

Summary of model fit:

1. *The baseline model with estimated IO structure performs substantially better in terms of predicting relative income levels and their variation than a model without productivity differences (which over-predicts relative income levels) and a model with productivity differences but without IO structure (which under-predicts relative income levels for most countries).*

2. *The above results hold for three different samples of countries: the WIOD dataset (36 countries), the GTAP dataset (65 countries) and the Penn World Tables dataset (155 countries).*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	GTAP sample				PWT sample			
	Naive model	No IO structure	GTAP IO structure	WIOD IO structure	Naive model	No IO structure	GTAP IO structure	WIOD IO structure
intercept	0.365***	-0.044***	0.054***	0.042***	0.342***	-0.018***	0.073***	0.051***
	(0.022)	(0.013)	(0.008)	(0.005)	(0.012)	(0.006)	(0.004)	(0.003)
slope	0.779***	0.804***	0.839***	0.918***	0.823***	0.763***	0.802***	0.897***
	(0.039)	(0.043)	(0.025)	(0.016)	(0.034)	(0.038)	(0.020)	(0.013)
R-squared	0.887	0.916	0.968	0.987	0.830	0.910	0.965	0.984
Observations	65	65	65	65	155	155	155	155

Table 4: Model Fit: GTAP and PWT Samples. Standard errors significant at 1% (***), 5% (**), 10% (*) significance level in parenthesis.

5 Robustness checks

In this section, we report the results of a number of robustness checks in order to show that our findings do not hinge on the specific restrictions imposed by the baseline model. We consider the following modifications of our benchmark setup. First, we allow IO multipliers to depend on implicit tax wedges. Second, we extend our model to sectoral CES production functions. Third, we present a more general version of our structural model, which does not impose any symmetry on IO coefficients. Fourth, we generalize the final demand structure by introducing expenditure shares that differ across countries and sectors. Fifth, we explicitly account for imported intermediate inputs. Finally, we allow for skilled and unskilled labor as separate production factors. We show that none of these modifications changes the basic conclusions of the baseline model. The formulas for aggregate income implied by these more general models as well as detailed derivations can be found in the Supplementary Appendix.

5.1 Wedges

One important concern is that empirically observed IO coefficients do not just reflect technological input requirements but also sector-specific distortions or wedges τ_i in the production of intermediates. To see this, consider the maximization problem of an intermediate producer:

$$\max_{\{d_{ji}\}}(1-\tau_i)p_i\Lambda_i\left(k_i^{\alpha}l_i^{1-\alpha}\right)^{1-\gamma_i}d_{1i}^{\gamma_{1i}}d_{2i}^{\gamma_{2i}}\cdot\ldots\cdot d_{ni}^{\gamma_{ni}}-\sum_{j=1}^{n}p_jd_{ji}-rk_i-wl_i,$$

taking $\{p_i\}$ as given (τ_i and Λ_i are exogenous). Sector-specific wedges are assumed to reduce the value of sector i's production by a factor $(1-\tau_i)$, so that $\tau_i>0$ means an implicit tax and $\tau_i<0$ means an implicit subsidy for the production of sector i's output.

The first-order condition w.r.t. d_{ji} is given by

$$(1-\tau_i)\gamma_{ji}=\frac{p_jd_{ji}}{p_iq_i}\quad j\in 1:n \tag{15}$$

Thus, a larger wedge in sector i implies lower observed IO coefficients in this sector since firms in a sector facing larger implicit taxes demand less inputs from all other sectors. Separately identifying wedges τ_i and technological IO coefficients γ_{ji} is an empirical challenge, which requires to impose some additional restrictions on the data. Observe that τ_i is the same for all inputs j demanded by a given sector i. Thus, introducing a country index c and summing across inputs j for a given country, we obtain

$$(1-\tau_{ic})\sum_j \gamma_{jic} \equiv (1-\tau_{ic})\gamma_{ic} = \sum_j \frac{p_{jc}d_{jic}}{p_{ic}q_{ic}} \quad j \in 1:n \tag{16}$$

Hence, if we restrict the total technological intermediate share of sector i, γ_{ic}, to be the same across countries for a given sector i, we can identify country-sector specific wedges as $(1-\tau_{ic}) = \sum_j \frac{p_{jc}d_{jic}}{p_{ic}q_{ic}}\frac{1}{\gamma_i}$. Observe that individual IO coefficients γ_{jic} are still allowed to differ across countries in an arbitrary way. According to equation (16), countries with below-average intermediate shares in a certain sector face an implicit tax in this sector, while countries with above-average intermediate shares receive an implicit subsidy. It is then straightforward to estimate γ_i using regression techniques. Taking logs of equation (16), we obtain:

$$\log\left(\sum_j \frac{p_{jc}d_{jic}}{p_{ic}q_{ic}}\right) = \log(\gamma_i) + \log(1-\tau_{ic}) \tag{17}$$

Hence we regress the intermediate input shares of each country-sector pair on a set of sector-specific dummies to obtain estimates of the technological intermediate shares $\log(\gamma_i)$ and then back out $\log(1-\tau_{ic})$ as the residual. The upper left panel of Figure 10 plots the distribution of intermediate input shares and the upper right panel plots the distribution of $\log(1-\tau_{ic})$ by income level for the WIOD sample. Average intermediate shares do not vary systematically with per capita income, but poor countries have a larger fraction of sectors with very low intermediate shares and a lower fraction with high intermediate shares. Correspondingly, poor countries have a larger fraction of sectors with relatively high wedges. Given wedges τ_{ic}, we construct IO coefficients adjusted for wedges as $\gamma_{ijc} = \frac{p_{jc}d_{jic}}{p_{ic}q_{ic}}\frac{1}{(1-\tau_{ic})}$. We then recompute sectoral productivities and IO multipliers using these adjusted IO coefficients. The lower panel of Figure 10 plots the resulting distribution of (log) IO multipliers adjusted for wedges by income level. Observe that the distribution remains very similar to the one without wedges (compare with Figure 3).

One can show that in the presence of wedges which are considered as pure waste,[41] and under the same simplifying restrictions used in our baseline model (cf. equation (10)), the expression for aggregate income can be written as:[42]

[41] In an unreported robustness check we verified that considering the revenues from tax wedges and rebating them lump sum to households does not make much difference for the results.

[42] With wedges equation (7) for aggregate income includes in addition the term $\sum_{i=1}^n \mu_i \log(1-\tau_i)$, which, for small enough τ_i, can be approximated by $-\sum_{i=1}^n \mu_i\tau_i = \sum_{i=1}^n \mu_i(1-\tau_i) - \sum_{i=1}^n \mu_i$. Then under the same simplifying restrictions as before, $\sum_{i=1}^n \mu_i \approx 1+\gamma$, and we obtain an equation very similar to (10).

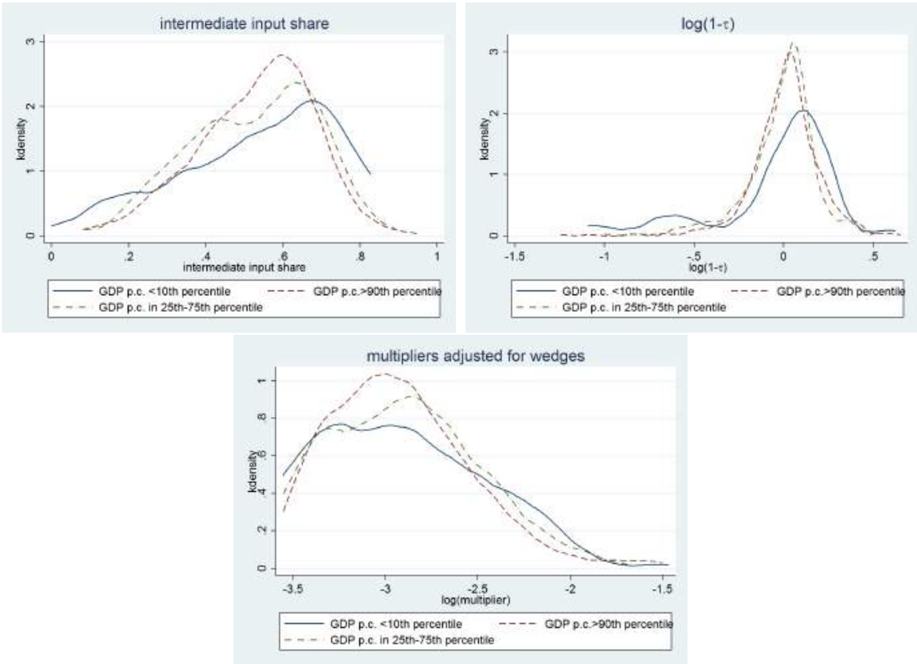


Figure 10: Intermediate input shares (upper left panel); wedges (upper right panel); IO coefficients adjusted for wedges (lower panel).

$$y = \sum_{i=1}^{n} \mu_i \Lambda_i^{rel} + \sum_{i=1}^{n} \mu_i (1-\tau_i) + \sum_{i=1}^{n} \mu_i \gamma \log(\widehat{\gamma}) + \log(1-\gamma) - \log n + \alpha \log K - 2(1+\gamma) + \sum_{i=1}^{n} \mu_i \log(\Lambda_i^{US}).$$

Now, assuming that sectoral multipliers, productivities and $(1-\tau_i)$ are stochastic, we obtain that expected aggregate output, $E(y)$, is given by:

$$\begin{aligned} E(y) &= n\Big(E(\mu)E(\Lambda^{rel}) + cov(\mu, \Lambda^{rel}) + E(\mu)E(1-\tau) + cov(\mu, 1-\tau)\Big) + (1+\gamma)(\gamma\log(\widehat{\gamma}) - 2) + \\ &+ \log(1-\gamma) - \log n + \alpha \log(K) + E(\mu) \sum_{i=1}^{n} \log\left(\Lambda_i^{US}\right). \end{aligned} \quad (18)$$

Again, this equation has an intuitive interpretation: higher average wedges τ are detrimental to aggregate income and more so if the average sector has a higher multiplier; moreover, the negative impact of high wedges is particularly distorting if wedges positively co-vary with multipliers. If we impose joint log normality on the triple $(\mu, \Lambda_i^{rel}, 1-\tau)$, we obtain:

$$\begin{aligned} E(y) &= n\left(e^{m_\mu + m_\Lambda + 1/2(\sigma_\mu^2 + \sigma_\Lambda^2) + \sigma_{\mu,\Lambda}} + e^{m_\mu + m_\tau + 1/2(\sigma_\mu^2 + \sigma_\tau^2) + \sigma_{\mu,\tau}}\right) + (1+\gamma)(\gamma\log(\widehat{\gamma}) - 2) + \\ &+ \log(1-\gamma) - \log n + \alpha \log(K) + e^{m_\mu + 1/2\sigma_\mu^2} \sum_{i=1}^{n} \log\left(\Lambda_i^{US}\right), \end{aligned} \quad (19)$$

where m_μ, m_Λ, m_τ are the means and σ_μ^2, σ_Λ^2, σ_τ^2, $\sigma_{\mu,\Lambda}$ and $\sigma_{\mu,\tau}$ are the elements of the variance-covariance matrix of the Normal distribution of $(\log(\mu), \log(\Lambda_i^{rel}), \log(1-\tau))$.

Given data on $(1-\tau)$, productivities Λ^{rel} and multipliers μ and imposing log-Normality on them, we re-estimate the parameters of their joint distribution separately for each country using Maximum Likelihood. We then regress these country-specific parameter estimates on (log) per-capita GDP. Table 5 reports the result.[43] While the point estimates are quantitatively somewhat different from those of the baseline model (compare with Table 2), the qualitative features remain very similar: the average log multiplier, m_μ, does not vary with income, while σ_μ decreases in (log) per capita GDP. Again, this result implies that in poor countries the distribution of log multipliers has more mass at the extremes. Average log productivity is again strongly increasing in income, while the variance of log productivity is decreasing. The mean of the distribution of $\log(1-\tau)$ does not change significantly with the income level while its variance decreases in (log) per capita GDP. Moreover, in rich countries wedges tend to be lower ($(1-\tau)$ is larger) in sectors with high multipliers, while the opposite is true in poor countries. Finally, productivity levels correlate positively with log multipliers in poor countries and negatively in rich ones.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	m_μ	σ_μ	m_Λ	σ_Λ	m_τ	σ_τ	$\sigma_{\mu,\Lambda}$	$\sigma_{\mu,\tau}$
intercept	-2.868***	0.847**	-10.483***	4.002***	-0.116	0.670***	0.607*	-0.105*
	(0.341)	(0.156)	(1.908)	(0.877)	(0.306)	(0.154)	(0.317)	(0.063)
slope	-0.026	-0.035**	1.009***	-0.314***	-0.009	-0.049**	-0.049*	0.012*
	(0.035)	(0.016)	(0.187)	(0.087)	(0.030)	(0.023)	(0.025)	(0.006)
R-squared	0.010	0.151	0.610	0.579	0.008	0.321	0.153	0.287
Observations	31	31	31	31	31	31	31	31

Table 5: Regression of estimated country-specific parameters on log(GDP p.c.). Bootstrapped standard errors significant at 1% (***), 5% (**), 10% (*) significance level in parenthesis.

Next, we plug the predicted parameter values into equation (19) to forecast relative income levels. The first column of Table 6 reports the result of regressing model-predicted per capita income relative to the U.S. on actual data of relative per capita GDPs. The intercept is -0.15, implying that the model under-predicts income somewhat for poor countries. The slope coefficient is 1.031 and the R-squared is 0.939, indicating a great model fit. Thus, the model with wedges performs only slightly worse in predicting income differences than the baseline model without wedges (compare with Table 3, column (3)). This implies that allowing wedges to affect the IO structure does not change our conclusion that it is foremost the interplay between sectoral productivities and IO structure that helps to predict cross-country income differences.

[43] Note that we have less observations than in Table 2 (31 instead of 36) because the Maximum Likelihood estimation does not converge for all countries.

	(1)	(2)	(3)	(4)	(5)
	wedges	exact	demand	open	skill
intercept	-0.151***	0.010	-0.191***	0.120***	0.092**
	(0.028)	(0.039)	(0.040)	(0.034)	(0.035)
slope	1.031***	0.932***	0.821***	0.897***	1.030***
	(0.028)	(0.059)	(0.083)	(0.053)	(0.069)
R-squared	0.939	0.899	0.838	0.887	0.832
Observations	36	36	36	36	36

Table 6: Robustness checks

5.2 CES production function

Another potential concern is that sectoral production functions are not Cobb-Douglas, but instead feature an elasticity of substitution between intermediate inputs different from unity. If this were the case, IO coefficients would no longer be sector-country-specific constants γ_{jic} but would instead be endogenous to equilibrium prices, which would reflect the underlying productivities of the upstream sectors. While it has been observed that for the U.S. the IO matrix has been remarkably stable over the last decades despite large shifts in relative prices (Acemoglu et al., 2012) – an indication of a unit elasticity – in this robustness check we briefly discuss the implications of considering a more general CES sectoral production function. The sectoral production functions are now given by:

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i} M_i^{\gamma_i}, \tag{20}$$

where $M_i \equiv \left(\sum_{j=1}^{N} \gamma_{ji} d_{ji}^{\frac{(\sigma-1)}{\sigma}}\right)^{\frac{\sigma}{(\sigma-1)}}$. The rest of the model is specified as in section 3.1.

With CES production functions the equilibrium cannot be analytically solved, so one has to rely on numerical solutions. However, it is straightforward to show how IO multipliers are related to sectoral productivities in this case. The relative expenditure of sector i on inputs produced by sector j relative to sector k is given by:

$$\frac{p_j d_{ji}}{p_k d_{ki}} = \left(\frac{p_j}{p_k}\right)^{1-\sigma} \left(\frac{\gamma_{ji}}{\gamma_{ki}}\right) \tag{21}$$

Thus, if $\sigma > 1$ ($\sigma < 1$), each sector i spends relatively more on the inputs provided by sectors that charge lower (higher) prices. These sectors then have higher (lower) multipliers, as multipliers are proportional (up to a shift by $1/n$) to the sector's out-degree $d_j^{out} = \sum_{i=1}^{n} \frac{p_j d_{ji}}{p_i q_i}$ (see equation (9)). Moreover, since prices are inversely proportional to productivities, sectors with higher productivity levels charge lower prices. Consequently, when $\sigma > 1$, sectoral multipliers and productivities should be positively correlated in *all* countries, while when $\sigma < 1$, the opposite should be true. We confirm these results in unreported simulations. Observe that these predictions are not consistent with our empirical finding that multipliers and productivities are positively correlated in low-income countries, while they are negatively correlated in high-income countries. Consequently – unless the elasticity of substitution differs systematically across countries – the data on IO tables and sectoral productivities are difficult to reconcile with CES production functions.

5.3 Log-Normally distributed IO coefficients

In the baseline model we imposed the restrictive and unrealistic assumption that all non-zero elements of the input-output matrix $\mathbf{\Gamma}$ are the same, that is, $\gamma_{ji} = \widehat{\gamma}$ for any i and j whenever $\gamma_{ji} > 0$. Here we consider a more general version of the model where γ_{ji}'s are independent random draws from a log-Normal distribution and are thus allowed to vary across countries and sectors. Note that this distribution is appropriate due to three observations: (i) by equation (9), sectoral multipliers can be approximated by the sum of IO coefficients in the corresponding row of the IO matrix (shifted and multiplied by $1/n$), (ii) sectoral multipliers are log-Normally distributed, and (iii) the sum of independent log-Normal random variables is approximately log-Normal according to the Fenton-Wilkinson method (Fenton, 1960).

When IO coefficients are not constant, the term $\sum_{i=1}^{n}\sum_{j\text{s.t. } \gamma_{ji}\neq 0}\mu_i\gamma_{ji}\log\gamma_{ji}$ in equation(7) is no longer equal to $\sum_{i=1}^{n}\mu_i\gamma\log(\widehat{\gamma})$ (as in (10)). Instead, it is given by a longer and more complex expression that we derive in the Supplementary Appendix. The expectation of this term is a function of the parameters of the Normal distribution of $\log\gamma_{ji}$, $(\mu_\gamma, \sigma_\gamma^2)$. These parameters, in turn, are related to the parameters of the Normal distribution of $\log(\mu)$, (m_μ, σ_μ^2), due to the relationship established in (9): $\mu_j \approx \frac{1}{n} + \frac{1}{n}\sum_{i=1}^{n}\gamma_{ji}$.[44] This then leads to the following expression for the expected aggregate income:

$$\begin{aligned} E(y) &= ne^{m_\mu + m_\Lambda + 1/2(\sigma_\mu^2+\sigma_\Lambda^2)+\sigma_{\mu,\Lambda}} - (1+\gamma) + E\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\gamma_{ji}\log\gamma_{ji}\right] + \\ &+ \log(1-\gamma) - \log n + \alpha\log(K) + e^{m_\mu+1/2\sigma_\mu^2}\sum_{i=1}^{n}\log\left(\Lambda_i^{US}\right) = \\ &= ne^{m_\mu + m_\Lambda + 1/2(\sigma_\mu^2+\sigma_\Lambda^2)+\sigma_{\mu,\Lambda}} - (1+\gamma) + \\ &+ x^{\frac{1}{2}}z[n + x^{\frac{1}{2}}z(n^2-1)](\log(x)+\log(z)) + x^2z^2(\log(z)+2\log(x)) + \\ &+ \log(1-\gamma) - \log n + \alpha\log(K) + e^{m_\mu+1/2\sigma_\mu^2}\sum_{i=1}^{n}\log\left(\Lambda_i^{US}\right), \end{aligned} \tag{22}$$

[44] Indeed, from this equation it follows that $E(\mu) = \frac{1}{n} + \frac{1}{n}\mu_{sum}$ and $var(\mu) = \frac{1}{n^2}\sigma_{sum}^2$, where μ_{sum}, σ_{sum}^2 are the mean and the variance of the distribution of the sum $\sum_{i=1}^{n}\gamma_{ji}$, which can be expressed in terms of $(\mu_\gamma, \sigma_\gamma^2)$, and $E(\mu)$, $var(\mu)$ can be expressed in terms of (m_μ, σ_μ^2) by means of the relationship between the Normal and log-Normal distributions ($E(\mu) = e^{m_\mu+1/2\sigma_\mu^2}$, $var(\mu) = e^{2m_\mu+m_\Lambda+\sigma_\mu^2}\cdot[e^{\sigma_\mu^2}-1]$). By the Fenton-Wilkinson method, the distribution of the sum $\sum_{i=1}^{n}\gamma_{ji}$ is approximately log-Normal with $\mu_{sum} = \log\left(ne^{\mu_\gamma}\right) + \frac{1}{2}\left(\sigma_\gamma^2 - \sigma_{sum}^2\right) = \log\left(ne^{\mu_\gamma}\right) + \frac{1}{2}\left(\sigma_\gamma^2 - \log\left(\frac{\left(e^{\sigma_\gamma^2}\right)-1}{n+1}.\right)\right)$, $\sigma_{sum}^2 = \log\left(\frac{\left(e^{\sigma_\gamma^2}\right)-1}{n+1}\right)$.

where x and z are functions of (m_μ, σ_μ^2), which are provided in the Supplementary Appendix. This expression for aggregate income depends only on the parameter estimates used in the baseline model without imposing any symmetry on the IO coefficients. It is similar to the one of the baseline model but includes additional terms that capture the effect of asymmetric linkages. We use it to predict cross-country income differences in this more general setting. While it is difficult to gain intuition for the expression summarizing the effect of asymmetric IO linkages, the predicted income levels from this model are very similar to those of the baseline model. Column (2) of Table 6 demonstrates that: the intercept in the regression of model-predicted on actual income is now 0.010 and not significantly different from zero, while the slope coefficient is 0.932. This justifies the use of the much simpler and more intuitive approximation in the baseline model.

5.4 Cross-country differences in final demand structure

So far we have abstracted from cross-country differences in the final demand structure, which also matter for the values of sectoral multipliers since sectors with higher final-expenditure shares will have a larger impact on GDP. In the next robustness check, we thus consider a more general demand structure. More specifically, we now model the production function for the aggregate final good as $Y = y_1^{\beta_1} \cdot \ldots \cdot y_n^{\beta_n}$, where β_i is allowed to be country-sector-specific. The advantage of this specification is that it picks up differences in the final demand structure that may have an impact on aggregate income. The drawback is that with this specification multipliers reflect both the IO structure and taste. Thus, this specification does not allow one to differentiate between the two channels. The vector of sectoral multipliers is now defined as $\boldsymbol{\mu} = \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}$, where $\boldsymbol{\beta} = (\beta_1, .., \beta_n)'$. So, holding constant the IO structure $\boldsymbol{\Gamma}$, sectors with larger final-expenditure shares have larger multipliers. The interpretation of IO multipliers is identical to the one before: each sectoral multiplier μ_i reveals how a change in productivity of sector i affects total value added in the economy. Given the new multipliers, we re-estimate their joint distribution and predict income levels using the formula presented in the Supplementary Appendix.

The results for regressing predicted on actual per capita income for this model can be found in column (3) of Table 6. The intercept is now -0.191, the slope coefficient is 0.821, and the R-squared is 0.838, which is somewhat worse than the performance of our baseline model. This indicates that – within the context of our model – modeling differences in final demand structure does not help to understand differences in aggregate income. The reason seems to be that modeling differences in final demand structure across countries introduces additional noise in the multiplier data, which makes it harder to estimate systematic features of the inter-industry linkages.

5.5 Imported intermediates

So far we have abstracted from international trade and we have assumed that all intermediate inputs have to be produced domestically. Here, we instead allow for both domestically produced and imported intermediates, which are imperfectly substitutable. We thus assume that sectoral production functions are given by:

$$q_i = \Lambda_i \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}} \cdot f_{1i}^{\sigma_{1i}} f_{2i}^{\sigma_{2i}} \cdot \ldots \cdot f_{ni}^{\sigma_{ni}}, \tag{23}$$

where d_{ji} are domestically produced intermediate inputs and f_{ji} are imported intermediate inputs. γ_{ji} and σ_{ji} denote the shares of each domestic and imported intermediate, respectively, in the value of sectoral gross output. We change the construction of the IO tables accordingly by separating domestically produced from imported intermediates. We then re-estimate the joint distributions of IO multipliers and productivities.

The results for model fit with this specification are given in column (4) of Table 6. The intercept is now 0.120, the slope coefficient is 0.897 and the R-squared is 0.887. The fit is thus only slightly worse than the one of the baseline model. The intuition for why results remain similar when considering imported inputs comes from the fact that most high-multiplier sectors tend to be services, which are effectively non-traded. Therefore, allowing for trade does not change the statistical distribution of multipliers and the implied predicted income much. We thus conclude that our results are quite robust to allowing for trade in intermediates.

5.6 Skilled labor

Finally, we split aggregate labor endowments into skilled and unskilled labor. Namely, let the technology of each sector $i \in 1:n$ in every country be described by the following Cobb-Douglas function:

$$q_i = \Lambda_i \left(k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta}\right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}, \tag{24}$$

where s_i and u_i denote the amounts of skilled and unskilled labor used by sector i, $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ is the share of intermediate goods in the total input use of sector i and α, δ, $1-\alpha-\delta \in (0,1)$ are the respective shares of capital, unskilled and skilled labor in the remainder of the inputs. The total supply of skilled and unskilled labor in the economy is fixed at the exogenous levels of S and U, respectively. We define skilled labor as the number of hours worked by workers with at least some tertiary education and we define unskilled labor as the number of hours worked by workers with less than tertiary education. Information on skilled and unskilled labor inputs by sector is from WIOD. We recompute productivities Λ^{rel} assuming production-functions as given by (24) and then re-estimate all parameter values. We calibrate $\delta = 1/6$ to fit the college skill premium of the U.S. The results for fitting relative income differences with this model are provided in column (5) of Table 6. The intercept is 0.092, the slope coefficient is 1.030 and the R-squared is 0.832, which is a slightly worse fit compared to the baseline model. This is not suprising: given the great fit of the baseline model, there is little room left for improving the explanatory power of the model by introducing human capital. We conclude that our results are not very sensitive to the definition of labor endowments.

6 Counterfactual experiments

We now present the results of a number of counterfactual experiments. We first investigate how differences in IO structure – as summarized by the distribution of multipliers – matter for cross-country income differences. Thus, in our first counterfactual exercise we set the distribution of log multipliers in all countries equal to the U.S. one by fixing m_μ and σ_μ^2 at the predicted values of a country at the U.S.-level of per capita income.[45] Given the Cobb-Douglas structure, our model allows us to separately identify sectoral productivities and IO structure and it thus makes sense to vary one of the two factors, while holding the other one fixed.[46] In this counterfactual we present numbers for the baseline model without wedges, but results remain very similar for the model with wedges, as wedges do not affect the estimated distribution of multipliers much. The result of this experiment can be grasped from the upper left panel of Figure 11, which plots the counterfactual percentage change in income per capita against GDP per capita relative to the U.S. It can be seen that virtually all countries would lose in terms of income if they had the U.S. IO structure. These losses are decreasing in income per capita and range from negligible levels for countries with income levels close to the U.S. one, to more than 60 percent of per capita income for very poor countries such as Congo (ZAR) or Zimbabwe (ZWE).

The reason why most countries lose in this counterfactual experiment is the shape of the distribution of multipliers in the U.S.: high-income countries have a distribution of multipliers with less mass at the extremes than poor countries but much more mass in the middle range of the distribution. This implies

[45]The experiment holds m_μ fixed and reduces σ_μ for virtually all countries, since, according to Table 2, σ_μ is a decreasing function of GDP per capita. For a log-normal distribution such a change shifts mass away from the lower and upper tails towards the center of the distribution.

[46]Note that productivity levels are also unaffected by changes in the distribution of IO multipliers even when technologies are not factor-neutral. To see this, note that labor-augmenting or intermediate-augmenting rather than Hicks-neutral technologies would imply:

$$\begin{aligned} q_i &= \left[k_i^\alpha (\Lambda_i l_i)^{1-\alpha}\right]^{1-\gamma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}}, \\ q_i &= \left(k_i^\alpha l_i^{1-\alpha}\right)^{1-\gamma_i} (\Lambda_i^{\gamma_i}) d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot \ldots \cdot d_{ni}^{\gamma_{ni}} \end{aligned} \tag{25}$$

In this case, a change in the γ_{ji}s (reflecting a change in the distribution of multipliers) would also affect measured productivity $\Lambda_i^{(1-\alpha)(1-\gamma_i)}$ or $\Lambda_i^{\gamma_i}$. While this is true in general, our counterfactual exercise remains valid even in this case due to the assumption that the intermediate share $\gamma_i = \sum_{j=1}^N \gamma_{ji}$ is constant across sectors. Therefore, any change in the IO structure that is implied by a change in the parameters m_μ or σ_μ leaves productivities unaffected.

that a typical sector in the U.S. is intermediately connected (the mode of the distribution is larger than in poor countries). Given the distribution of productivities in low-income countries – with a low mean, high variance and a positive correlation with multipliers – they perform much worse with their new IO structure: now their typical sector – which is much less productive than in the U.S. – has a higher multiplier and thus is more of a drag on aggregate performance. Moreover, they can no longer benefit much from the fact that their super-star, high-multiplier sectors are relatively productive because the relative importance of these sectors for the economy has been reduced. To put it differently, recall that in low-income economies, a few sectors, such as Energy, Transport and Trade, provide inputs for most other sectors, while the typical sector provides inputs to only a few sectors. Thus, it suffices to have comparatively high productivity levels in those crucial sectors in order to obtain a relatively satisfactory aggregate outcome. By contrast, in the industrialized countries most sectors provide inputs for several other sectors (the IO network is quite dense), but there are hardly any sectors that provide inputs to most other sectors. Thus, with such a dense IO structure increasing productivity levels in a few selected sectors is no longer enough to achieve a relatively good aggregate performance.

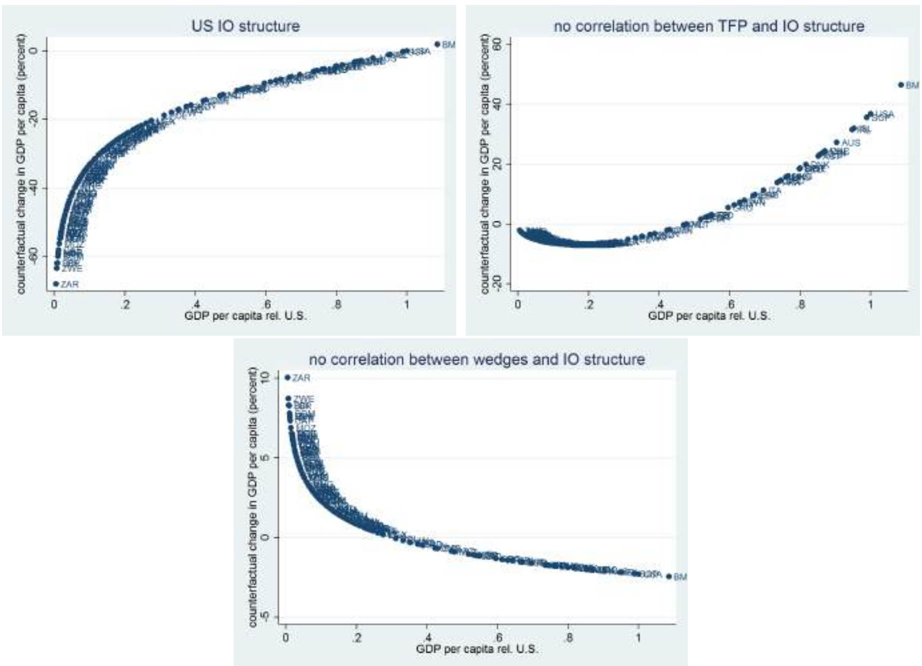


Figure 11: Counterfactuals

In the second counterfactual exercise, we keep the mean and the variance of log multipliers fixed and instead set the covariance between log multipliers and log productivities, $\sigma_{\mu,\Lambda}$, to zero. We can see from the upper right panel of Figure 11 that poor countries (up to around 40 percent of the U.S. level of income per capita) would lose significantly (up to 10 percent) in terms of their initial income, while rich countries would gain up to 40 percent from this change. Why is this the case? From our estimates, poor countries have a positive covariance between log multipliers and log productivities, while rich countries have a negative one. This implies that poor countries are doing relatively well despite their low average productivity levels, because they perform significantly better than average precisely in those sectors that have a large impact on aggregate performance. The opposite is true in rich countries, where the same covariance tends to be negative, so that highly connected sectors perform below average. Eliminating this link improves aggregate outcomes in rich economies further, while hurting poor countries.

Next, we consider the model including wedges to check if varying the covariance between wedges and log multipliers has important quantitative implications. Remember that this covariance is positive in poor countries and negative in rich ones. We thus set $\sigma_{\mu,\tau}$, the covariance between log multipliers and $\log(1-\tau)$, to zero for all countries. The lower panel of Figure 11 plots the resulting changes in per capita income (in percent) against GDP relative to the U.S. level. Poor countries – which empirically exhibit a positive covariance between multipliers and wedges – experience moderate increases in income (up to 10 percentage points for Congo (ZAR)), while rich countries – which empirically have a negative covariance between multipliers and wedges – lose around one to two percentage points of per capita income. This implies that removing the positive covariance between wedges and multipliers in poor economies can lead to significant gains for them. However, cross-country income changes are smaller than those induced by removing the covariance between productivities and multipliers.

Summary of counterfactual experiments:

1. *Imposing the dense IO structure of the U.S. on poor economies would reduce their income levels by up to 60 percent because a typical sector, which has a lower productivity level than the high-multiplier sectors in these economies, would become more connected.*

2. *If poor economies did not have above-average productivity levels in high-multiplier sectors, their income levels would be reduced by up to 10 percent.*

3. *If poor economies did not have above-average wedges in high-multiplier sectors, their income levels would increase by up to 10 percent.*

7 Optimal taxation

The model with wedges employed in section 5.1 considers wedges as exogenously given and wasteful. In this section, we introduce an active role for the government and address the problem of optimal taxation by interpreting wedges τ_i as taxes imposed by the government to finance its expenditures and, possibly, also proceed to redistribution. To do that, we should specify the objective function of the government or social planner that is to be maximized by the choice of tax rates. As there are no other frictions, the redistribution motive is likely to be absent. Then we analyze the problem of optimal taxation for exogenously specified government expenditures. The appealing feature of analyzing such semi-optimal taxation schemes (with exogenously fixed government expenditures) is that they are much less dependent on the specific welfare function. Indeed, as long as welfare increases with individual consumption C, any welfare function would generate the same outcome for exogenously fixed government consumption G. In short, we will designate this analysis as GDP per capita maximization with exogenous G.[47]

7.1 Optimal taxes: setup

To derive characteristics of optimal tax scheme, we use the equilibrium expression for log GDP modified to account for government revenues. The logarithm of GDP per capita, y, is given by

$$\begin{aligned} y &= \sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \mu_i \log(1-\tau_i) + \sum_{i=1}^{n} \sum_{j \text{ s.t. } \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) - \log n + \\ &+ \log\left(1 + \sum_{i=1}^{n} \tau_i \bar{\mu}_i\right) + \alpha \log K, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\tau} &= \{\tau_i\}_i, && n \times 1 \text{ vector of sector-specific taxes} \\ \bar{\boldsymbol{\mu}} &= \{\bar{\mu_i}\}_i = \frac{1}{n}[\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}]^{-1}\mathbf{1}, && n \times 1 \text{ vector of multipliers corresponding to } \bar{\boldsymbol{\Gamma}} \\ \bar{\boldsymbol{\Gamma}} &= \{\bar{\gamma_{ji}}\}_{ji} = \{\frac{\tau_i}{n} + (1-\tau_i)\gamma_{ji}\}_{ji}, && n \times n \text{ input-output matrix adjusted for taxes} \end{aligned}$$

This expression is very similar to the one in (7) of the baseline model but includes two extra terms that capture the effects of taxation: taxes, on the one hand, are distortionary and more so in sectors with larger multipliers, but on the other hand, they also contribute to government expenditures and thereby increase GDP.[48]

We consider the optimization problem in which this expression is maximized subject to a given level of government consumption. To solve that problem, we follow the statistical approach, in line with the rest of the paper. That is, instead of considering actual values of taxes, we focus on the first and second moments of their distribution that generate the highest predicted aggregate output $E(y)$ for a given level of expected tax revenues/government consumption as computed from the data.[49] The expected values of aggregate output and tax revenues/government consumption are computed via a Monte Carlo optimization method under the assumption that sectoral IO multipliers, productivities and $(1-\tau_i)$ follow

[47] In unreported simulations we have considered the case with endogenous government expenditures. There we assumed that government expenditures enter households utility in a Cobb-Douglas fashion. The results were very similar to those of the model that takes government expenditure as given.

[48] The detailed proof is available from the authors.

[49] An analytical solution in terms of actual values of tax rates (that maximize y subject to a given level of tax revenues) appears feasible only under some strong simplifying assumptions, which eventually lead to trivial or corner values of tax rates. We therefore resort to the statistical approach, which is also consistent with our approach in the prior empirical analysis.

a trivariate log-Normal distribution. All parameters of this distribution, apart from those that relate to the distribution of taxes, are fixed at the levels of their empirical estimates. Then by varying the mean, variance and covariance of the tax distribution,[50] we derive the features of the optimal tax scheme. The results of this numerical analysis can be briefly summarized as follows.

7.2 Optimal taxes: results

We assume that for each country, government consumption is fixed at the level generated by the estimated distributions. We find that the optimal tax distribution is degenerate with variance $\sigma_\tau^2 \to 0$. The correlation between taxes and IO multipliers is not relevant in the limit. Empirically, the optimal mean tax rate in poor countries is substantially higher than the estimated ones (for some poor countries the optimal mean tax rate can be larger by a factor of 10). For rich countries, the optimal tax rate is only marginally larger. In fact, the estimated distribution of tax rates in rich countries turns out to be close to optimum, featuring low variance and reasonable mean. In poor countries, instead, the variance is high and the estimated mean tax rate is substantially lower than the optimal one. Moreover, there is a large positive correlation between tax rates and sectoral IO multipliers in poor countries, which ensures that high-multiplier sectors are taxed more. The latter is precisely the reason why a given level of tax revenues in poor countries can be reached with a lower mean tax rate than prescribed in optimum. Indeed, under the optimal tax scheme all sectors should be taxed evenly, and then raising the same amount of tax revenues requires a higher mean tax. Still, we find that the distortion loss associated with high (optimal) mean tax is small compared to the loss associated with taxing high-multiplier sectors more. The left panel of Figure 12 plots welfare gains (in terms of percentage gains in GDP) of moving to a uniform tax rate that generates the same revenue as the current tax system against GDP per capita. The welfare gains are basically zero for all high-income countries but they can rise to up to 10% of GDP for some of the poorest countries in the world.

We also perform a more unusual experiment. Indeed, as there might be reasons why tax rates cannot be uniform, we want to explore the role of the covariance between taxes and IO multipliers for a given variation in tax rates. We set the variance of the tax rate distribution to be equal to the estimated value in each country and examine the role of choosing the optimal correlation between the distribution of tax rates and sectoral IO multipliers and the mean tax rate that keeps tax revenue constant. We find that the optimal tax distribution has negative correlation with sectoral IO multipliers, so that consistently with the findings of our empirical analysis, more central sectors should be taxed less. The right panel of Figure 12 plots the percentage gains in GDP per capita of moving to the optimal correlation between

[50] By covariance we mean the covariance between the distribution of taxes and IO multipliers, as the covariance between taxes and productivities does not affect the calculated values.

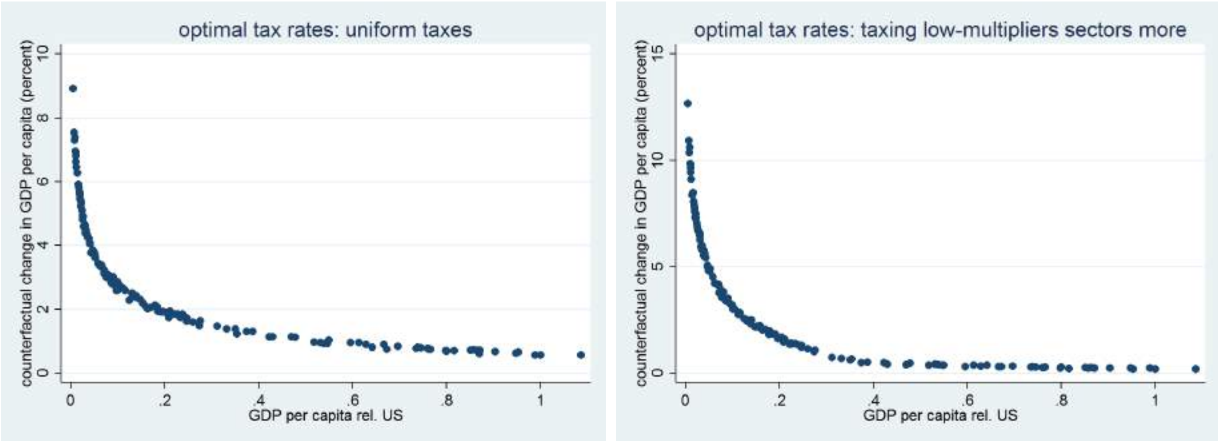


Figure 12: Optimal taxation

taxes and multipliers that keeps tax revenue constant. Again, welfare gains are substantial for very poor countries. Moreover, moving to a negative correlation between taxes and multipliers and increasing average tax rates would imply gains which are almost as large as those of moving to a uniform tax rate.

8 Conclusions

In this paper we have studied the role of input-output structure and its interaction with sectoral productivity levels in explaining income differences across countries. In contrast to the typical approach in the literature on development accounting, dual economies and structural transformation, we explicitly model input-output linkages between sectors and the difference in these linkages across countries. Moreover, our approach is to a large extent empirical, which complements the predominantly theoretical analysis of previous studies on cross-country differences in IO structure.

We first develop and analytically solve a multi-sector general equilibrium model with IO linkages and sector-specific productivities. We then estimate this model using a statistical approach that allows us to derive a simple closed-form expression of aggregate per capita income as a function of the first and second moments of the joint distribution of IO multipliers and sectoral productivities. We estimate country-specific parameters of this distribution to fit the corresponding empirical distributions of sectoral IO multipliers and productivities for a large cross-section of countries at all stages of development. The estimates imply important cross-country differences in countries' IO structure as well as in the interaction between IO structure and sectoral productivities. First, in low-income countries the distribution of sectoral IO multipliers is more extreme: while most sectors have very low multipliers, the multipliers of a small number of sectors are very high compared to the average. By contrast, the distribution of sectoral multipliers in rich countries allocates a relatively large weight to intermediate values of multipliers. Moreover, while in poor countries sectoral IO multipliers and productivities are positively correlated, in rich countries this correlation is negative.

These cross-country differences in the distribution of IO multipliers and their interaction with productivities lead to differences in predicted income levels. We find that our (over-identified) model predicts cross-country income differences extremely well both within and out of sample. In fact, the generated predictions are much more accurate than those of a model that measures aggregate productivity as an average of the estimated sectoral productivities and ignores IO structure. Such a model overpredicts the variation in per capita income. The reason is that the empirically large sectoral TFP differences are actually mitigated by IO structure. In particular, since very low-productivity sectors in poor countries tend to be badly connected, they are not that relevant for the aggregate economy.

Our counterfactual experiments suggest that if we impose the much denser IO structure of the U.S. on poor countries and thereby increase the overall significance of their low-productivity sectors, the per capita income of these countries could decline by as much as 60%. That is, given the very low productivity levels of many sectors in poor countries, having these sectors largely isolated effectively benefits these economies. Similarly, eliminating the correlation between sectoral multipliers and productivities would hurt poor countries but benefit the rich ones, due to the fact that the correlation of multipliers and productivities is positive in poor countries and negative in rich economies. At last, reducing distortions arising from sectoral wedges would improve the aggregate economic performance of poor countries; however, the associated per capita income changes would be relatively small.

Finally, we study the problem of optimal taxation and analyze the welfare gains from moving to an optimal tax system in all countries, while keeping tax revenues constant. Our findings suggest that when the government aims at maximizing GDP per capita for a given level of tax revenue, the actual distribution of tax rates is close to optimum in rich countries, but in poor countries, the mean of the distribution is too low and the variance is too high relative to the optimum. We also find that for a given value of tax variance, a negative correlation of taxes with IO multipliers is optimal, which once again suggests a relative advantage of the tax scheme implemented in rich countries. We find that some of the poorest countries in the world could gain up to 10% in terms of income per capita by moving to an optimal tax system, while benefits for the rich countries would be negligible.

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Appendix: Proofs for the benchmark model and its extensions

Proposition 1 and formulas for aggregate output in the main text are particular cases of Proposition 2 that applies in a generic setting – with imported intermediates, division of labor into skilled and unskilled labor inputs and unequal demand shares. A brief description of this economy, as well as Proposition 2 and its proof are provided below.

- The technology of each of n competitive sectors is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta} \right)^{1-\gamma_i-\sigma_i} d_{1i}^{\gamma_{1i}} d_{2i}^{\gamma_{2i}} \cdot ... \cdot d_{ni}^{\gamma_{ni}} \cdot f_{1i}^{\sigma_{1i}} f_{2i}^{\sigma_{2i}} \cdot ... \cdot f_{ni}^{\sigma_{ni}},$$

where s_i and u_i are the amounts of skilled and unskilled labor, d_{ji} is the quantity of the domestic good j and f_{ji} is the quantity of the imported good j used by sector i. $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ and $\sigma_i = \sum_{j=1}^{n} \sigma_{ji}$ are the respective shares of domestic and imported intermediate goods in the total input use of sector i and α, δ, $1 - \alpha - \delta$ are the respective shares of capital, unskilled and skilled labor in the remainder of the inputs.

- A good produced by sector i can be used for final consumption, y_i, or as an intermediate good:

$$y_i + \sum_{j=1}^{n} d_{ij} = q_i \qquad i = 1 : n$$

- Final consumption goods are aggregated into a single final good through another Cobb-Douglas production function:

$$Y = y_1^{\beta_1} \cdot ... \cdot y_n^{\beta_n},$$

where $\beta_i \geq 0$ for all i and $\sum_{i=1}^{n} \beta_i = 1$.

- This aggregate final good can itself be used in one of two ways, as households' consumption or export to the rest of the world:

$$Y = C + X.$$

- Exports pay for the imported intermediate goods, and we impose a balanced trade condition:

$$X = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{p}_j f_{ji},$$

where $\overline{p}_j$ is the exogenous world price of the imported intermediate goods.

- Households finance their consumption through income:

$$C = w_U U + w_S S + rK.$$

- The total supply of physical capital, unskilled and skilled labor are fixed at the exogenous levels of K, U and S, respectively:

$$\begin{aligned} \sum_{i=1}^{n} k_i &= K, \\ \sum_{i=1}^{n} u_i &= U, \\ \sum_{i=1}^{n} s_i &= S. \end{aligned}$$

For this "generic" economy, the competitive equilibrium with distortions is defined by analogy with the definition in section 3.1. The solution is described by Proposition 2.

Proposition 2. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of GDP per capita, $y = \log(Y/(U+S))$, is given by*

$$\begin{aligned} y \;=\; & \frac{1}{\sum_{i=1}^{n} \mu_i(1-\gamma_i-\sigma_i)} \Bigg[\sum_{i=1}^{n} \mu_i\lambda_i + \sum_{i=1}^{n} \sum_{j \; s.t.\gamma_{ij}\neq 0} \mu_i\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^{n} \sum_{j \; s.t.\sigma_{ij}\neq 0} \mu_i\sigma_{ji}\log\sigma_{ji} - \\ & - \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\sigma_{ji}\log\bar{p_j} + \sum_{i=1}^{n}\beta_i log\beta_i + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)log(1-\gamma_i-\sigma_i)\Bigg] + \log\left(1+\sum_{i=1}^{n}\sigma_i\bar{\mu_i}\right) + \\ & + \alpha\log K + \delta logU + (1-\alpha-\delta)logS - \log(U+S). \qquad (26) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\mu} &= \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}, && n\times 1 \text{ vector of multipliers} \\ \boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\log\Lambda_i\}_i, && n\times 1 \text{ vector of sectoral log-productivity coefficients} \\ \bar{\boldsymbol{\mu}} &= \{\bar{\mu_i}\}_i = [\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}]^{-1}\boldsymbol{\beta}, && n\times 1 \text{ vector of multipliers corresponding to } \bar{\boldsymbol{\Gamma}} \\ \bar{\boldsymbol{\Gamma}} &= \{\bar{\gamma_{ji}}\}_{ji} = \{\beta_j\sigma_i + \gamma_{ji}\}_{ji}, && n\times n \text{ input-output matrix adjusted for trade} \end{aligned}$$

Proof. Part I: Calculation of $\log w_U$.

Consider the profit maximization problems of a representative firm in the final goods market and in each sector. For a representative firm in the final goods market the FOCs allocate to each good a spending share that is proportional to the good's demand share β_i:

$$p_i y_i = \beta_i Y = \beta_i(C+X) = \beta_i(w_U U + w_S S + rK) + \beta_i \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{p}_j m_{ji} \quad \forall i \in 1:n$$

where the price of the final good is normalized to 1, $p = 1$. For a firm in sector i the FOCs are:

$$\alpha(1-\gamma_i-\sigma_i)\frac{p_i q_i}{r} = k_i \quad (27)$$
$$\delta(1-\gamma_i-\sigma_i)\frac{p_i q_i}{w_U} = u_i \quad (28)$$
$$(1-\alpha-\delta)(1-\gamma_i-\sigma_i)\frac{p_i q_i}{w_S} = s_i \quad (29)$$
$$\gamma_{ji}\frac{p_i q_i}{p_j} = d_{ji} \quad j \in 1:n \quad (30)$$
$$\sigma_{ji}\frac{p_i q_i}{\bar{p_j}} = f_{ji} \quad j \in 1:n \quad (31)$$

Substituting the left-hand side of these equations for the values of k_i, u_i, s_i, $\{d_{ji}\}$ and $\{f_{ji}\}$ in firm i's log-production technology and simplifying the obtained expression, we derive:

$$\begin{aligned}\delta \log w_U = & \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \log p_i - \sum_{j=1}^{n}\gamma_{ji}\log p_j + \sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\gamma_{ji}\log\gamma_{ji} - \\ & - \sum_{j=1}^{n}\sigma_{ji}\log\bar{p_j} + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\sigma_{ji}\log\sigma_{ji}\Big) - \alpha\log r - (1-\alpha-\delta)\log(w_S) + \\ & + \log(1-\gamma_i-\sigma_i) + \alpha\log(\alpha) + \delta\log\delta + (1-\alpha-\delta)\log(1-\alpha-\delta) \end{aligned} \quad (32)$$

Next, we use FOCs (27) – (31) and market clearing conditions for labor and capital to express r and w_S in terms of w_U:

$$w_U = \frac{1}{U}\delta\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(p_i q_i) \quad (33)$$
$$w_S = \frac{1}{S}(1-\alpha-\delta)\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(p_i q_i) = \frac{w_U U}{S}\frac{1-\alpha-\delta}{\delta} \quad (34)$$
$$r = \frac{1}{K}\alpha\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(p_i q_i) = \frac{w_u U}{K}\frac{\alpha}{\delta} \quad (35)$$

Substituting these values of r and w_S in (32) we obtain:

$$\begin{aligned}\log w_U = & \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \log p_i - \sum_{j=1}^{n}\gamma_{ji}\log p_j + \sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^{n}\sigma_{ji}\log\bar{p_j} + \\ & + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\sigma_{ji}\log\sigma_{ji}\Big) + \alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log(1-\gamma_i-\sigma_i) + \log\delta \end{aligned}$$

Multiplying this equation by the ith element of the vector $\boldsymbol{\mu}'\boldsymbol{Z} = \boldsymbol{\beta}'\mathbf{1}'[\boldsymbol{I}-\boldsymbol{\Gamma}']^{-1}\cdot\boldsymbol{Z}$, where $\boldsymbol{Z}$ is a diagonal matrix with $Z_{ii} = 1 - \gamma_i - \sigma_i$, and summing over all sectors i gives

$$\sum_{i=1}^{n} \mu_i(1-\gamma_i-\sigma_i)\log w_U = \sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\beta_i \log p_i + \sum_{i=1}^{n}\sum_{j \text{ s.t. } \gamma_{ji}\neq 0} \mu_i\gamma_{ji}\log\gamma_{ji} -$$
$$-\sum_{j=1}^{n}\mu_i\sigma_{ji}\log\bar{p}_j + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\mu_i\sigma_{ji}\log\sigma_{ji} + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) +$$
$$+\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\left(\alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log\delta\right)$$

Next, we use the relation between the price of the final good p (normalized to 1) and prices of each sector goods, derived from a profit maximization of the final good firm that has Cobb-Douglas technology.[51] This relation implies that $\prod_{i=1}^{n}(p_i)^{\beta_i} = \prod_{i=1}^{n}(\beta_i)^{\beta_i}$, so that $\sum_{i=1}^{n}\beta_i\log p_i = \sum_{i=1}^{n}\beta_i\log\beta_i$, and the above equation becomes:

$$\begin{aligned}\log w_U &= \frac{1}{\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)}\Big[\sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\beta_i\log\beta_i + \sum_{i=1}^{n}\sum_{j \text{ s.t. } \gamma_{ji}\neq 0}\mu_i\gamma_{ji}\log\gamma_{ji} - \\ &- \sum_{j=1}^{n}\mu_i\sigma_{ji}\log\bar{p}_j + \sum_{j \text{ s.t. } \sigma_{ji}\neq 0}\mu_i\sigma_{ji}\log\sigma_{ji} + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i)\Big] + \\ &+ \alpha\log K - (1-\delta)\log U + (1-\alpha-\delta)\log S + \log\delta \end{aligned} \tag{36}$$

Part II: Calculation of y.

Recall that our ultimate goal is to find $y = \log(Y/(U+S)) = \log(C+X) - \log(U+S)$. From the households' budget constraint and from the balanced trade condition, $C + X = w_U U + w_S S + rK + \sum_{i=1}^{n}\sum_{j=1}^{n}\overline{p}_j f_{ji}$, where in the last term, $\overline{p}_j f_{ji} = \sigma_{ji}p_i q_i$ (cf. (31)). Below we show that $p_i q_i$ can be expressed as a product of $w_U U + w_S S + rK$ and another term that involves structural characteristics. Then using (34) and (35), we obtain the representation of $C + X$ as a product of w_U and another term determined by exogenous variables. This representation, together with (36), will then allow us to solve for y.

Consider the resource constraint for sector j, with both sides multiplied by p_j:

$$p_j y_j + \sum_{i=1}^{n} p_j d_{ji} = p_j q_j$$

Using FOCs of the profit maximization problem of the final good's firm and a firm in sector i, this can

[51] Profit maximization of the final good's firm implies that $\frac{\partial Y}{\partial y_i} = \frac{p_i}{p}$. On the other hand, since $Y = y_1^{\beta_1}\cdot\ldots\cdot y_n^{\beta_n}$, we have $\frac{\partial Y}{\partial y_i} = \beta_i\frac{Y}{y_i}$. Hence, $\beta_i\frac{Y}{y_i} = \frac{p_i}{p}$, or $y_i = \beta_i\frac{pY}{p_i}$. Substituting this in the production technology of the firm in final good market, we obtain:

$$Y = \prod_{i=1}^{n}\left(\beta_i\frac{pY}{p_i}\right)^{\beta_i} = pY\prod_{i=1}^{n}\left(\beta_i\frac{1}{p_i}\right)^{\beta_i}.$$

So, $p\prod_{i=1}^{n}\left(\beta_i\frac{1}{p_i}\right)^{\beta_i} = 1$. Now, since we used the normalization $p = 1$, it must be that $\prod_{i=1}^{n}(p_i)^{\beta_i} = \prod_{i=1}^{n}(\beta_i)^{\beta_i}$.

be written as:

$$\beta_j Y + \sum_{i=1}^{n} \gamma_{ji} p_i q_i = p_j q_j$$

or

$$\beta_j (w_U U + w_S S + rK) + \sum_{i=1}^{n} \gamma_{ji} p_i q_i + \beta_j \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ji} p_i q_i = p_j q_j.$$

Using the fact that $\sum_{j=1}^{n} \sigma_{ji} = \sigma_i$ and combining terms, we obtain:

$$\beta_j (w_U U + w_S S + rK) + \sum_{i=1}^{n} \left[\beta_j \sigma_i + (1-\tau_i)\gamma_{ji}\right] p_i q_i = p_j q_j.$$

Denote by $a_j = p_j q_j$ and by $\bar{\gamma}_{ji} = \beta_j \sigma_i + \gamma_{ji}$. Then the above equation in the matrix form is:

$$(w_U U + w_S S + rK)\, \boldsymbol{\beta} + \bar{\boldsymbol{\Gamma}} \boldsymbol{a} = \boldsymbol{a}$$

where $\boldsymbol{\beta} = (\beta_1, .., \beta_n)'$, $\bar{\boldsymbol{\Gamma}} = \{\bar{\gamma}_{ji}\}_{ji}$ and $\boldsymbol{a} = \{a_j\}_j$. Hence,

$$\boldsymbol{a} = (\boldsymbol{I} - \bar{\boldsymbol{\Gamma}})^{-1} (w_U U + w_S S + rK)\, \boldsymbol{\beta} = (w_U U + w_S S + rK) \bar{\boldsymbol{\mu}}$$

where $\bar{\boldsymbol{\mu}} = \left(\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}\right)^{-1} \boldsymbol{\beta}$.[52] So, $a_i = p_i q_i = (w_U U + w_S S + rK)\bar{\mu}_i$ and therefore,

$$\begin{aligned} Y &= C + X = w_U U + w_S S + rK + \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ji} p_i q_i = \\ &= (w_U U + w_S S + rK) \left(1 + \sum_{i=1}^{n} \sigma_i \bar{\mu}_i\right) \end{aligned}$$

Using (34) and (35), this leads to

$$Y = \frac{w_U U}{\delta} \left(1 + \sum_{i=1}^{n} \sigma_i \bar{\mu}_i\right).$$

so that

$$y = \log Y - \log(U+S) = \log w_U + \log U + \log\left(1 + \sum_{i=1}^{n} \sigma_i \bar{\mu}_i\right) - \log\delta - \log(U+S).$$

Finally, substituting $\log w_U$ with (36) yields our result:

$$\begin{aligned} y &= \frac{1}{\sum_{i=1}^{n} \mu_i (1-\gamma_i-\sigma_i)} \Bigg[\sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \sum_{j \text{ s.t.} \gamma_{ij} \neq 0} \mu_i \gamma_{ji} \log\gamma_{ji} + \sum_{i=1}^{n} \sum_{j \text{ s.t.} \sigma_{ij} \neq 0} \mu_i \sigma_{ji} \log\sigma_{ji} - \\ &\quad - \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i \sigma_{ji} \log\bar{p_j} + \sum_{i=1}^{n} \mu_i (1-\gamma_i-\sigma_i) log(1-\gamma_i-\sigma_i) + \sum_{i=1}^{n} \beta_i log \beta_i \Bigg] + \log\left(1 + \sum_{i=1}^{n} \sigma_i \bar{\mu}_i\right) + \\ &\quad + \alpha \log K + \delta log U + (1-\alpha-\delta) log S - \log(U+S). \end{aligned}$$

[52]Notice that $\left(\boldsymbol{I} - \bar{\boldsymbol{\Gamma}}\right)^{-1}$ exists because the sum of elements in each column of $\bar{\boldsymbol{\Gamma}}$ is less than 1 for any $\sigma_i + \gamma_i < 1$: $\sum_{j=1}^{n} (\beta_j \sigma_i + \gamma_{ji}) = \sigma_i + \gamma_i < 1$.

This completes the proof. □

Application of Proposition 2 to the case of the benchmark economy:

Proof. (Proposition 1) In case of our benchmark economy, we assume that: i) there is no distinction between skilled and unskilled labor, so that $\delta = 1-\alpha$ and the total supply of labor is normalized to 1; ii) demand shares for all final goods are the same, that is, $\beta_i = \frac{1}{n}$ for all i; iii) the economies are closed, so that no imported intermediate goods are used in sectors' production, that is, $\sigma_{ji} = 0$ for all $i, j \in 1:n$ and $\sigma_i = 0$ for all i. This simplifies the expression for y in Proposition 2 and produces:

$$y = \frac{1}{\sum_{i=1}^{n} \mu_i (1-\gamma_i)} \left(\sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \sum_{j \text{ s.t. } \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) - \log n \right) + \alpha \log K.$$

Now, observe that $\sum_{i=1}^{n} \mu_i (1-\gamma_i) = \mathbf{1}'[\boldsymbol{I} - \boldsymbol{\Gamma}] \cdot \frac{1}{n} [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1} \mathbf{1} = \frac{1}{n} \mathbf{1}'\mathbf{1} = 1$. Then the expression simplifies even further and leads to the result of Proposition 1:

$$y = \sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \sum_{j \text{ s.t. } \gamma_{ji} \neq 0} \mu_i \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \mu_i (1-\gamma_i) log(1-\gamma_i) - \log n + \alpha \log K,$$

where

$$\boldsymbol{\mu} = \{\mu_i\}_i = \frac{1}{n} [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1} \mathbf{1}, \qquad n \times 1 \text{ vector of multipliers}$$

$$\boldsymbol{\lambda} = \{\lambda_i\}_i = \{\log \Lambda_i\}_i, \qquad n \times 1 \text{ vector of sectoral log-productivity coefficients.}$$

□