Income Differences, Productivity and Input-Output Networks *

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Abstract

We study the importance of input-output (IO) linkages and sectoral productivity (TFP) levels in determining cross-country income differences. Using data on IO tables and sectoral TFP levels for 38 countries, we uncover important differences in the interaction of IO structure with sectoral TFP levels across countries: while highly connected sectors are more productive than the typical sector in poor countries, the opposite is true in rich ones. To assess the quantitative role of linkages and sectoral TFP differences in cross-country income differences, we decompose cross-country variation in real GDP per worker using a multi-sector general equilibrium model. We find that these features explain between 8 and 10 percent of cross-country income variation.

KEY WORDS: input-output structure, productivity, cross-country income differences, development accounting

JEL CLASSIFICATION: O11, O14, O47, C67, D85

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1 Introduction

The development accounting literature[1] has established that cross-country differences in income per capita come from two sources: from aggregate productivity differences and from differences in physical production factors. This paper takes this a step further and decomposes aggregate productivity differences into sectoral productivity variation and differences in the interaction of countries' input-output (IO) structure with sectoral productivities. IO linkages between sectors can potentially dampen or amplify sectoral productivity differences, as noted by a literature in development economics initiated by Hirschman (1958), with more recent contributions provided by Ciccone (2002) and Jones (2011 a,b). In this paper we contribute to this literature by establishing systematic and empirically relevant cross-country differences in the interaction of IO structure with sectoral TFP levels. We then show, theoretically and quantitatively, that these differences are of first-order importance for explaining cross-country income variation.

Countries' IO structure, by means of the linkages between sectors, determines each sector's importance or "weight" in aggregate TFP. It can be effectively summarized using the distribution of sectoral IO *multipliers*. The (first-order) IO multiplier of a sector depends on the value-added share of that sector, the number of sectors to which the sector supplies and the intensity with which its output is used as an input by other sectors.[2] It measures by how much aggregate income changes if productivity of a given sector changes by one percent. Thus, TFP levels in sectors with high multipliers have a larger impact on aggregate income compared to sectors with low multipliers.

To quantitatively assess the role of IO linkages and sectoral TFP levels for cross-country income differences, we first build a neoclassical multi-sector model that admits a closed-form solution for GDP per worker as a log-linear function of sectoral IO multipliers, sectoral TFP levels and the capital stock per worker.[3] Higher average IO multipliers, higher average sectoral TFP and a positive correlation between sectoral IO multipliers and TFP levels all have a positive effect on income per worker.

We then use data from the World Input-Output Database (Timmer, 2012) to construct a unique dataset of IO tables and sectoral TFP levels (relative to those of the U.S.) for 38 low and high-income countries and 35 sectors. The empirical distribution of sectoral multipliers has a fat right tail in all countries, so that the TFP levels of a few high-multiplier sectors have a large impact on

[1] See, e.g., Klenow and Rodriguez-Clare (1997), Hall and Jones (1999), Caselli (2005).

[2] The intensity of input use is measured by the IO coefficient, which states the cents spent on that input per dollar of output produced. There are also higher-order effects, which depend on the IO coefficients of the sectors to which the sectors that use the initial sector's output as an input supply.

[3] In our baseline model, we take variation in IO structure across countries as exogenous. Due to Cobb-Douglas technology, the IO coefficients correspond to the coefficients of the sectoral Cobb-Douglas production functions, which are independent of TFP levels. In robustness checks we account for possible endogeneity of IO linkages by: (i) allowing for sector-country-specific tax wedges; (ii) introducing CES production functions, which makes IO linkages endogenous to sectoral TFP levels.

aggregate outcomes. This feature is more pronounced in low-income countries. Most importantly, in low-income countries, sectoral IO multipliers and TFP levels are *positively* correlated, while they are *negatively* correlated in rich economies.

When feeding the empirical values of sectoral IO multipliers, sectoral TFP levels and aggregate capital stocks per worker into our model, it is able to explain roughly 90 percent of the income variation observed in the data, which is remarkable.[4] To understand the channels of cross-country income differences in our model, we then provide an exact variance decomposition of log GDP per worker. The model splits income variation into (i) variation in the capital stock per worker, (ii) variation in average sectoral multipliers and average sectoral TFP levels and (iii) variation in the covariance between sectoral TFPs and multipliers across countries.[5] Variation in capital stocks per worker, and in average sectoral TFP levels and multipliers explain roughly equal proportions of the variation in GDP per worker – about 49% each. Importantly, variation in the covariance term between TFP levels and multipliers *reduces* the variation in GDP per worker by 8-10%. Intuitively, the large average sectoral TFP differences are mitigated by countries' IO structures: in low-income countries, low-productivity sectors tend to be poorly connected (have low multipliers) and are thus not too harmful, while sectors with high multipliers have relatively high productivity levels and thus boost aggregate income.[6] By contrast, in high-income countries, high-multiplier sectors tend to have below-average productivity levels, which reduces income of rich countries significantly. Thus, since (iii) is a part of aggregate TFP, relative to the 90% of the total variation explained by the model, cross-country income differences can be split roughly into 45% due to aggregate TFP differences and 55% due to differences in production factors per worker.[7]

In our baseline model, differences in IO structure across countries are exogenously given. However, one may be concerned that observed IO linkages are affected by (implicit) tax wedges. In an extension, we thus identify sector-country-specific wedges as deviations of sectoral intermediate input shares from their cross-country average value: a below-average intermediate input share in a given sector identifies a positive implicit tax wedge. We show that poor countries have higher average tax wedges and also tax their high-multiplier sectors relatively more, while the opposite is the case in rich economies. Introducing wedges into our model reduces the role of average productivity

[4]The residual variation is arguably due to measurement error, which is more severe in low-income economies.

[5]In the light of Hulten's (1978) results, one may be skeptical whether using a structural general equilibrium model and considering the features of the IO matrices adds much compared to computing aggregate TFP as a weighted average of sectoral TFPs (where the adequate 'Domar' weights correspond to the shares of sectoral gross output in GDP). Absent distortions, Domar weights equal sectoral IO multipliers and summarize the direct and indirect effect of IO linkages. However, such a reduced-form approach does not allow to assess which features of the IO structure matter for aggregate outcomes or to compute counter-factual outcomes due to changes in IO structure or productivities, as we do. Finally, as Basu and Fernald (2002) show, in the presence of sector-specific distortions (that we consider in an extension) the simple reduced-form connection between sectoral productivities and aggregate TFP breaks down.

[6]An important exception is agriculture, which in low-income countries has a high IO multiplier and a below-average productivity level.

[7]$45\% \approx (49 - 8) * 100/90$.

differences in explaining cross-country income differences by around 5%, while leaving the role of the covariance term between sectoral TFPs and multipliers unaffected (it reduces income variation by 9%). Moreover, variation in average wedges and the covariance between multipliers and wedges across countries explain an additional 7% of income variation. Overall, the message that according to the model cross-country income variation can be decomposed into roughly 45% due to aggregate TFP differences (including wedges), and 55% due to differences in production factors per worker continues to hold.

In a further robustness check, we relax the assumption of a unit elasticity of substitution between intermediate inputs, so that IO linkages become endogenous to prices. We show that an elasticity of substitution between intermediate inputs different from unity is hard to reconcile with the data because – depending on whether intermediates are substitutes or complements – it implies that sectoral IO multipliers and TFP levels should either be positively or negatively correlated in *all* countries. Instead, we observe a positive correlation between these variables in poor economies and a negative one in rich economies.

Moreover, we also extend our baseline model to incorporate trade in intermediate inputs. This model explains 94% of cross-country income variation: higher relative prices of imported intermediates in poor countries account for an additional 10% of cross-country income variation, and the remaining variation is split according to 49% due to capital stock per worker, 46% due to average multiplier and average TFP, −12% due to the covariance term between multipliers and TFP. In the last robustness check, we differentiate between skilled and unskilled labor inputs, which increases the role of physical factors by 5%, while keeping the role of the interaction between TFP and multipliers unaffected.

Finally, we carry out a number of simple counterfactuals. First, we eliminate TFP differences between countries and set all sectoral TFP levels equal to those of the U.S. Not surprisingly, virtually all countries would gain if they had the U.S. productivity levels. Low-income countries would benefit most, with some of them almost doubling their income per worker. Second, we impose that sectoral IO multipliers and productivities are uncorrelated. This scenario would hurt low-income countries significantly: they would lose up to 20% of income per worker, because they would no longer experience the advantage of having above-average TFP levels in high-multiplier sectors. By contrast, high-income countries would benefit, since for them the correlation between multipliers and TFP levels would no longer be negative. In the last counterfactual we eliminate the correlation between sectoral wedges and multipliers. This would benefit a number of low-income countries and raise their income by around 10%. On the other hand, the income of rich countries would fall, since these countries tend to have below average tax wedges in high-multiplier sectors.

1.1 Literature

We now turn to a discussion of the related literature.

Our work is related to the literature on development accounting, which aims at quantifying the importance of cross-country variation in factor endowments – such as physical, human or natural capital – relative to aggregate productivity differences in explaining disparities in income per capita across countries. This literature typically finds that both are roughly equally important in accounting for cross-country income differences.[8] The approach of development accounting is to specify an aggregate production function for value added (typically Cobb-Douglas) and to back out productivity differences as residual variation that reconciles the observed income differences with those predicted by the model given the observed variation in factor endowments. Thus, this aggregate production function abstracts from cross-country differences in the underlying IO structure. We contribute to this literature by: (i) showing how an aggregate production function for value added can be derived in the presence of IO linkages; (ii) proving that differences in the interaction between IO structure and sectoral TFPs are of first-order importance for explaining cross-country income differences.

The importance of linkages and IO multipliers for aggregate income differences has been highlighted by Fleming (1955), Hirschmann (1958), and, more recently, by Ciccone (2002) and Jones (2011 a,b). These authors point out theoretically that if the intermediate share in gross output is sizable, there exist large multiplier effects: small firm (or industry-level) productivity differences or distortions that lead to misallocation of resources across sectors or plants can add up to large aggregate effects. While our setup in principle allows for a mechanism whereby intermediate linkages amplify small sectoral productivity differences, we find little empirical evidence for this channel: cross-country TFP differences at the sector level are actually larger than aggregate TFP differences, and the tendency of low-income countries to have above-average productivity in sectors that are highly connected, in fact, helps to reduce aggregate productivity differences.

Our finding that sectoral productivity differences between rich and poor countries are large compared to aggregate ones is instead similar to the result of the literature on dual economies and sectoral productivity gaps in agriculture.[9] Also closely related to our work is a literature on structural transformation. It emphasizes sectoral productivity gaps and transitions from agriculture to manufacturing and services as a reason for cross-country income differences (see, e.g., Duarte and Restuccia, 2010 for a recent contribution). However, most of this literature abstracts from the role of linkages between industries.

In terms of modeling approach, our paper adopts the framework of the multi-sector real business

[8] See, e.g., Klenow and Rodriguez-Clare (1997), Hall and Jones (1999), Caselli (2005), Hsieh and Klenow (2010).

[9] See, e.g., Caselli (2005), Chanda and Dalgaard (2008), Restuccia, Yang, and Zhu (2008), Vollrath (2009), Gollin et al.(2014).

cycle model with IO linkages of Long and Plosser (1983); in addition, we model the input-output structure quite similarly to the setup of Acemoglu, Carvalho and Ozdaglar (2012).[10] In contrast to these studies, which deal with the relationship between sectoral productivity shocks and aggregate economic fluctuations, we are interested in the question how sectoral TFP *levels* interact with the IO structure to determine aggregate income *levels* and we provide corresponding empirical evidence.

Other recent related contributions are Oberfield (2018) and Carvalho and Voigtländer (2015), who develop an abstract theory of endogenous input-output network formation, and Boehm (2018), who focuses on the role of contract enforcement on aggregate productivity differences in a quantitative structural model with IO linkages. Differently from these papers, we do not try to model the IO structure as arising endogenously and we take sectoral productivity differences as exogenous. Instead, we aim at understanding how given differences in IO structure and sectoral productivities translate into aggregate income differences.

The number of empirical studies investigating cross-country differences in IO structure is quite limited. In the most comprehensive study up to that date, Chenery, Robinson, and Syrquin (1986) find that the intermediate input share of manufacturing increases with industrialization and that IO matrices become denser as countries industrialize. Most closely related to our paper is the contemporaneous work by Bartelme and Gorodnichenko (2015). They also collect data on IO tables for many countries and investigate the relationship between IO linkages and aggregate income.[11] In reduced-form regressions of *aggregate* IO multipliers on income per worker, they find a positive correlation between the two variables. Moreover, they investigate how distortions affect IO linkages and income levels. Differently from the present paper, they do not use data on sectoral productivities nor disaggregated IO tables. As a consequence, they do not investigate how differences in the interaction of sectoral multipliers and productivities impact on aggregate income, which is the focus of our work.

The outline of the paper is as follows. In the next section, we lay out our theoretical model and derive an expression for aggregate GDP per worker in terms of sectoral IO multiplers and TFP levels. In the following section, we describe our dataset and present some descriptive statistics. Subsequently, we turn to the empirical quantification of our model. We then present a number of robustness checks and the results of the counterfactuals. The final section presents our conclusions.

[10] Related to Acemoglu et al. (2012) empirical work by Barrot and Sauvagnat (2016) provides reduced-form evidence for the short-run propagation of exogenous firm-specific shocks in the production network of U.S. firms.

[11] Grobovsek (2018) performs a development accounting exercise in a more aggregate structural model with two final and two intermediate sectors.

2 Theoretical Framework

2.1 Model

In this section we present a simple model of an economy with intersectoral linkages (based on Long and Plosser, 1983 and Jones, 2011b) that will be used in the remainder of our analysis. Consider a static multi-sector economy. n competitive sectors each produce a distinct good that can be used either for final consumption or as an input for production in any of the other sectors. The technology of sector $i \in 1 : n$ is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(\frac{1}{1-\gamma_i} k_i^{\alpha} l_i^{1-\alpha} \right)^{1-\gamma_i} \left(\frac{d_{1i}}{\gamma_{1i}} \right)^{\gamma_{1i}} \left(\frac{d_{2i}}{\gamma_{2i}} \right)^{\gamma_{2i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}} \right)^{\gamma_{ni}} \tag{1}$$

where Λ_i is the exogenous total factor productivity of sector i, k_i and l_i are the quantities of capital and labor used by sector i and d_{ji} is the quantity of good j used in production of good i (intermediate good produced by sector j).[12] The exponent $\gamma_{ji} \in [0, 1)$ represents the output elasticity of good j in the production technology of firms in sector i, which also corresponds to the cost share of sector j's output, $p_j d_{ji}/p_i q_i$. $\gamma_i = \sum_{j=1}^{n} \gamma_{ji} \in (0, 1)$ is the total share of intermediate goods in gross output of sector i, and parameters α, $1-\alpha \in (0, 1)$ are the shares of capital and labor in the remainder of the inputs (value added). This specification allows for arbitrary asymmetries in linkages between sector pairs ij but fixes the output elasticities of labor and capital to be the same across sectors.

Given the Cobb-Douglas technology in (1) and competitive markets, the γ_{ji}s also correspond to the entries of the IO matrix, measuring the value of spending on input j per dollar of production of good i. We denote this IO matrix by $\mathbf{\Gamma}$. The entries of the j'th row of matrix $\mathbf{\Gamma}$ represent the values of spending on a given input j per dollar of production of each sector in the economy. By contrast, the elements of the i'th column of matrix $\mathbf{\Gamma}$ are the values of spending on inputs from each sector in the economy per dollar of production of a given good i.[13]

The output of sector i can be used either for final consumption, c_i, or as an input in sector j:

$$c_i + \sum_{j=1}^{n} d_{ij} = q_i, \qquad i = 1 : n \tag{2}$$

Consumers have Cobb-Douglas utility:

$$u(c_1, ..., c_n) = \prod_{i=1}^{n} \left(\frac{c_i}{\beta_i} \right)^{\beta_i}, \tag{3}$$

[12] In section 5 we consider the case of an open economy, where each sector's production technology employs both domestic and imported intermediate goods that are imperfectly substitutable.

[13] According to our notation, the sum of elements in the i'th column of matrix $\mathbf{\Gamma}$ is equal to γ_i, the total intermediate goods' share of sector i.

where $\beta_i \geq 0$ for all i and $\sum_{i=1}^{n} \beta_i = 1$. β_i corresponds to consumers' expenditure share on sector i. Consumers own all production factors and spend all their income I on consumption. Aggregate expenditure E of consumers can be written as $E = \sum_i p_i c_i = P \cdot u$, where u is a given utility level and P is the expenditure minimizing price index for this given utility (ideal price index). It is easy to show that $P = \prod_{i=1}^{n} (p_i)^{\beta_i}$.[14]

Finally, the total supply of capital and labor are exogenous and fixed at the levels of K and 1, respectively, implying that all aggregate variables can be interpreted in per-worker terms:

$$\sum_{i=1}^{n} k_i = K, \tag{4}$$

$$\sum_{i=1}^{n} l_i = 1. \tag{5}$$

To complete the description of the model, we provide a formal definition of a competitive equilibrium.

Definition A competitive equilibrium is a collection of quantities q_i, k_i, l_i, c_i, d_{ij}, Y and prices p_i, P, w, and r for $i \in 1:n$ such that

1. $\{c_i\}_{i \in 1:n}$ solve the utility maximization problem of a consumer subject to the budget constraint $\sum_i p_i c_i = I$, taking prices $\{p_i\}$ as given.

2. $\{d_{ij}\}$, k_i, l_i solve the profit maximization problem of the representative firm in each perfectly competitive sector i for $i \in 1:n$, taking $\{p_i\}$ of all goods and prices of labor and capital, w and r, as given (Λ_i is exogenous).

3. Aggregate expenditure equals income: $E = P \cdot u = w + rK$.

4. Markets clear:

   (a) capital market clearing: $\sum_{i=1}^{n} k_i = K$,

   (b) labor market clearing: $\sum_{i=1}^{n} l_i = 1$,

   (c) market clearing in sector i: $c_i + \sum_{j=1}^{n} d_{ij} = q_i$, for $i = 1, ..., n-1$.

5. Numeraire: $P = \prod_{i=1}^{n} (p_i)^{\beta_i} = 1$.

6. Definition of real GDP per worker: $Y = \sum_{i=1}^{n} p_i c_i = u$.

The choice of the aggregate consumer price index P as numeraire converts nominal consumption expenditure E into utility. Since consumption expenditure equals GDP per worker (total value

[14] Indeed, the solution of $\min_{c_i} \sum_i p_i c_i$ s.t. $\prod_{i=1}^{n} \left(\frac{c_i}{\beta_i}\right)^{\beta_i} = u$ is $c_i = \beta_i u \prod_{j \neq i} \left(\frac{p_j}{p_i}\right)^{\beta_j}$. Then $E = \sum_i p_i c_i = P \cdot u$.

added), we obtain that real GDP per worker Y is equal to utility: $Y = \sum_{i=1}^{n} p_i c_i = u$. We take it as our welfare measure.

2.2 Equilibrium

The system of optimality conditions for the utility and profit maximization problems together with the market clearing conditions can be solved analytically and allows deriving an explicit expression for welfare in terms of exogenous variables. The following proposition characterizes the equilibrium value of the logarithm of real GDP per worker.

Proposition 1. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of real GDP per worker, $y = \ln(Y)$, is given by*

$$y = \sum_{i=1}^{n} \mu_i \lambda_i + \alpha \ln K, \tag{6}$$

where

$$\boldsymbol{\mu} = \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}, \qquad n \times 1 \text{ vector of multipliers}$$

$$\boldsymbol{\lambda} = \{\lambda_i\}_i = \{\ln \Lambda_i\}_i, \qquad n \times 1 \text{ vector of sectoral log-productivity coefficients}$$

Proof. The proof of Proposition 1 is provided in the Appendix.

Due to the Cobb-Douglas structure of our economy, log real GDP per worker can be represented by an aggregate log-linear production function akin to the one used in standard development accounting (see, e.g., Caselli, 2005). It depends in a log-linear fashion on (i) aggregate TFP and (ii) the capital share in GDP α multiplied by the log capital stock per worker. In contrast to standard development accounting, aggregate log TFP is not a blackbox but instead depends on the underlying (exogenous) economic structure. It is given by a weigthed average of sectoral log TFPs λ_i with sectoral IO multipliers μ_i as weights. Thus, the impact of each sector's productivity on aggregate output is proportional to the value of the sectoral IO multiplier μ_i. This means that the positive effect of higher sectoral productivity on aggregate value added is stronger in sectors with larger multipliers.

The vector of sectoral multipliers, in turn, is determined by the features of the IO matrix through the Leontief inverse,[15] $[\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}$, and the vector of value-added shares $\boldsymbol{\beta}$. A typical element l_{ji} of the Leontief inverse can be interpreted as the percentage increase in the output of downstream

[15] Observe that in this model the Leontief inverse matrix is well-defined since CRS technology of each sector implies that $\gamma_i < 1$ for any $i \in 1 : n$. According to the Frobenius theory of non-negative matrices, this means that the maximal eigenvalue of $\boldsymbol{\Gamma}$ is bounded above by 1. This, in turn, implies the existence of $[\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}$.

sector i following a one-percent increase in productivity of upstream sector j.[16] Multiplying the Leontief inverse by the vector of value-added weights $\boldsymbol{\beta}$ adds up the effects of sector j on all the other sectors in the economy, weighting each using sector by its share in aggregate valued added β_i. Thus, a typical element of the resulting vector of IO multipliers reveals how a one-percent increase in productivity of sector j affects the overall value added in the economy.

The vector of sectoral multipliers can be written as

$$\boldsymbol{\mu} = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta} = \left(\sum_{k=0}^{+\infty} \boldsymbol{\Gamma}^k\right)\boldsymbol{\beta} = \boldsymbol{\beta} + \boldsymbol{\Gamma}\boldsymbol{\beta} + (\boldsymbol{\Gamma})^2\boldsymbol{\beta} + ... \tag{7}$$

where the jth element is the sector-j multiplier. Each sectoral multiplier is an infinite sum: the first term β_j is the direct impact of a shock to sector j on aggregate value added. Thus, ceteris paribus, sectors with higher value added shares have larger multipliers. The other terms of the infinite sum correspond to effects that travel through the IO network. In particular, the first-order term is the direct impact of the sector-j shock on the using sectors: $\sum_{i=1}^{n} \gamma_{ji}\beta_i$ is a weighted average of the $i = 1, .., n$ using sectors' cost shares γ_{ji} for sector j's output, with weights corresponding to the value-added shares of the using sectors. Thus, sectors whose output is more important as an input of all other sectors have larger sectoral multipliers. The higher-order terms correspond to the indirect effects of productivity shocks: e.g., if sector j supplies to k which in turn supplies to l, the second-order effect of raising productivity in sector j is the impact on l (and all other sectors indirectly linked to j): j's productivity shock increases the output of the downstream sector k and hence raises the output in sector l, which uses k's output as an input. The multiplication with β_l converts the increase in output of sector l into value added.

2.3 Conceptual Issues of Cross-country Welfare Comparisons

Suppose we have data on real GDP per worker Y_s, sectoral multipliers μ_s, sectoral log TFP levels λ_s, and aggregate capital stocks per worker K_s for $s = 1, ..., m$ countries. In order to match exactly the data, both the IO matrices $\boldsymbol{\Gamma}_s$ and the vectors of final expenditure shares $\boldsymbol{\beta}_s$ need to be country-specific. At the same time, expenditure-based real GDP per worker in country s, Y_s, conceptually corresponds to an empirical measure of utility of consumers in each country, $Y_s = u_s$. Indeed, as we showed in section 2.1, in each country s, the expenditure $E_s = P_s \cdot u_s$ and hence, $Y_s = E_s/P_s = u_s$ with $P_s = \prod_i (p_{is})^{\beta_{is}}$ and $u_s = \prod_i (\frac{c_{is}}{\beta_{is}})^{\beta_{is}}$. When consumers residing in different countries do not have the same utility function, welfare comparisons across countries become a tricky issue because cardinal utility comparisons across agents who do not share a common utility function are not

[16] In general, sectoral shocks also affect upstream production through a price and a quantity effect. For instance, with a negative shock to a sector, (i) its output price increases, raising its demand for inputs; and (ii) its production decreases, reducing its demand for inputs. With Cobb-Douglas production technologies, however, these two effects cancel out.

meaningful. In fact, in order to measure the utility a country-s consumer would get from residing in country k, we would need to deflate the expenditure of country k with the country-s consumer's optimal price index.[17] With m countries this procedure would give a different set of welfare levels (real GDPs) for each utility function (m different measures of real GDP for each country) whose ranking across countries is not necessarily the same.

Faced with this problem, we need to abandon the possibility of preference heterogeneity across countries and construct instead an artificial *reference* consumer as an average of the individual countries' consumers.[18] Of course, this leads to a discrepancy between the actual and constructed expenditure shares in each country and hence, will not allow fitting the data perfectly. However, we believe that this is an acceptable price to pay for making cross-country welfare comparisons possible, which is a key goal of this paper. Moreover, we show that the effect of mis-measurement is small empirically and that our results are not sensitive to the precise way of constructing the preferences of the reference consumer.

In defining this reference consumer, it seems reasonable to give consumers in each country the same weight. We thus use, alternatively, the arithmetic $\boldsymbol{\beta^*} = 1/n \sum_s \boldsymbol{\beta}_s$ and the geometric average $\boldsymbol{\beta^*} = \prod_s \boldsymbol{\beta}_s^{1/n}$ of the expenditure shares $\boldsymbol{\beta}_s$ across countries. This means that the expenditure share allocated to each given sector corresponds to the cross-country average of the expenditure shares for this sector. The so-defined $\boldsymbol{\beta^*}$ determines the preferences of the reference household and is used to construct multipliers $\boldsymbol{\mu^*} = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta^*}$.[19] Observe that the Penn World Table also uses implicitly the concept of a reference consumer when constructing PPP price indices of GDP with the Geary-Khamis methodology.[20] The Geary-Khamis approach uses each country's quantities as weights and thus gives more weight to consumers from larger economies. To match this approach, as a third alternative, we also use a quantity-weighted average of countries' expenditure shares to

[17]To give an example, suppose there are two countries, Italy and Germany. Italians care more about food than about cars $C_I = c^{1/3} f^{2/3}$, while for Germans it's the other way round $C_G = c^{2/3} f^{1/3}$. Assume that Germany produces 3 cars and 2 tons of food, and Italy 3 tons of food and 2 cars. Then the utility of Germans residing in Germany $C_{GG} = 3^{2/3} 2^{1/3}$, which equals the utility of Italians residing in Italy C_{II}. If we want to compare welfare across countries, we would need to evaluate Germans' utility if they resided in Italy, $U_{GI} = 2^{2/3} 3^{1/3} < U_{GG}$ (Germans don't care that much about food) and the utility Italians would derive from living in Germany $U_{IG} = 2^{2/3} 3^{1/3} < U_{II}$ (Italians don't care that much about cars).

[18]In the Italian-German example above, a reference consumer has the utility function that equals an average of the preferences of each country: $U_r = c^{1/2} f^{1/2}$. This reference consumer would be indifferent between living in Germany and living in Italy since $U_{rG} = 3^{1/2} 2^{1/2} = U_{rI} = 2^{1/2} 3^{1/2}$.

[19]To theoretically rationalize our approach of using average expenditure shares, we could assume that consumers in each country have a common utility function but that actual expenditure shares correspond to expected expenditure shares plus a random preference shock with mean zero. One could then use expected utility as a welfare measure. In this case (log-)utility is given by

$$\ln u = \sum_i (\beta_i^* + \varepsilon_i) \ln(c_i) - \sum_i \beta_i^* \ln(\beta_i^*), \tag{8}$$

where $E(\varepsilon_i) = 0$. The reference household maximizes expected utility $E(\ln u) = \sum_i \beta_i^* \ln(c_i) - \sum_i \beta_i^* \ln(\beta_i^*)$, where β_i^* is the expected expenditure share of sector i.

[20]See Feenstra et al. (2015) for a description of the the price indices used in the Penn World Table and Diewert (1999) for an in-depth discussion of the relationship between different methodologies for international price comparisons and the existence of a reference consumer.

compute the expenditure shares of the reference consumer. In this average, the weight of each country corresponds to its share in world's expenditure for sector i.

2.4 Decomposing Variation in Real GDP per Worker

For a reference household with preferences $u = \prod_{i=1}^{n}\left(\frac{c_i}{\beta_i^*}\right)^{\beta_i^*}$ log real income of a given country predicted by the model can be written as

$$y_{model} = \sum_{i=1}^{n} \lambda_i \mu_i + \alpha \ln(K) = n\bar{\mu}\bar{\lambda} + nCov(\lambda, \mu) + \alpha \ln(K), \tag{9}$$

where $\bar{\mu} = 1/n \sum_{i=1}^{n} \mu_i$ is the arithmetic average of sectoral multipliers, $\bar{\lambda} = 1/n \sum_{i=1}^{n} \lambda_i$ is the arithmetic average of sectoral log TFPs and $Cov(\lambda, \mu)$ is the covariance between sectoral log TFPs and multipliers within a given country.[21] Thus, according to the model, ceteris paribus countries have higher GDP per worker when average log TFP levels and average multipliers are larger. Importantly, income per worker is also higher when TFP is larger than average in high-multiplier sectors, so that log TFP levels and multipliers are positively correlated.

While the model can hopefully explain a large part of the variation in GDP per worker across countries, it will certainly not be able to fit the data perfectly. There are two main reasons for this. Most importantly, there is measurement error: all of the different objects that appear in (9), i.e. multipliers, sectoral TFP levels, capital stock and income per worker are likely to suffer from measurement error. Second, we impose some restrictions that are counterfactual: final expenditure shares $\boldsymbol{\beta}^*$ and the capital share in GDP α are imposed to be identical across countries. We therefore add a residual $\epsilon = y_{data} - y_{model}$.

$$y_{data} = n\bar{\mu}\bar{\lambda} + nCov(\lambda, \mu) + \alpha \ln(K) + \epsilon \tag{10}$$

The measurement error is probably more severe for low-income countries, leading to a negative correlation between y_{data} and ϵ.

Next we would like to decompose the variation of log GDP per worker into the various components of (10). Since the terms on the right-hand side are correlated, there exists no unique variance decomposition. A convenient way to decompose the variance of log GDP per worker is to use regressions. In particular,[22]

$$Var(y_{data}) = Cov(n\bar{\mu}\bar{\lambda}, y_{data}) + Cov[nCov(\lambda, \mu), y_{data}] + Cov[\alpha \ln(K), y_{data}] + Cov[\epsilon, y_{data}] \tag{11}$$

[21] The empirical covariance is computed as $Cov(\lambda, \mu) = \frac{1}{n-1} \sum_{i=1}^{n} (\mu_i - \bar{\mu})(\lambda_i - \bar{\lambda})$.

[22] We use $Var(X) = Cov(X, X)$ and $Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z)$.

Thus,

$$1 = \frac{Cov(n\bar{\mu}\bar{\lambda}, y_{data})}{Var(y_{data})} + \frac{Cov[nCov(\lambda, \mu), y_{data}]}{Var(y_{data})} + \frac{Cov[\alpha \ln(K), y_{data}]}{Var(y_{data})} + \frac{Cov[\epsilon, y_{data}]}{Var(y_{data})} \tag{12}$$

This decomposition is equivalent to looking at the coefficients obtained from independently regressing each term on the right-hand side of (10) on y_{data}. Since the terms on the right-hand side of (10) sum to y_{data} and OLS is a linear operator, the coefficients sum to one. So the decomposition amounts to asking, "When we see a one percent higher y_{data} in one country relative to the average of the countries in the sample, how much higher is our conditional expectation of $\alpha \ln K$, how much higher is our conditional expectation of $n\bar{\mu}\bar{\lambda}$, and how much does our conditional expectation of $nCov(\lambda, \mu)$ change?"

2.5 Measuring Sector-specific Productivity

By our assumption, production functions in a given sector vary across countries due to differences in the importance of sectoral linkages – the γ_{ji}s vary across countries for a given sector-pair ij. Computing a measure of productivity (TFP) that is comparable across countries when countries have different production functions in a given sector is methodologically challenging. A set of basic requirements for TFP comparisons across countries is the following: (i) the productivity measure should be unique when holding constant the reference country; (ii) it should be invariant to changes in units; (iii) it should be transitive, i.e., computing the productivity of country j relative to l should give the same number as the one obtained by first comparing j to k and then k to l. To provide an example for this problem, note that just taking ratios of outputs and inputs for a given pair of countries – like in the development-accounting literature (e.g. Caselli, 2005) – is not invariant to changes in units when the two countries have different output elasticities of inputs. Thus, productivity of any two countries in a given sector has to be compared while holding the production function constant. But this raises another problem: productivities can be computed with the production function of country k, the one of country l or the one of any other country. With m countries, this gives m productivity measures for a given country-sector pair, and thus, the so-obtained productivity is not unique. To address these problems, we borrow the approach from Caves, Christensen and Diewert (1982) who have devised a methodology that satisfies requirements (i)-(iii) for translog production functions. Since Cobb-Douglas is a special case of the translog function when the second-order terms are zero,[23] we can use their methodology and adapt it to our

[23]In general, the translog production function for an economic entity (country or sector) s that produces the vector of outputs $\{q_k^s\}_{k=1}^K$ using the vector of inputs $\{X_i^s\}_{i=1}^n$ can be written as

$$\alpha_0^s + \sum_{j=1}^{J} \alpha_j^s \ln q_j^s + \sum_{i=1}^{n} \beta_i^s \ln X_i^s + \mathit{2nd\ order\ terms} = 1.$$

special case, so as to derive our measure of sector-specific productivity.

Without loss of generality, consider for simplicity the Cobb-Douglas technology with a composite input X_{is}, $q_{is} = \Lambda_{is}\left(\frac{X_{is}}{\alpha_{Xis}}\right)^{\alpha_{Xis}}$, where i denotes a sector and s denotes a country. Let us define the productivity of country k relative to l in sector i using country l's production function as a base as follows: $\lambda_{ik} = \Lambda_{ikl}/\Lambda_{ill}$, where Λ_{ikl} is defined by $q_{ik} = \Lambda_{ikl}(X_{ik}/\alpha_{Xil})^{\alpha_{Xil}}$, and $\Lambda_{ill} = \Lambda_{il}$. Essentially, Λ_{ikl} is a TFP parameter that makes the sector i's output of country k producible with own input levels of country k and the production function of l. Similarly, we can define $\lambda_{il} = \Lambda_{ikk}/\Lambda_{ilk}$, the productivity of country k relative to l in sector i using country k's production function as a base. Then $\ln\lambda_{ik} \equiv (\ln q_{ik} - \ln q_{il}) - \alpha_{Xil}(\ln X_{ik} - \ln X_{il})$ and $\ln\lambda_{il} = (\ln q_{ik} - \ln q_{il}) - \alpha_{Xik}(\ln X_{ik} - \ln X_{il})$. In this way, we can construct m pairs of different productivity indices (λ_{ik}, λ_{is}) for each sector-country pair ik (using country k and country s as a base, $s \in 1:m$). Next, for each of these pairs we define λ_{iks} as the geometric mean of λ_{ik} and λ_{is}. This is then the *bilateral base-country invariant* definition of the productivity of k relative to s in sector i:

$$\ln\lambda_{iks} = (\ln\lambda_{ik} + \ln\lambda_{is})/2$$

Plugging in the defined $\ln\lambda_{ik}$ and $\ln\lambda_{is}$, we obtain:

$$\ln\lambda_{iks} = (\ln q_{ik} - \ln q_{is}) - \frac{1}{2}(\alpha_{Xis} + \alpha_{Xik})(\ln X_{ik} - \ln X_{is}) \tag{13}$$

However, the so-defined λ_{iks} is not transitive, i.e. $\ln\lambda_{iks} \neq \ln\lambda_{ikl} - \ln\lambda_{isl}$. Therefore, we next define $\overline{\ln\lambda}_{ik}$ – the productivity of country k in sector i relative to an average of all other countries $s = 1, ..., m$ – as the geometric average of λ_{iks}. It corresponds to the geometric average of all bilateral base-country invariant productivity comparisons for a given country k.

$$\overline{\ln\lambda}_{ik} = \frac{1}{m}\sum_{s=1}^{m}\ln\lambda_{iks} \tag{14}$$

Finally, we define the *multilateral productivity index* as:

$$\ln\lambda^*_{ikl} \equiv \overline{\ln\lambda}_{ik} - \overline{\ln\lambda}_{il} = \frac{1}{m}\sum_{s=1}^{m}\ln\lambda_{iks} - \frac{1}{m}\sum_{s=1}^{m}\ln\lambda_{ils} \tag{15}$$

This multilateral productivity index corresponds to log TFP of country k relative to country l in sector i, where both countries' productivity is measured as the geometric average of all bilateral productivity comparisons.

Plugging (13) into this definition gives our measure of the multilateral sector-specific Cobb-

Douglas productivity index:[24]

$$\ln \lambda^*_{ikl} = \ln q_{ik} - \ln q_{il} - \frac{1}{2}(\alpha_{Xik} + \overline{\alpha}_{Xi})(\ln X_{ik} - \overline{\ln X}_i) + \frac{1}{2}(\alpha_{Xil} + \overline{\alpha}_{Xi})(\ln X_{il} - \overline{\ln X}_i), \qquad (16)$$

where $\overline{\alpha}_{Xi} = \frac{1}{m}\sum_{s=1}^{m} \alpha_{Xis}$, and $\overline{\ln X}_i = \frac{1}{m}\sum_{s=1}^{m} \ln X_{is}$.

Generalizing the production function to many inputs, and assuming (i) constant returns to scale and (ii) perfect competition without distortions, we note that the output elasticities α_{Xis} correspond to the cost shares $\{\gamma_{ji}\}_j$, α and $1-\alpha$ of individual inputs. These can be directly taken from the data: IO coefficients and sectoral factor shares in gross output. In our empirical application, we will take the U.S. as the base country ($l = U.S.$). Thus, the resulting set of productivity indices $\{\ln \lambda^*_{isl}\}$ will represent log TFP of each country $s \in 1:m$ relative to the U.S. in each sector $i \in 1:n$.

3 Dataset and Descriptive Analysis

3.1 Data Sources and Description

IO tables measure the flow of intermediate products between different plants, both within and between sectors. The ji'th entry of the IO table is the value of output from establishments in industry j that is purchased by different establishments in industry i for use in production.[25] Dividing the flow of industry j to industry i in the IO table by gross output of industry i, one obtains the IO coefficient γ_{ji}, which states the cents of industry j's output used in the production of each dollar of industry i's output.

In order to construct a dataset of IO tables for a range of low- and high-income countries, to compute sectoral TFP levels, and to obtain information on countries' GDP per worker and factor endowments, we combine information from two datasets: the World Input-Output Database (WIOD), February 2012 release (Timmer, 2012), and the Penn World Table (PWT), Version 8.0 (Feenstra et al., 2015).

The first dataset, WIOD, contains IO data and sectoral socio-economic accounts for 38 countries classified into 35 sectors. We use WIOD data for year 2005 because for this year we have PPP price indices. The list of countries and sectors is provided in Appendix Tables A-1 and A-2.[26]

WIOD IO tables are available in current national currency at basic prices.[27] In our main specification, we compute IO coefficients as the value of domestically produced plus imported intermediates divided by the value of gross output at basic prices.[28] Sectoral multipliers are computed

[24] See Appendix for details.

[25] Note that intermediate outputs must usually be traded between establishments in order to be recorded in the IO tables. Therefore, flows that occur within a given plant are not measured.

[26] We drop Indonesia from the sample because the data reported by WIOD for this country are problematic.

[27] Basic prices exclude taxes and transport margins.

[28] In a robustness check, we separate domestically produced from imported intermediates and define domestic

as $\boldsymbol{\mu} = \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}^*$. The WIOD data also contain all the necessary information to compute gross-output-based sectoral total factor productivity for 35 sectors: nominal gross output and material use, sectoral capital stocks and labor inputs, sectoral factor payments to labor, capital and intermediates disaggregated into 35 inputs. Crucially, WIOD also provides purchasing power parity (PPP) deflators (in purchasers' prices) for sector-level gross output for the year 2005 that we use to convert nominal values of outputs and inputs into real units that are comparable internationally. This allows us to compute TFP levels at the sector level using the methodology explained above.[29] The PPP deflators have been constructed by Inklaar and Timmer (2012) and are consistent in methodology and outcome with the PWT 8.0. They combine expenditure prices and levels collected as part of the International Comparison Program (ICP) with data on industry output, exports and imports and relative prices of exports and imports from Feenstra and Romalis (2014). The authors use export and import values and prices to correct for the problem that the prices of goods consumed or invested domestically do not take into account the prices of exported products, while the prices of imported goods are included. To our knowledge, WIOD combined with these PPP deflators is the best available cross-country dataset for computing sector-level productivities using production data.

The second dataset, PWT, includes data on real GDP in PPP, the number of workers, as well as information on aggregate PPP price indices for exportables and importables for the same set of countries as WIOD in the year 2005. Our main measure of real GDP is RGDPE, real GDP in PPP prices computed from the expenditure side. This measure is most appropriate to compute welfare-relevant real GDP because it measures differences in the standard of living across countries (Feenstra, et al., 2015). Alternatively, we have used RGDPO, real GDP in PPP prices computed from the production side. This variable measures the production capacity of each country. For our sample, the difference between these measures is negligible and our results are basically identical with both measures. To construct aggregate physical capital stocks and employment of each country, we add up the sectoral capital stocks and employment numbers from WIOD. Results are very similar if information on the number of workers and capital stocks is instead taken directly from the PWT. We prefer aggregating information from WIOD since this guarantees that the sectoral values are consistent with the aggregate values. Finally, we use aggregate price indices for exports and imports in the open-economy extension of our model, which we discuss in a robustness check.

IO coefficients as the value of domestically produced intermediates divided by the value of gross output, while IO coefficients for imported intermediates are defined as the value of imported intermediates divided by the value of gross output. We show in the robustness section that this choice does not affect our results .

[29]The WIOD data comprise socio-economic accounts that are defined consistently with the IO tables. We use sector-level data on gross output and physical capital stocks in constant 1995 prices, the price series for investment, and labor inputs (employment). Using the sector-level PPPs for gross output, we convert nominal gross output and inputs into constant 2005 PPP prices. Furthermore, using price series for investment from WIOD and the PPP price index for investment from PWT, we convert sector-level capital stocks from WIOD into constant 2005 PPP prices.

3.2 Descriptives of IO Structure

We now provide some descriptive statistics of IO structure, as summarized by the distributions of sectoral multipliers. We report these statistics by income level, classifying countries with a per-capita GDP of less than 5000 PPP Dollars as low-income, those with 5,000-2,0000 PPP Dollars as medium-income, and those with more than 20,000 PPP Dollars as high-income. Figure 1 reports kernel density plots of the distribution of multipliers pooled across countries and sectors. For all income levels, the distributions are skewed with a long right tail: while most sectors have low multipliers, there are a few high-multiplier sectors. In addition, low-income countries' distribution has more mass in the right tail.[30] Table 1 reports moments of the distribution of multipliers. The mean sectoral multiplier is 0.057, the median multiplier is 0.049, and the 95th-percentile of multipliers is 0.133. In Appendix Figure A-1, we plot average multipliers by sector.[31]

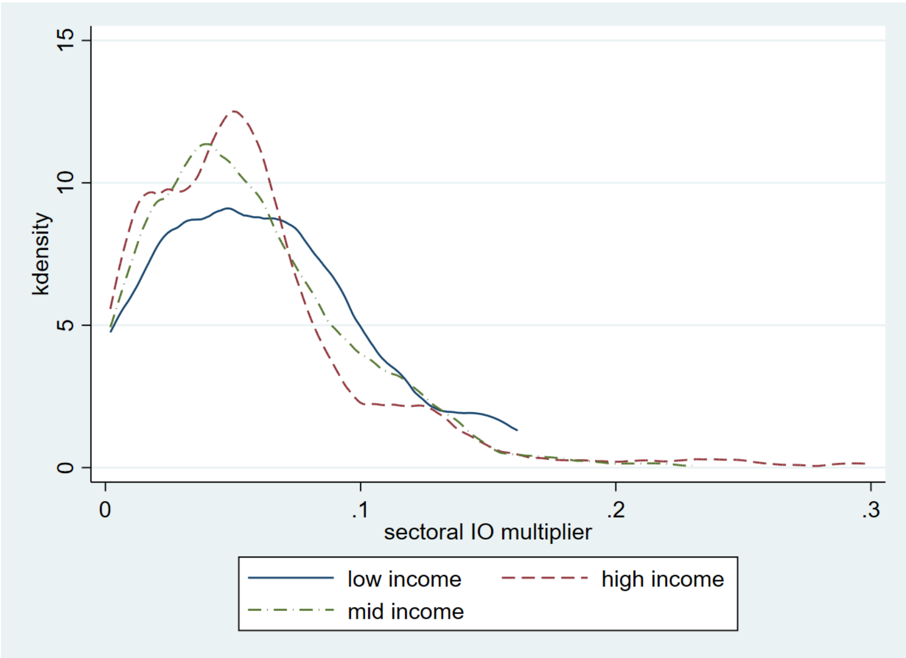

Figure 1: Distribution of sectoral IO multipliers by income level.

Sample	Mean	Std.	5th Pct.	10th Pct.	Median	90th Pct.	95th Pct.
all	0.057	0.042	0.003	0.011	0.049	0.112	0.133
low income	0.061	0.400	0.006	0.011	0.057	0.115	0.143
med income	0.057	0.039	0.004	0.011	0.049	0.110	0.130
high income	0.056	0.045	0.003	0.011	0.049	0.116	0.136

Table 1: Summary statistics of sectoral IO multipliers.

[30] In the working paper version, we also report descriptive statistics for GTAP data, which comprises a larger sample and includes many more low-income countries. These features of the multipliers' distribution also hold in the larger GTAP sample and are even more pronounced.

[31] The high-multiplier sectors in all countries are mostly service sectors such as Business Services, Real Estate, Financial Services, Wholesale Trades that provide inputs to most other sectors of the economy.

3.3 Descriptives of TFP

Next, we report descriptive statistics of sectoral TFP levels. Figure 2 provides kernel density plots of sectoral log TFP relative to the U.S. by income level. The distribution of log TFP is approximately normal. Moreover, low-income countries have a distribution of log TFPs with a significantly lower mean and a larger variation across sectors than high-income countries. Table 2 reports means and standard deviation of log TFP relative to the U.S., as well as the within-country correlation between log TFPs and multipliers. While in low-income countries mean TFP is around 60 percent of the U.S. level (0.6=exp(-0.517)), with a large standard deviation across sectors, mean sectoral TFP in high-income countries is around 90 percent of the U.S. level (0.9=exp(-0.104)) with much less dispersion across sectors. Interestingly, in low-income countries, log TFP levels of high-multiplier sectors are above their average TFP level relative to the U.S. (the correlation between log TFPs and multipliers is positive), while in rich countries log TFP levels are below average in high-multiplier sectors (the correlation between log TFPs and multipliers is negative).

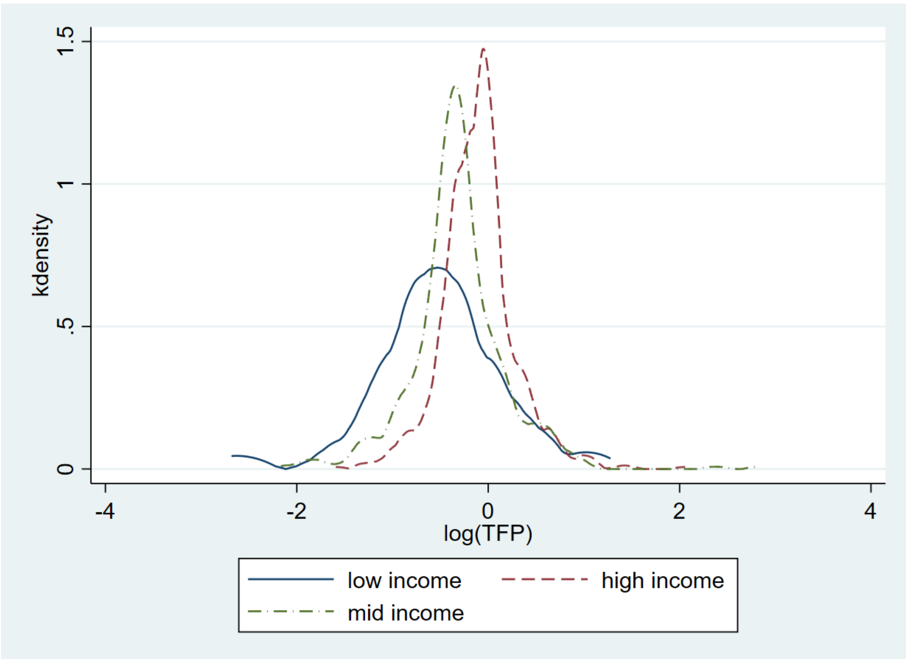

Figure 2: Distribution of log TFP by income level

Sample	Obs.	Mean log TFP	Std. log TFP (within)	Corr. log TFP, mult. (within)
all	1,295	-0.206	0.413	-0.060**
low income	70	-0.517	0.676	0.363***
mid income	490	-0.316	0.475	-0.015
high income	735	-0.104	0.347	-0.224***

Table 2: Summary statistics of sectoral log TFPs. *** (**) indicates statistical significance at the 1-percent (5-percent) level.

4 Empirical Analysis

We now decompose the variation of log real GDP per worker into its different components. In the calibration we set the capital share in GDP $\alpha = 1/3$, as standard in the development accounting literature (see Caselli, 2005). Moreover, we set n, the number of sectors, equal to 35 because this corresponds to the number of sectors in WIOD. We first present plots of each of the components on the right-hand side of equation (9) against log real GDP per worker (relative to the U.S.). Figure 3 plots $\alpha \ln K$ against log real GDP per worker relative to the U.S., while Figure 4 plots $\bar{\lambda}$, mean log TFP relative to the U.S. of each country, against log real GDP per worker. Not surprisingly, both capital stock per worker and average log TFP are strongly positively correlated with log GDP per worker. Figure 5 presents a similar plot for average multipliers $\bar{\mu}$. Average multipliers tend to be somewhat larger in poor countries, but the relationship between average multipliers and income per worker is not very strong. There are also some low-income countries with low average multipiers, such as Brazil (BRA) and India (IND). Figure 6 plots the within-country covariance between log TFP and multipliers $Cov(\lambda, \mu)$ against log real income per worker: this relationship is strongly negative. While low-income countries, such as China (CHN) and India, tend to have higher than average TFP levels in high-multiplier sectors, in rich countries, sectors with high multipliers tend to have below-average TFP levels.

Table 3 reports the result of decomposing the variance of log GDP per worker using (11). The first row reports results for for the case when $\boldsymbol{\beta^*}$ is defined by an arithmetic average of countries' expenditure shares, while the second row reports the results for the geometric average and the third one for a weighted average where the weights correspond to each country's produced quantities. The model with the arithmetic-average expenditure shares explains a remarkable 90% of the variance of log GDP per worker in the data. The 90% of variance explained by the model can be split into 49% due to variation in capital per worker, 49% due to variation in the product of average log TFP and average multiplier and minus 8% due to variation in the covariance term between log TFP and multipliers. The magnitude of the negative covariance term implies that if poor countries did not have above average productivity levels and rich countries did not have below average productivity levels in high-multiplier sectors, the actual variation in GDP per worker in this sample would be 8% larger than it actually is. Note that poor countries have a very large variation in relative TFP levels across sectors. The very low TFP levels of these countries in some of their sectors are however mitigated by the fact that these sectors have low multipliers, i.e. they are not very connected to the rest of the economy. At the same time, those sectors that are particularly important for other sectors (high-multiplier sectors) have above-average productivity levels. By contrast, in most rich countries (Western Europe and Japan) TFP levels relative to the U.S. are lower than average in high-multiplier sectors, which significantly reduces their real GDP per worker. Finally, the remaining 10% residual variation is due to measurement error, which is negatively correlated with income per worker.

Note that compared to the 50-50 split of income variation into production factors and productivity found in the development accounting literature, the role of productivity is reduced, since the negative covariance term is part of aggregate TFP. We find that relative to the total variation explained by the model, cross-country income differences can be split roughly into 45% due to aggregate TFP differences and 55% due to differences in production factors per worker.[32]

The model with $\boldsymbol{\beta}^*$ computed as geometric average gives similar results and further reduces the role of aggregate productivity. In particular, the covariance term continues to be minus 8%, while the variation explained by average TFP and multipliers is reduced to 42%. Finally, when $\boldsymbol{\beta}^*$ is computed as a quantity-weighted average, the covariance between TFP and multipliers is a bit less important compared to the other cases (it reduces income variation by 6%). Thus, overall, the results are robust to the specific way of computing expenditure weights of the reference consumer.

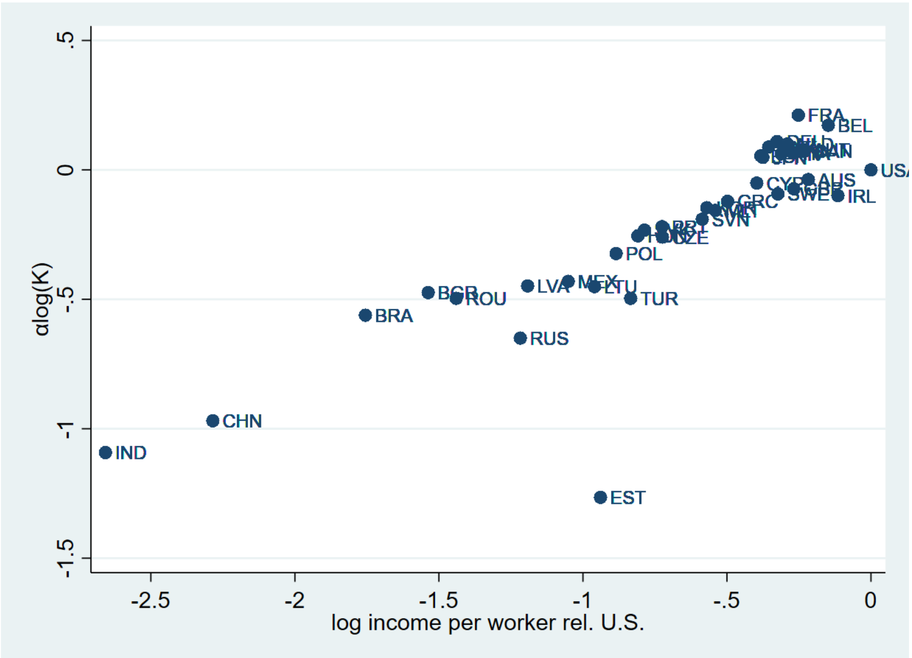

Figure 3: $\alpha \ln(K)$ vs. log income per worker rel. U.S.

Table 3: Variance decomposition of log real GDP per worker – baseline model

	fraction of variance explained by				
	model	$\alpha \ln K$	$n * \bar{\lambda}\bar{\mu}$	$n * cov(\lambda, \mu)$	residual
arithmetic mean	0.90	0.49	0.49	-0.08	0.10
geometric mean	0.83	0.49	0.42	-0.08	0.17
weighted mean	0.92	0.49	0.49	-0.06	0.08

[32] $45\% \approx (49 - 8) * 100/90$.

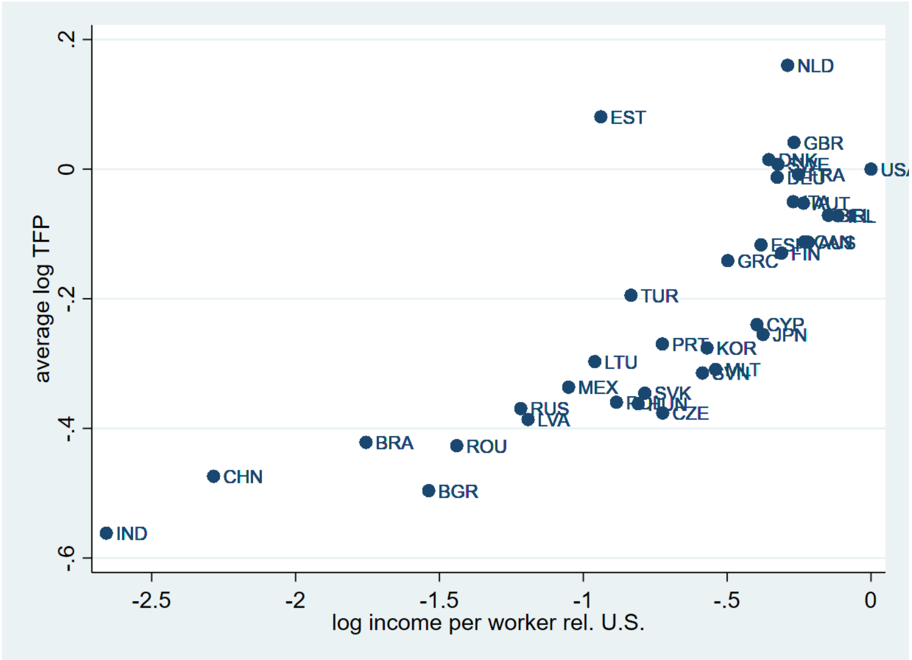

Figure 4: $\bar{\lambda}$ vs. log income per worker rel. U.S.

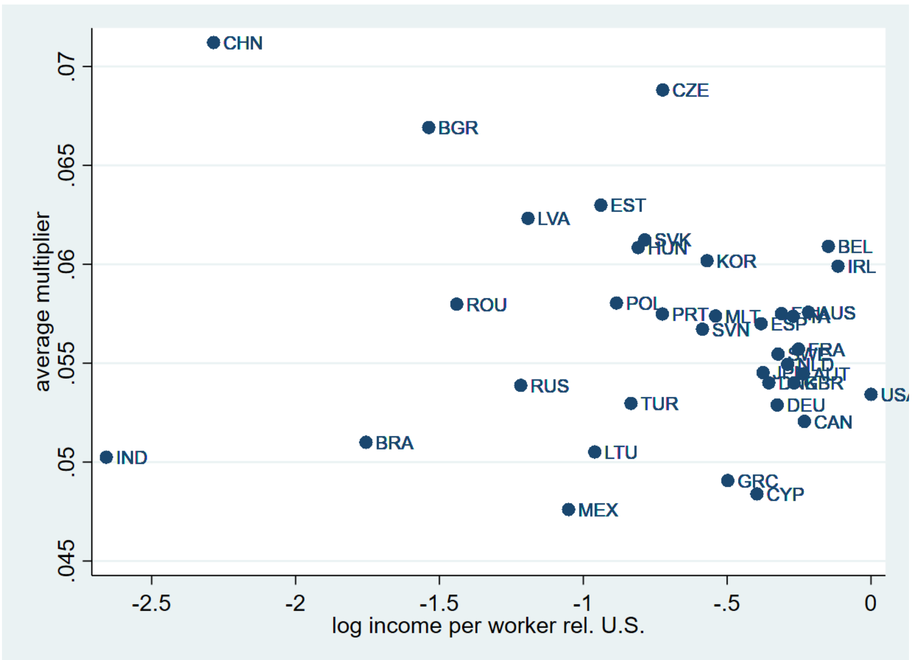

Figure 5: $\bar{\mu}$ vs. log income per worker rel. U.S.

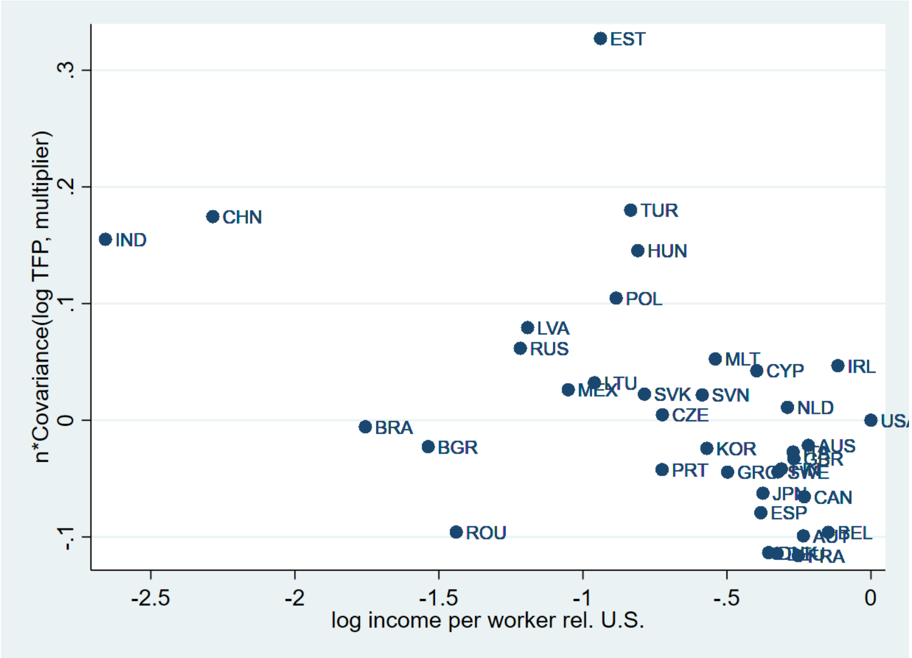

Figure 6: $n * Cov(\lambda, \mu)$ vs. log income per worker rel. U.S.

5 Robustness Checks

In this section, we report the results of a number of robustness checks in order to show that our results do not hinge on the specific assumptions adopted in the model. We consider the following modifications of our benchmark setup. First, we allow IO multipliers to depend on implicit tax wedges or distortions. Second, we account for imported intermediate inputs. Third, we extend our model to sectoral CES production functions. Finally, we allow for skilled and unskilled labor as separate production factors. We show that none of these modifications changes the basic conclusions of the baseline model.

5.1 Wedges

One important concern is that the empirically observed IO coefficients do not just reflect technological input requirements but also sector-specific distortions or wedges τ_i in the production of intermediates. To see this, consider the following maximization problem of an intermediate producer:

$$\max_{\{d_{ji},k_i,l_i\}} (1-\tau_i)p_i\Lambda_i \left(\frac{1}{1-\gamma_i}k_i^{\alpha}l_i^{1-\alpha}\right)^{1-\gamma_i} \left(\frac{d_{1i}}{\gamma_{1i}}\right)^{\gamma_{1i}} \left(\frac{d_{2i}}{\gamma_{2i}}\right)^{\gamma_{2i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}}\right)^{\gamma_{ni}} - \sum_{j=1}^{n} p_j d_{ji} - rk_i - wl_i,$$

where $\{p_i\}$ is taken as given (τ_i and Λ_i are exogenous). Sector-specific wedges are assumed to reduce the value of sector i's production by a factor $(1-\tau_i)$, so that $\tau_i > 0$ implies an implicit tax and $\tau_i < 0$ corresponds to an implicit subsidy on the production of sector i. The first-order condition w.r.t. d_{ji} is given by

$$(1-\tau_i)\gamma_{ji} = \frac{p_j d_{ji}}{p_i q_i}, \quad j \in 1:n$$

Thus, a larger wedge in sector i implies lower observed IO coefficients in this sector since firms in sectors facing larger implicit taxes demand less inputs from all other sectors. Separately identifying wedges τ_i and technological IO coefficients γ_{ji} is an empirical challenge, which requires to impose additional restrictions on the data. Observe that τ_i is the same for all inputs j demanded by a given sector i. Thus, introducing a country index s and summing across inputs j for a given country, we obtain

$$(1-\tau_{is})\sum_j \gamma_{jis} \equiv (1-\tau_{is})\gamma_{is} = \sum_j \frac{p_{js} d_{jis}}{p_{is} q_{is}}, \quad i \in 1:n$$

Now, if we restrict the total technological intermediate share of a given sector i, γ_{is}, to be the same across countries, we can identify country-sector-specific wedges as

$$(1-\tau_{is}) = \sum_j \frac{p_{js} d_{jis}}{p_{is} q_{is}} \frac{1}{\gamma_i}, \quad i \in 1:n \tag{17}$$

Observe that individual IO coefficients γ_{jis} are still allowed to differ across countries in an arbitrary way. According to equation (17), countries with below-average intermediate shares $\sum_j \frac{p_{js} d_{jis}}{p_{is} q_{is}}$ in a certain sector face an implicit tax in this sector, while countries with above-average intermediate shares receive an implicit subsidy. Taking logs of equation (17), we obtain:

$$\ln\left(\sum_j \frac{p_{js} d_{jis}}{p_{is} q_{is}}\right) = \ln(\gamma_i) + \ln(1-\tau_{is}) \tag{18}$$

Given (18), we regress log intermediate input shares of each country-sector pair on a set of sector-specific dummies to obtain estimates of the technological intermediate shares $\ln(\gamma_i)$ and then back out $\ln(1-\tau_{is})$ as the residual. Average intermediate shares γ_i are slightly lower for low-income countries. Low-income countries also have a larger fraction of sectors with very low intermediate shares and a lower fraction with high intermediate shares. Consequently, they have a larger fraction of sectors with relatively high wedges, which corresponds to more mass in the left tail of the distribution of $\ln(1-\tau_{is})$. This is clear from Figure 7, which plots the distribution of $\ln(1-\tau_{is})$ by income level for the WIOD sample. Given wedges τ_{is}, we construct IO coefficients adjusted for wedges as $\gamma_{jis} = \frac{p_{js} d_{jis}}{p_{is} q_{is}} \frac{1}{(1-\tau_{is})}$. We then recompute sectoral productivities and IO multipliers using these adjusted coefficients.

In the presence of wedges the expression for log GDP per worker also needs to be modified since wedges distort decisions and thus reduce income per worker. In particular, there is now an additional term $\sum_{i=1}^n \mu_i \ln(1-\tau_i)$. Higher distortions (lower values of $\ln(1-\tau_i)$) reduce income per

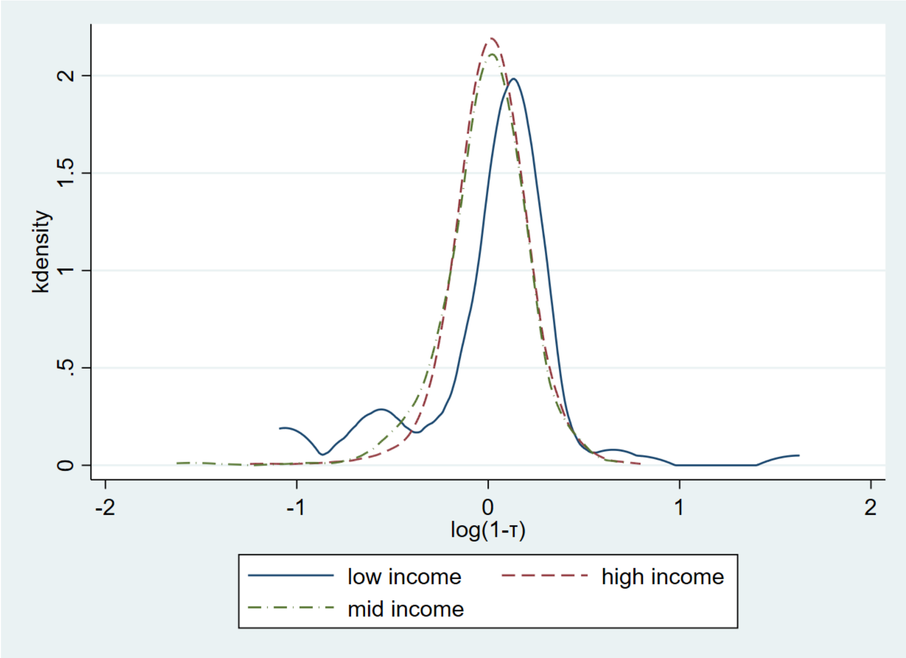

Figure 7: Distribution of log(1-wedges) by income level

worker by more if they occur in high-multiplier sectors.

Proposition 2. *In the unique competitive equilibrium the logarithm of real GDP per worker, y, is given by*

$$y = \sum_{i=1}^{n} \mu_i \lambda_i + \sum_{i=1}^{n} \mu_i \ln(1-\tau_i) + \alpha \ln K, \tag{19}$$

where

$$\begin{aligned} \boldsymbol{\mu} &= \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}^*, && n \times 1 \textit{ vector of multipliers} \\ \boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\ln \Lambda_i\}_i, && n \times 1 \textit{ vector of sectoral log-productivity coefficients} \\ \boldsymbol{\tau} &= \{\tau_i\}_i, && n \times 1 \textit{ vector of sectoral wedges} \end{aligned}$$

This expression can be further decomposed as:

$$y_{model} = n\bar{\mu}\bar{\lambda} + nCov(\lambda, \mu) + n\bar{\mu} \ln \overline{(1-\tau)} + nCov(\ln(1-\tau), \mu) + \alpha \ln(K) \tag{20}$$

Figure 8 plots the covariance of $\ln(1-\tau)$ and multipliers μ against log GDP per worker: while rich countries tend to have lower implicit taxes or even provide implicit subsidies to their high-multiplier sectors, low-income countries tend to have high implicit taxes in these sectors.

Finally, in Table 4 we provide a variance decomposition similar to (11), with two additional terms, that account for the role of wedges. We first discuss results for the model where the reference consumer's expenditure shares are given by the arithmetic average of countries' expenditure shares.

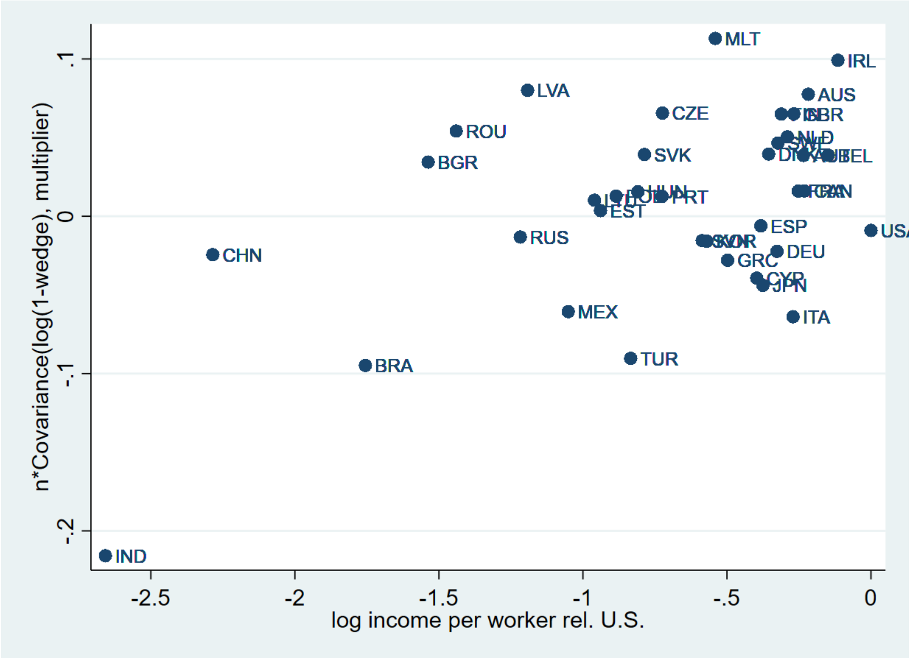

Figure 8: $n * Cov(ln(1-\tau), \mu)$ vs. log income per worker rel. U.S.

The fraction of the total variance of log GDP per worker explained by this model is very similar to the one of the baseline model (91% compared to 90%). The fraction of the variance explained by variation in the covariance between TFP and multipliers is also very similar (minus 9% vs. minus 8%). Like in the baseline model, the negative contribution of this term is a consequence of TFP and multipliers being positively correlated in poor countries and negatively in rich ones. However, while the baseline model attributes 49% of the variance to differences in the product of average log TFP and average multiplier, the model with wedges attributes only 44% to this term. In addition, the positive correlation between wedges and multipliers in poor countries increases the variance of log GDP per worker by 5%. Finally, the fact that average wedges are also higher in poor countries increases income differences by another 2%. Thus, if we attribute the fraction of income variance due to wedges as being part of variation due to aggregate TFP, we conclude that the variance of model-based income is split roughly according to 46% due to aggregate TFP differences and 54% due to production factors,[33] which is close to the analogous finding in our benchmark setting. The results are quantitatively very similar for the models where instead of the arithmetic average of countries' expenditure shares, we use the geometric average or the quantity-weighted average.

Table 4: Variance decomposition of log GDP per worker – model with wedges

		fraction of variance explained by					
	model	$\alpha \ln K$	$n\bar{\lambda}\bar{\mu}$	$nCov(\lambda, \mu)$	$n\overline{\ln(1-\tau)}\bar{\mu}$	$nCov(\ln(1-\tau), \mu)$	residual
arith. mean	0.91	0.49	0.44	-0.09	0.02	0.05	0.09
geo. mean	0.84	0.49	0.37	-0.09	0.02	0.05	0.16
w. mean	0.94	0.49	0.44	-0.08	0.02	0.05	0.06

[33] $46\% \approx (44 - 9 + 2 + 5) * 100/91$.

5.2 CES Production Function

Another potential concern is that sectoral production functions are not Cobb-Douglas, but instead feature an elasticity of substitution between intermediate inputs different from unity. If this were the case, IO coefficients would no longer be sector-country-specific constants γ_{jis} but would instead be endogenous to equilibrium prices, which would reflect the underlying productivities of the upstream sectors. While it has been observed that for the U.S. the IO matrix has been remarkably stable over the last decades despite large shifts in relative prices (Acemoglu et al., 2012) – an indication of a unit elasticity, – in this robustness check we briefly discuss the implications of considering a more general CES sectoral production function. The sectoral production functions are now given by:

$$q_i = \Lambda_i \left(\frac{1}{1-\gamma_i} k_i^{\alpha} l_i^{1-\alpha} \right)^{1-\gamma_i} M_i^{\gamma_i}, \tag{21}$$

where $M_i \equiv \left(\sum_{j=1}^{N} \gamma_{ji} d_{ji}^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}}$. The rest of the model is specified as in section 2.1.

With CES production functions the equilibrium cannot be solved analytically, so one has to rely on numerical solutions. However, it is straightforward to show how IO multipliers are related to sectoral productivities in this case. From the first-order conditions it follows that the relative expenditure of sector i on inputs produced by sector j relative to sector k is given by:

$$\frac{p_j d_{ji}}{p_k d_{ki}} = \left(\frac{p_j}{p_k} \right)^{1-\sigma} \left(\frac{\gamma_{ji}}{\gamma_{ki}} \right) \tag{22}$$

Thus, if $\sigma > 1$ ($\sigma < 1$), each sector i spends relatively more on the inputs provided by sectors that charge lower (higher) prices. Recall that sectors whose output accounts for a larger fraction of other sectors' spending have higher multipliers (see equation (7)). Moreover, since prices are inversely proportional to productivities, sectors with higher productivity levels charge lower prices. Consequently, when $\sigma > 1$, sectoral multipliers and productivities should be positively correlated in *all* countries, while when $\sigma < 1$, the opposite should be true. We confirm these results in unreported simulations. However, these predictions are not consistent with our empirical finding that multipliers and productivities are positively correlated in low-income countries, while they are negatively correlated in high-income ones. Consequently – unless the elasticity of substitution differs systematically across countries – the data on IO tables and sectoral productivities are difficult to reconcile with CES production functions.

5.3 Traded Intermediate Goods

So far, we have treated all intermediate inputs as being domestically produced. Here, we extend our model and differentiate between domestically produced and imported intermediate inputs, while keeping the Cobb-Douglas structure of sectoral production functions. The technology of sector i is now given by

$$q_i = \Lambda_i \left(\frac{1}{1-\gamma_i-\sigma_i} k_i^{\alpha} l_i^{1-\alpha} \right)^{1-\gamma_i-\sigma_i} \left(\frac{d_{1i}}{\gamma_{1i}} \right)^{\gamma_{1i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}} \right)^{\gamma_{ni}} \cdot \left(\frac{f_{1i}}{\sigma_{1i}} \right)^{\sigma_{1i}} \cdot \ldots \cdot \left(\frac{f_{ni}}{\sigma_{ni}} \right)^{\sigma_{ni}},$$

where d_{ji} is the quantity of the domestic good j used in the production of sector i and f_{ji} is the quantity of imported good j used by sector i. $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ and $\sigma_i = \sum_{j=1}^{n} \sigma_{ji}$ are the respective shares of domestic and imported intermediate goods in the total input use of sector i and α is the share of capital in sectoral value added. We assume that output of sector i can be used either for final consumption, c_i, as a domestic intermediate input d_{ij}, or as an exportable x_i.

$$q_i = c_i + \sum_{j=1}^{n} d_{ij} + x_i \qquad i = 1:n$$

We impose balanced trade, so that the value of exported intermediates must be equal to the value of imported intermediates.

$$\sum_{j=1}^{n} p_j x_j = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{p}_j f_{ji},$$

where p_j is the domestic and export price of intermediate good j and $\overline{p}_j$ is the import price of intermediate good j. Because the domestic economy is assumed to be small, these prices are exogenous. Let us denote by $\rho_j = \frac{\overline{p}_j}{P}$ the ratio of the import price of intermediate good j relative to the aggregate consumer price index.[34] Because we only have data on the aggregate import price index from the Penn World Table, we assume that import prices do not vary across sectors: $\rho_j = \rho$. In the Appendix, we show that with these modifications the aggregate production function for log GDP per worker can be expressed as follows:

Proposition 3. *In the unique competitive equilibrium, the logarithm of real GDP per worker, y, is*

$$y = \frac{1}{\sum_{i=1}^{n} \mu_i (1-\sigma_i-\gamma_i)} \left(\sum_{i=1}^{n} \mu_i \lambda_i - \ln \rho \sum_{i=1}^{n} \mu_i \sigma_i \right) + \alpha \ln K, \tag{23}$$

[34] We continue to normalize P to unity. In the empirical analysis we use the price index of imports relative to the aggregate consumer price index, as provided in the data.

where

$$
\begin{aligned}
\boldsymbol{\mu} &= \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}^*, && n \times 1 \textit{ vector of multipliers} \\
\boldsymbol{\lambda} &= \{\lambda_i\}_i = \{\ln \Lambda_i\}_i, && n \times 1 \textit{ vector of sectoral log-productivity coefficients} \\
\boldsymbol{\Gamma} &= \{\gamma_{ji}\}_{ji}, && n \times n \textit{ input-output matrix for domestic intermediates} \\
\boldsymbol{\sigma} &= \{\sigma_i\}, && n \times 1 \textit{ vector of imported intermediate shares} \\
\boldsymbol{\gamma} &= \{\gamma_i\}, && n \times 1 \textit{ vector of domestic intermediate shares} \\
& \rho && \textit{relative price of imported intermediates}
\end{aligned}
$$

Compared to the baseline model, there are a few modifications. First, sectoral multipliers $\boldsymbol{\mu}$ depend only on the domestic IO coefficients γ_{ji}, since foreign production is unaffected by changes in domestic productivity. Second, while $\sum_{i=1}^{n} \mu_i(1-\gamma_i) = 1$ in the model with only domestic intermediates, the new term $\sum_{i=1}^{n} \mu_i(1-\sigma_i-\gamma_i)$ is smaller than one,[35] and this amplifies the effect of sectoral multipliers $\boldsymbol{\mu}$. The intuition for this is as follows. What matters for the effect of multipliers is not just the share of domestic intermediates γ_i but the total share of intermediates $\sigma_i+\gamma_i$. Indeed, imported intermediates do not dilute multipliers because of our assumption of balanced trade: an increase in productivity of a given sector increases exports, which in turn increases imports. Third, income now depends negatively on ρ, the relative price of imported intermediates. When they become more expensive, GDP is reduced because an increase in the price of imported intermediates acts effectively as a negative supply shock. The magnitude of this effect depends on the weighted average of imported intermediate shares σ_i, with multipliers μ_i as weights.

Figure 9 plots the term $-\ln\rho\sum_{i=1}^{n}\mu_i\sigma_i$ against log GDP per worker: poor countries have a much higher relative price of imported intermediates, leading to a positive correlation between this term and log GDP per worker.

In Table 5 we report the results of our variance decomposition. It now has an additional term which accounts for the effect of imported intermediates. We first discuss results for the model with arithmetic-average expenditure shares. The model explains 94% of the variance of GDP per worker in the data, which is 4% more than the baseline model. The fraction of variance explained by capital per worker and average multiplier times average productivity is almost identical to the one in the baseline model, while the role of the covariance term between multipliers and productivities increases in absolute terms by 4% to minus 12%. The term reflecting the role of the price of intermediates, $-\ln\rho\sum_i\sigma_i\mu_i$, is also important: it increases the variance of GDP per worker across countries by 10%. This is driven mostly by the fact that for low-income countries the relative price of imports

[35] Note that (a) this term is positive, and (b) by definition of multipliers, $\sum_i \mu_i(1-\gamma_i) = 1$. Thus, $\sum_i \mu_i(1-\gamma_i-\sigma_i) = 1-\sum_i \mu_i\sigma_i < 1$.

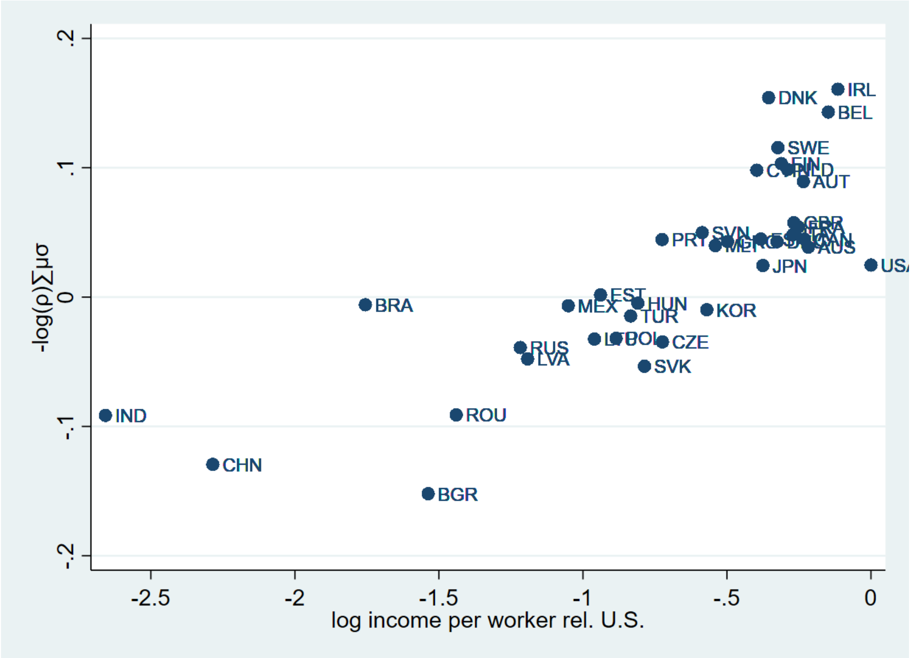

Figure 9: $-ln(\rho)\sum_i \mu_i\sigma_i$ vs. log income per worker rel. U.S.

is much higher than for rich countries, which reduces their level of GDP per worker significantly. Again, computing expenditure shares as a geometric mean (row 2) or as a quantity-weighted mean (row 3) gives very similar results. In the last case, though, the role of the covariance term between TFP and multipliers is a bit smaller in magnitude (minus 9%).

Table 5: Variance decomposition of log GDP per worker – model with traded intermediates

	fraction of variance explained by					
	model	$\alpha\ln(K)$	$An\bar{\mu}\bar{\lambda}$	$AnCov(\lambda,\mu)$	$-A\ln\rho\sum_i\sigma_i\mu_i$	residual
arith. mean	0.94	0.49	0.47	-0.12	0.10	0.06
geo. mean	0.91	0.49	0.46	-0.13	0.09	0.09
w. mean	0.97	0.49	0.47	-0.09	0.10	0.03

$A = [\sum_{i=1}^n \mu_i(1-\sigma_i-\gamma_i)]^{-1}$

5.4 Human Capital

In a final robustness check, we account for variation in human capital levels across countries and sectors to make sure that our results are not biased by the omission of this factor. We thus modify the sectoral production functions as follows:

$$q_i = \Lambda_i \left(\frac{1}{1-\gamma_i}k_i^\alpha u_i^\delta s_i^{1-\alpha-\delta}\right)^{1-\gamma_i} \left(\frac{d_{1i}}{\gamma_{1i}}\right)^{\gamma_{1i}} \left(\frac{d_{2i}}{\gamma_{2i}}\right)^{\gamma_{2i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}}\right)^{\gamma_{ni}}, \tag{24}$$

where u_i is the number of unskilled workers and s_i is the number of skilled workers in sector i, and where δ and $1-\alpha-\delta$ are, respectively, the income shares of unskilled and skilled workers in sectoral value added. The rest of the model is assumed to be the same as in the baseline case. Denoting the aggregate amount of unskilled workers by U, the aggregate amount of skilled workers by S and normalizing the total size of the workforce to unity, we obtain the following expression for log real GDP per worker:

Proposition 4. *In the unique competitive equilibrium, the logarithm of real GDP per worker,* $y = \ln(Y)$, *is*

$$y = \sum_{i=1}^{n} \mu_i \lambda_i + \alpha \ln K + \delta \ln U + (1 - \alpha - \delta) \ln S. \tag{25}$$

In order to assess how the introduction of skilled and unskilled labor as separate production factors affects our results quantitatively, we proceed as follows. We follow Caselli, Coleman and John (2006) and define unskilled labor as workers with primary and lower secondary education and skilled labor as workers with more than lower secondary education. WIOD provides for each sector and country the factor inputs and income shares of workers separated by education category. We recompute sectoral TFP levels with the methodology exposed in section 2.5 but we now separate labor inputs of each sector into skilled and unskilled workers. To calibrate δ and $(1 - \alpha - \delta)$, we first compute for each country the income share of unskilled and skilled workers in GDP and then take the arithmetic average across countries. Assuming that $\alpha = 1/3$, this gives $\delta = 0.22$ and $1 - \alpha - \delta = 0.44$. We also calculate aggregate stocks of unskilled and skilled workers by aggregating sectoral labor inputs by skill level from WIOD.

Table 6 presents the results for variance decomposition of log real GDP per worker. Here, ykh denotes the fraction of variance of log real GDP per worker explained by variation in the amount of physical production factors per worker $ykh = \alpha \ln K + \delta \ln U + (1 - \alpha - \delta) \ln S$. The remaining terms are the same as in the baseline model. Using arithmetic averages of expenditure shares for the reference consumer, we obtain that the model with human capital can explain 93% of the variance in GDP per worker, a bit more than the baseline model. Compared to the baseline model, the fraction of income variation explained by production factors also increases from 49 to 54%. By contrast, the fraction of variation explained by average productivity times average multipliers is reduced a bit, from 49 to 46%. Crucially, the negative contribution of the covariance term between sectoral productivities and multipliers is unafffected: like in the baseline model, this term reduces the variance in log GDP per worker by 8%. The other rows report results for the model with expenditure shares obtained as the geometric mean and the quantity weighted mean. Results remain very similar. We conclude that our findings are robust to accounting for variation in human capital across countries.

Table 6: Variance decomposition of log real GDP per worker – model with human capital

	fraction of variance explained by				
	model	ykh	$n\bar{\mu}\bar{\lambda}$	$nCov(\lambda, \mu)$	residual
arithmetic mean	0.92	0.54	0.46	-0.08	0.06
geometric mean	0.86	0.54	0.40	-0.08	0.14
weighted mean	0.94	0.54	0.46	-0.06	0.06

6 Counterfactual Experiments

We now present the results of a number of counterfactual experiments. We first investigate how differences in TFP levels affect cross-country income differences before turning to the effects of differences in IO linkages. For the first two counterfactuals we go back to our baseline model, while for the third counterfactual we use the model with wedges.

In our first counterfactual exercise we eliminate all TFP differences between countries by setting all sectoral productivities equal to the U.S. level. The result of this experiment is shown in Figure 10. It plots the counterfactual percentage change in income per worker of each country against log GDP per worker. As can be seen from the figure, virtually all countries would gain if they had the U.S. TFP levels. While gains are relatively modest for most high-income countries, bringing sectoral TFPs to U.S. levels would almost double income per worker in countries like China (CHN) or Romania (ROU).

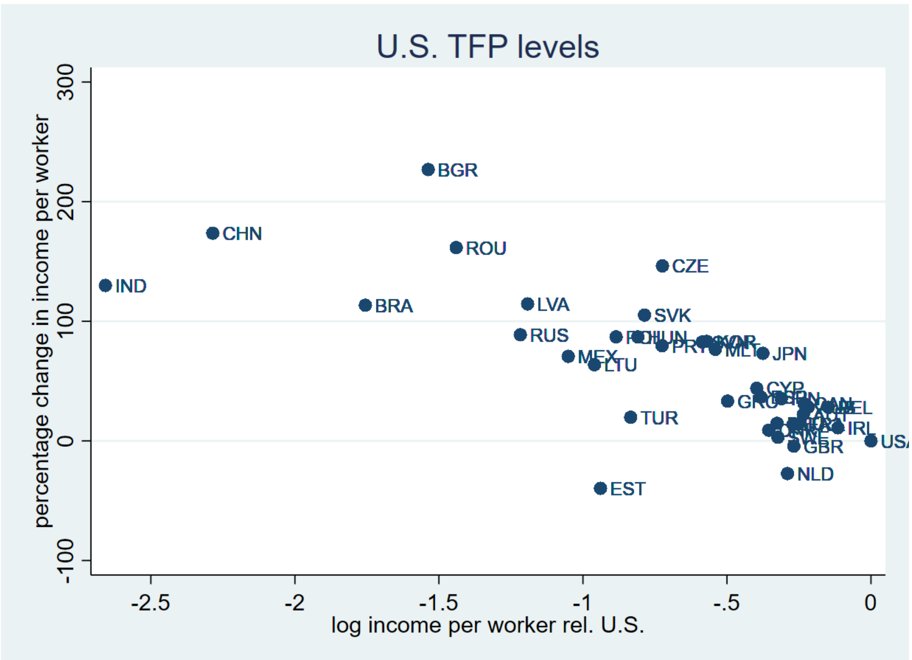

Figure 10: Counterfactuals 1

In the second counterfactual exercise, we hold sectoral productivity levels fixed and instead set the covariance between multipliers and log productivities, $Cov(\mu, \lambda)$, to zero in all countries. Figure 11 makes clear that a number of low-income countries, such as India and China would lose more than

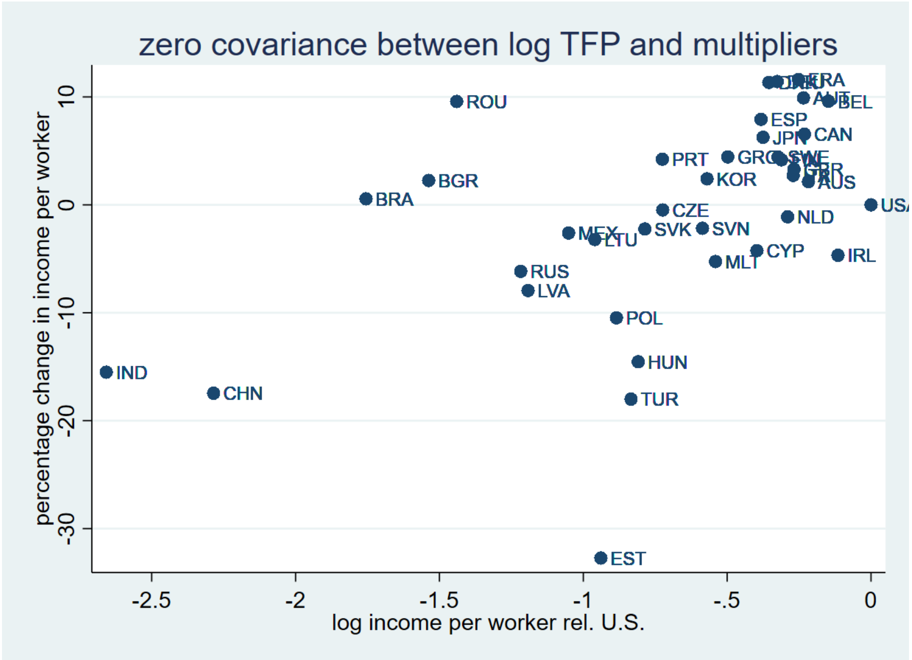

Figure 11: Counterfactuals 2

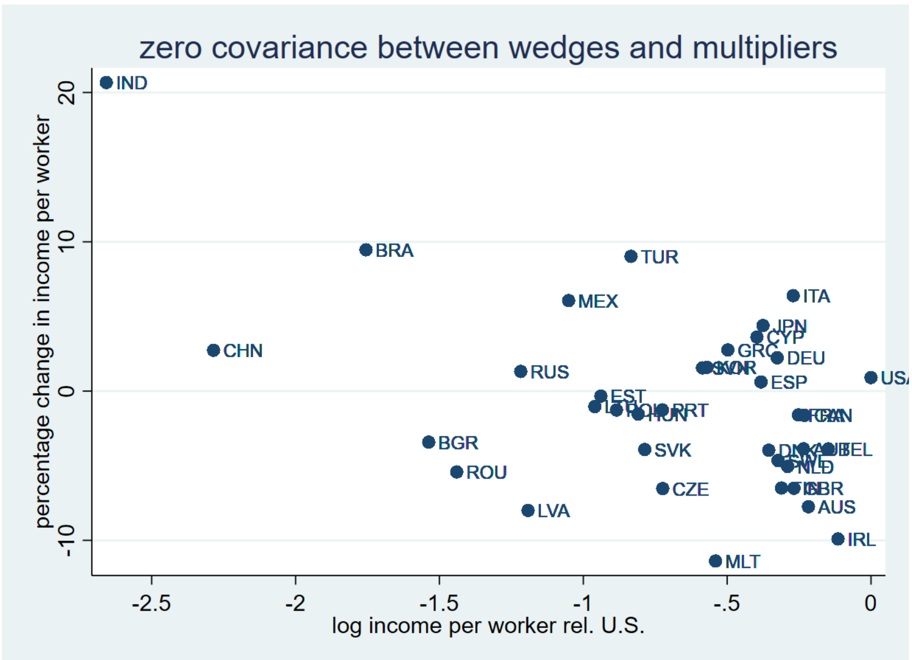

Figure 12: Counterfactuals 3

15% of their income, with a number of Eastern European countries, like Poland (POL), Hungary (HUN) and Estonia (EST) also affected very negatively. Instead many rich countries would gain up to 10% of GDP per worker from this change. Why is this the case? From our estimates, poor countries tend to have a positive covariance between multipliers and log TFPs, while rich countries tend to have a negative one. This implies that poor countries are doing relatively well despite their low average productivity levels, because they perform significantly better than average precisely in those sectors that have a large impact on aggregate performance. The opposite is true in rich countries, where highly connected sectors perform below average. Eliminating this link improves aggregate outcomes in rich economies further, while hurting poor countries. The main reason for negative correlations in rich countries is that they tend to have particularly large productivity gaps with the U.S. in high-multiplier sectors, such as services. Setting the covariance between TFP and multipliers to zero then effectively means bringing European productivity levels in the service sectors to the U.S. level.

Finally, in the last counterfactual we use the model with wedges (see section 5.1) and set the covariance between sectoral wedges and multipliers to zero. Figure 12 describes the result of this exercise. On average low-income countries would gain in this counterfactual. In particular, countries like India, Brazil (BRA), Mexico (MEX) and Turkey (TUR) would see their income improve significantly because they have large wedges in high-multiplier sectors that are very distortive. By contrast, a number of high-income countries, such as Australia (AUS) and Ireland (IRL), would see a significant reduction of their income because these countries currently provide implicit subsidies to high-multiplier sectors that would vanish in the counterfactual.[36]

7 Conclusions

In this paper we have studied the role of IO structure and its interaction with sectoral productivity levels in explaining income differences across countries. We have described and formally modeled cross-country differences in the interaction of sectoral IO multipliers and productivities and shown that they are important for understanding variation in real GDP per worker across countries. Poor countries rely on a few highly connected sectors, which tend to have higher-than-average productivity levels. Their typical, low-productivity sectors are not strongly linked to the rest of the economy, mitigating their impact on aggregate income. By contrast, in rich countries highly connected sectors tend to have below-average productivity levels. At the same time, in low-income countries highly connected sectors tend to be more distorted through high implicit tax rates, which significantly reduces aggregate income. These insights have important consequences for the design of development

[36]This positive effect of subsidies has to be interpreted cautiously because for simplicity wedges are modeled as a pure waste, which implies that subsidies do not reduce resources available to other sectors.

policies, which should focus on increasing productivity and reducing distortions in these key sectors.

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Appendix A: Proofs for the benchmark model and its extensions

Propositions 1 – 4 are particular cases of Proposition 5 that applies in a generic setting – with sector-specific wedges, traded intermediates and division of labor into skilled and unskilled labor inputs. A brief description of this economy, together with Proposition 5, its proof and conditions on parameters that result in each of the particular cases (Propositions 1 – 4) are provided below.

- The technology of each of n competitive sectors is Cobb-Douglas with constant returns to scale. Namely, the output of sector i, denoted by q_i, is

$$q_i = \Lambda_i \left(\frac{1}{1-\gamma_i-\sigma_i} k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta} \right)^{1-\gamma_i-\sigma_i} \left(\frac{d_{1i}}{\gamma_{1i}}\right)^{\gamma_{1i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}}\right)^{\gamma_{ni}} \cdot \left(\frac{f_{1i}}{\sigma_{1i}}\right)^{\sigma_{1i}} \cdot \ldots \cdot \left(\frac{f_{ni}}{\sigma_{ni}}\right)^{\sigma_{ni}},$$

  where s_i and u_i are the amounts of skilled and unskilled labor, d_{ji} is the quantity of the domestic good j and f_{ji} is the quantity of the imported good j used by sector i. $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ and $\sigma_i = \sum_{j=1}^{n} \sigma_{ji}$ are the respective shares of domestic and imported intermediate goods in the total input use of sector i and α, δ, $1-\alpha-\delta$ are the respective shares of capital, unskilled and skilled labor in the remainder of the inputs.

- A good produced by sector i can be used for final consumption, c_i, as an intermediate good or exported abroad:

$$c_i + \sum_{j=1}^{n} d_{ij} + x_i = q_i \qquad i = 1:n$$

- Exports pay for the imported intermediate goods, and we impose a balanced trade condition:

$$\sum_{j=1}^{n} p_j x_j = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{p}_j f_{ji},$$

  where p_j is the domestic and export price of intermediate good j and $\overline{p}_j$ is the import price of intermediate good j.

- Consumers have Cobb-Douglas utility:

$$u(c_1, ..., c_n) = \prod_{i=1}^{n} \left(\frac{c_i}{\beta_i}\right)^{\beta_i},$$

  where $\beta_i \geq 0$ for all i and $\sum_{i=1}^{n} \beta_i = 1$.

- Consumers own all production factors, and use their income to finance consumption:

$$\sum_i p_i c_i = I = w_U U + w_S S + rK.$$

- Consumers maximize utility subject to their budget constraint $\sum_i p_i c_i = I$, taking prices $\{p_i\}$ as given.

- Intermediate good producers maximize profits:

$$\max_{\{d_{ji}\},\{f_{ji}\},k_i,l_i} (1-\tau_i) p_i \Lambda_i \left(\frac{1}{1-\gamma_i-\sigma_i} k_i^{\alpha} u_i^{\delta} s_i^{1-\alpha-\delta} \right)^{1-\gamma_i-\sigma_i} \left(\frac{d_{1i}}{\gamma_{1i}}\right)^{\gamma_{1i}} \cdot \ldots \cdot \left(\frac{d_{ni}}{\gamma_{ni}}\right)^{\gamma_{ni}} \cdot \left(\frac{f_{1i}}{\sigma_{1i}}\right)^{\sigma_{1i}} \cdot \ldots \cdot \left(\frac{f_{ni}}{\sigma_{ni}}\right)^{\sigma_{ni}}$$
$$- \sum_{j=1}^{n} p_j d_{ji} - \sum_{j=1}^{n} \overline{p}_j f_{ji} - rk_i - wl_i, \qquad i \in 1:n$$

  taking prices $\{p_j\}$, $\{\overline{p}_j\}$ of all goods and prices of labor and capital, w and r, as given (τ_i and Λ_i are exogenous). τ_i is a sector-specific wedge that reduces the value of sector i's production by a factor $(1-\tau_i)$.

- The total supply of physical capital, unskilled and skilled labor are fixed at the exogenous levels of K, U and S, respectively, and we normalize $U + S = 1$:

$$\sum_{i=1}^{n} k_i = K,$$
$$\sum_{i=1}^{n} u_i = U,$$
$$\sum_{i=1}^{n} s_i = S.$$

- Numeraire: $P = \prod_{i=1}^{n} (p_i)^{\beta_i} = 1$.
- Definition of real GDP: $Y = \sum_{i=1}^{n} p_i c_i = u$.

For this "generic" economy, the competitive equilibrium is described by the following proposition.

Proposition 5. *There exists a unique competitive equilibrium. In this equilibrium, the logarithm of GDP per capita,* $y = \ln(Y)$*, is given by*

$$y = \frac{1}{\sum_{i=1}^{n} \mu_i (1-\gamma_i-\sigma_i)} \left[\sum_{i=1}^{n} \mu_i \lambda_i - \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i \sigma_{ji} \ln \bar{p_j} + \sum_{i=1}^{n} \mu_i \ln(1-\tau_i) \right] + \\ + \alpha \ln K + \delta \ln U + (1-\alpha-\delta) \ln S, \tag{A-1}$$

where

$$\boldsymbol{\mu} = \{\mu_i\}_i = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\beta}, \quad n \times 1 \textit{ vector of multipliers}$$
$$\boldsymbol{\Gamma} = \{\gamma_{ji}\}_{ji}, \quad n \times n \textit{ input-output matrix for domestic intermediates}$$
$$\boldsymbol{\lambda} = \{\lambda_i\}_i = \{\ln \Lambda_i\}_i, \quad n \times 1 \textit{ vector of sectoral log-productivity coefficients}$$

Proof. Part I: Calculation of $\ln w_U$*.*
Consider a profit maximization problem of the representative firm in each sector i. The FOCs are:

$$\alpha(1-\gamma_i-\sigma_i)(1-\tau_i)\frac{p_i q_i}{r} = k_i \tag{A-2}$$
$$\delta(1-\gamma_i-\sigma_i)(1-\tau_i)\frac{p_i q_i}{w_U} = u_i \tag{A-3}$$
$$(1-\alpha-\delta)(1-\gamma_i-\sigma_i)(1-\tau_i)\frac{p_i q_i}{w_S} = s_i \tag{A-4}$$
$$\gamma_{ji}(1-\tau_i)\frac{p_i q_i}{p_j} = d_{ji} \quad j \in 1:n \tag{A-5}$$
$$\sigma_{ji}(1-\tau_i)\frac{p_i q_i}{\bar{p_j}} = f_{ji} \quad j \in 1:n \tag{A-6}$$

Substituting the left-hand side of these equations for the values of k_i, u_i, s_i, $\{d_{ji}\}$ and $\{f_{ji}\}$ in firm i's log-production technology and simplifying the obtained expression, we derive:

$$\delta \ln w_U = \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \ln p_i - \sum_{j=1}^{n} \gamma_{ji} \ln p_j - \sum_{j=1}^{n} \sigma_{ji} \ln \bar{p_j} + \ln(1-\tau_i)\Big) - \\ - \alpha \ln r - (1-\alpha-\delta)\ln(w_S) + \alpha \ln \alpha + \delta \ln \delta + (1-\alpha-\delta)\ln(1-\alpha-\delta). \tag{A-7}$$

Next, we use FOCs (A-2) – (A-6) and market clearing conditions for labor and capital to express

r and w_S in terms of w_U:

$$w_U = \frac{1}{U}\delta\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(1-\tau_i)(p_iq_i) \tag{A-8}$$

$$w_S = \frac{1}{S}(1-\alpha-\delta)\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(1-\tau_i)(p_iq_i) = \frac{w_U U}{S}\frac{1-\alpha-\delta}{\delta} \tag{A-9}$$

$$r = \frac{1}{K}\alpha\sum_{i=1}^{n}(1-\gamma_i-\sigma_i)(1-\tau_i)(p_iq_i) = \frac{w_u U}{K}\frac{\alpha}{\delta} \tag{A-10}$$

Substituting these values of r and w_S in (A-7) we obtain:

$$\ln w_U = \frac{1}{1-\gamma_i-\sigma_i}\Big(\lambda_i + \ln p_i - \sum_{j=1}^{n}\gamma_{ji}\ln p_j - \sum_{j=1}^{n}\sigma_{ji}\ln \bar{p_j} + \ln(1-\tau_i)\Big) + \\ +\alpha\ln K - (1-\delta)\ln U + (1-\alpha-\delta)\ln S + \ln\delta$$

Multiplying this equation by the ith element of the vector $\boldsymbol{\mu'D} = \boldsymbol{\beta'}[\boldsymbol{I}-\boldsymbol{\Gamma'}]^{-1}\cdot\boldsymbol{D}$, where $\boldsymbol{D}$ is a diagonal matrix with $D_{ii} = 1-\gamma_i-\sigma_i$, and summing over all sectors i gives

$$\ln w_U \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i) = \sum_{i=1}^{n}\mu_i\lambda_i + \sum_{i=1}^{n}\beta_i\ln p_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\sigma_{ji}\ln\bar{p_j} + \sum_{i=1}^{n}\mu_i\ln(1-\tau_i) + \\ + \sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)\Big(\alpha\ln K - (1-\delta)\ln U + (1-\alpha-\delta)\ln S + \ln\delta\Big)$$

Next, we use the price index normalization $P = \prod_{i=1}^{n}(p_i)^{\beta_i} = 1$, which implies that $\sum_{i=1}^{n}\beta_i\ln p_i = 0$. Then we can write the above equation as follows:

$$\ln w_U = \frac{1}{\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)}\left[\sum_{i=1}^{n}\mu_i\lambda_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\sigma_{ji}\ln\bar{p_j} + \sum_{i=1}^{n}\mu_i\ln(1-\tau_i)\right] + \\ +\alpha\ln K - (1-\delta)\ln U + (1-\alpha-\delta)\ln S + \ln\delta \tag{A-11}$$

Part II: Calculation of y.
Recall that our ultimate goal is to find $y = \ln(Y) = \ln(\sum_i p_ic_i)$. Since consumers' expenditure is financed through income, $Y = \sum_i p_ic_i = w_UU + w_SS + rK$.

Using (A-9) and (A-10), this leads to

$$Y = \frac{w_UU}{\delta}.$$

so that

$$y = \ln Y = \ln w_U + \ln U - \ln\delta.$$

Finally, substituting (A-11) for $\ln w_U$ yields our result:

$$y = \frac{1}{\sum_{i=1}^{n}\mu_i(1-\gamma_i-\sigma_i)}\left[\sum_{i=1}^{n}\mu_i\lambda_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\sigma_{ji}\ln\bar{p_j} + \sum_{i=1}^{n}\mu_i\ln(1-\tau_i)\right] + \\ +\alpha\ln K - (1-\delta)\ln U + (1-\alpha-\delta)\ln S + \ln\delta + \ln U - \ln\delta$$

that is,

$$y = \frac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \left[\sum_{i=1}^n \mu_i\lambda_i - \sum_{i=1}^n \sum_{j=1}^n \mu_i\sigma_{ji} \ln \bar{p_j} + \sum_{i=1}^n \mu_i \ln(1-\tau_i) \right] + \\ + \alpha \ln K + \delta \ln U + (1-\alpha-\delta) \ln S.$$

This completes the proof. □

Application of Proposition 5 to the case of the benchmark economy (Proposition 1) and robustness checks (Propositions 2 – 4):

- *Benchmark economy, Proposition 1:* In case of our benchmark economy, we assume that: i) there is no distinction between skilled and unskilled labor, so that $\delta = 1-\alpha$ and the total supply of labor is normalized to 1; ii) the economies are closed, so that no imported intermediate goods are used in sectors' production, that is, $\sigma_{ji} = 0$ for all $i, j \in 1:n$ and $\sigma_i = 0$ for all i; iii) there are no wedges, that is, $\tau_i = 0$ for all i. This simplifies the expression for y in Proposition 5 and produces the result of Proposition 1:[37]

$$y = \sum_{i=1}^n \mu_i\lambda_i + \alpha \ln K.$$

- *Wedges, Proposition 2:* For the economy with sector-specific wedges, we assume, in addition to the benchmark model, that there exist non-zero distortions, or wedges $\tau_i \neq 0$. Then the expression for y in Proposition 5 turns into

$$y = \sum_{i=1}^n \mu_i\lambda_i + \sum_{i=1}^n \mu_i \ln(1-\tau_i) + \alpha \ln K.$$

- *Traded intermediate goods, Proposition 3:* In the economy, where we differentiate between domestically produced and imported intermediates, $\sigma_{ji} \neq 0$ and $\sigma_i \neq 0$. But, as in the benchmark model, there is no distinction between skilled and unskilled labor, and no wedges. In addition, due to restrictions imposed by the data, we assume that import prices do not vary across sectors, that is, $\rho_j = \rho$, where $\rho_j = \bar{p_j}/P$, and P is normalized to 1. Then $\sum_{i=1}^n \sum_{j=1}^n \mu_i\sigma_{ji} \ln \bar{p_j} = \ln\rho \sum_{i=1}^n \mu_i\sigma_i$, and the expression for y in Proposition 5 becomes:

$$y = \frac{1}{\sum_{i=1}^n \mu_i(1-\sigma_i-\gamma_i)} \left(\sum_{i=1}^n \mu_i\lambda_i - \ln\rho \sum_{i=1}^n \mu_i\sigma_i \right) + \alpha \ln K,$$

- *Human capital, Proposition 4:* The model where we introduce two types of labor, skilled and unskilled, is identical to the benchmark model in all other respects. So, the expression for y is

$$y = \sum_{i=1}^n \mu_i\lambda_i + \alpha \ln K + \delta \ln U + (1-\alpha-\delta) \ln S.$$

[37]Note that $\sum_{i=1}^n \mu_i(1-\gamma_i) = \mathbf{1}'[\boldsymbol{I}-\boldsymbol{\Gamma}] \cdot \frac{1}{n}[\boldsymbol{I}-\boldsymbol{\Gamma}]^{-1}\mathbf{1} = \frac{1}{n}\mathbf{1}'\mathbf{1} = 1$.

Appendix B: Derivation of the productivity index

In section 2.5, the multilateral sector-specific Cobb-Douglas productivity index $\ln\lambda^*_{ikl}$ in (16) is obtained as follows. Using (15) and definition of $\ln\lambda_{iks}$ in (13), we obtain:

$$\begin{aligned}
\ln\lambda^*_{ikl} \quad & \equiv \overline{\ln\lambda}_{ik} - \overline{\ln\lambda}_{il} = \frac{1}{m}\sum_{s=1}^{m}\ln\lambda_{iks} - \frac{1}{m}\sum_{s=1}^{m}\ln\lambda_{ils} = \\
& = (\ln q_{ik} - \frac{1}{m}\sum_{s=1}^{m}\ln q_{is}) - \frac{1}{2}\left[\alpha_{Xik}\left(\ln X_{ik} - \frac{1}{m}\sum_{s=1}^{m}\ln X_{is}\right) + \frac{1}{m}\sum_{s=1}^{m}\alpha_{Xis}\left(\ln X_{ik} - \ln X_{is}\right)\right] - \\
& -(\ln q_{il} - \frac{1}{m}\sum_{s=1}^{m}\ln q_{is}) + \frac{1}{2}\left[\alpha_{Xil}\left(\ln X_{il} - \frac{1}{m}\sum_{s=1}^{m}\ln X_{is}\right) + \frac{1}{m}\sum_{s=1}^{m}\alpha_{Xis}\left(\ln X_{il} - \ln X_{is}\right)\right] = \\
& = (\ln q_{ik} - \ln q_{il}) - \frac{1}{2}\left[\alpha_{Xik}\left(\ln X_{ik} - \frac{1}{m}\sum_{s=1}^{m}\ln X_{is}\right) - \alpha_{Xil}\left(\ln X_{il} - \frac{1}{m}\sum_{s=1}^{m}\ln X_{is}\right) + \right. \\
& \left. + \frac{1}{m}\sum_{s=1}^{m}\alpha_{Xis}\left(\ln X_{ik} - \ln X_{il}\right)\right].
\end{aligned}$$

Combining the terms, we derive (16):

$$\ln\lambda^*_{ikl} = \ln q_{ik} - \ln q_{il} - \frac{1}{2}\left(\alpha_{Xik} + \overline{\alpha}_{Xi}\right)\left(\ln X_{ik} - \overline{\ln X}_i\right) + \frac{1}{2}\left(\alpha_{Xil} + \overline{\alpha}_{Xi}\right)\left(\ln X_{il} - \overline{\ln X}_i\right),$$

where $\overline{\alpha}_{Xi} = \frac{1}{m}\sum_{s=1}^{m}\alpha_{Xis}$ and $\overline{\ln X}_i = \frac{1}{m}\sum_{s=1}^{m}\ln X_{is}$.

Appendix C: Additional Tables

Table A-1: Countries: WIOD Sample

countries	
AUS	IND
AUT	IRL
BEL	ITA
BGR	KOR
BRA	JPN
CAN	LTU
CHN	LVA
CYP	MEX
CZE	MLT
DEU	NLD
DNK	POL
ESP	PRT
EST	ROM
FIN	RUS
FRA	SVK
GBR	SVN
GRC	SWE
HUN	TUR
IDN	USA

Table A-2: Sector List

WIOD sectors	
1	Agriculture
2	Mining
3	Food
4	Textiles
5	Leather
6	Wood
7	Paper
8	Refining
9	Chemicals
10	Plastics
11	Minerals
12	Metal products
13	Machinery
14	Elec. equip.
15	Transport equip.
16	Manufacturing nec
17	Electricity
18	Construction
19	Car retail.
20	Wholesale trade
21	Retail trade
22	Restaurants
23	Inland transp.
24	Water transp.
25	Air transp.
26	Transp. nec.
27	Telecomm.
28	Fin. serv.
29	Real est.
30	Business serv.
31	Pub. admin.
32	Education
33	Health
34	Social serv.
35	Household empl.

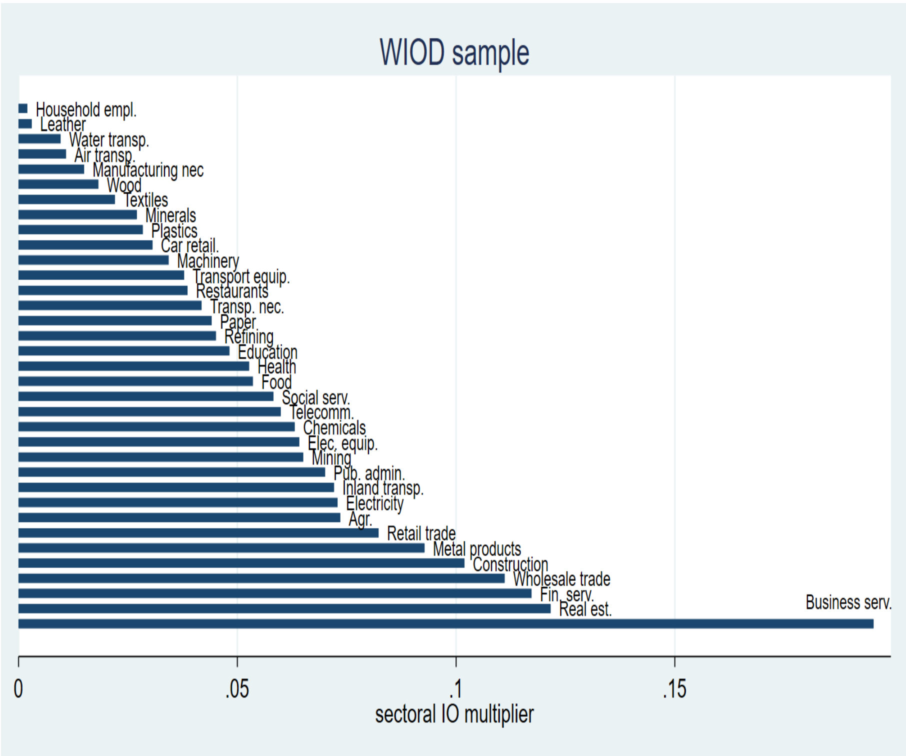

Figure A-1: Sectoral IO multipliers