Trade and Domestic Policies in Models with Monopolistic Competition*

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Abstract

We consider unilateral and strategic trade and domestic policies in single and multi-sector versions of models with CES preferences and monopolistic competition featuring homogeneous (Krugman, 1980) or heterogeneous firms (Melitz, 2003). We first solve the world-planner problem to identify the efficiency wedges between the planner and the market allocation. We then derive a common welfare decomposition in terms of macro variables that incorporates all general-equilibrium effects of trade and domestic policies and decomposes them into consumption and production-efficiency wedges and terms-of-trade effects. We show that the Nash equilibrium when both domestic and trade policies are available is characterized by first-best-level labor subsidies that achieve production efficiency, and inefficient import subsidies and export taxes that aim at improving domestic terms of trade. Since the terms-of-trade externality is the only beggar-thy-neighbor motive, it remains the only reason for signing trade agreements in this general class of models. Finally, we show that when trade agreements only limit the strategic use of trade taxes but do not require coordination of domestic policies, the latter are set inefficiently in the Nash equilibrium in order to manipulate the terms of trade.

Keywords: Heterogeneous Firms, Trade Policy, Domestic Policy, Trade Agreements, Terms of Trade, Efficiency, Tariffs and Subsidies

JEL classification codes: F12, F13, F42

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1 Introduction

A fundamental challenge in the research on trade policy is to understand the purpose of trade agreements. In particular, as a prerequisite for the optimal design of trade agreements, one needs to characterize the international externalities that can be solved by coordinating countries' decisions (Bagwell and Staiger, 2016). In the context of perfectly competitive neoclassical trade models, a large literature has shown that trade agreements solve a terms-of-trade externality. Uncoordinated individual-country policy makers try use trade taxes, such as tariffs, to manipulate international prices in their favor. This leads to a Prisoners' Dilemma and countries end up with inefficiently high trade taxes in equilibrium (Bagwell and Staiger, 1999, 2016). Surprisingly, it is still not well understood if these results on the purpose of trade agreements also apply to the workhorse model of modern international trade theory – the monopolistic competition model with heterogeneous firms (Melitz, 2003).

Moreover, with the fall of tariff barriers during the last decades, the focus of recent trade negotiations – both at the multilateral[1] and at the regional level[2] – has shifted away from trade tax reductions towards coordination of domestic policies (e.g. sector-specific policies, product market regulation, labor standards,...). The Melitz (2003) model provides a natural framework for studying the role of these policies in the context of trade agreements, since distortions induced by imperfect competition may lead to inefficiencies in the market allocation, thus calling for domestic regulation (e.g., sector-specific subsidies). We investigate if domestic policies induce additional motives for signing trade agreements beyond the classical terms-of-trade externality (Bagwell and Staiger, 2016). Importantly, we also investigate the distortions that arise from limiting trade agreements to the coordination of trade taxes and study if clauses requiring coordination of domestic policies or proscribing their use should be incorporated into trade agreements.

Our first main result is that neither the presence of firm heterogeneity nor domestic policies affect the standard wisdom from perfectly competitive models that the only motive for signing trade agreements are terms-of-trade externalities. When countries can set both domestic and trade policies strategically, domestic policies are set efficiently, while trade taxes are used to manipulate the terms of trade. Our second main result is that when trade agreements only limit countries' ability to use trade taxes strategically but do not require coordination of domestic policies, individual-country policy makers set domestic policies inefficiently because they face a trade-off between correcting domestic inefficiencies and manipulating their terms of trade. In order to achieve these conclusions, we proceed as follows.

Our theoretical setup features two countries, CES preferences and either a single or multiple sectors. Firms operate under monopolistic competition and are potentially heterogeneous in terms of productivity. In the version with heterogeneous firms we allow firm-specific productivity levels to be drawn from arbitrary productivity

[1]The Doha round of multilateral WTO negotiations deals with issues such as environmental regulation and intellectual property rights and initially also included competition policy and government procurement.

[2]E.g., the European Union's strategy is to sign "deep" bilateral trade agreements that cover a host of areas in addition to classical tariff reduction, such as domestic regulation, foreign direct investment and intellectual property rights. Recent examples are the trade agreements with Canada, Korea and Japan.

distributions. Policy makers in each country can set both sector-specific trade policies (import and export taxes) and domestic policies (labor taxes). We use the insight of Arkolakis, Costinot and Rodríguez-Clare (2012) that monopolistic competition models with CES preferences have a common macro representation in terms of sectoral aggregate bundles. This representation also makes clear that in this class of models the welfare-relevant terms of trade are defined in terms of aggregate price indices of importables and exportables. As a consequence, policy instruments can affect the terms of trade both by changing the international prices of individual varieties (directly and indirectly via selection) and by impacting on the measure of firms active in foreign markets.

We first solve for the allocation chosen by a social planner whose objective is to maximize world welfare in order to identify the welfare wedges present in the market allocation. Following the approach of Costinot, Rodríguez-Clare and Werning (2016), we separate the planner problem into different stages: a micro stage; a macro stage within sectors; and a cross-sector macro stage. At the micro stage, the planner chooses how much to produce of each differentiated variety given the sectoral aggregates. We show that given CES preferences, the solution to the micro problem always corresponds to the market allocation, i.e., relative firm size is always optimal. In the within-sector macro stage, the planner chooses for each sector how much to produce of the aggregate bundle that is domestically produced and consumed and how much to produce of the aggregate exportable bundle. Here, a consumption-efficiency wedge arises between the planner and the market allocation whenever trade policy instruments are used.[3] Finally, the cross-sector macro stage is present only in the multi-sector version of the model. At this stage, the planner determines the optimal allocation across macro sectors. The cross-sectoral allocation corresponds to the market allocation if and only if trade taxes are not used and monopolistic markups are offset with labor subsidies, otherwise too little labor is allocated to the monopolistically competitive sector and a production-efficiency wedge is present.

We then turn to the problem of a benevolent policy maker who is concerned with maximizing world welfare and can set labor and trade taxes. By using the total-differential approach to optimization, we are able to derive an exact welfare decomposition in terms of macro aggregates that decomposes general-equilibrium welfare effects of trade and domestic policies and simultaneously identifies the optimality conditions of the world policy maker. Welfare effects can be decomposed into three terms: (i) a consumption-efficiency wedge, (ii) a production-efficiency wedge and (iii) terms-of-trade effects. This decomposition is useful for the following reasons: The efficiency terms in the welfare decomposition correspond exactly to the wedges between the planner and the market allocation mentioned above. As a result, solving the problem of the world policy maker is equivalent to setting both consumption and production-efficiency wedges in the welfare decomposition equal to zero. Moreover, in the symmetric equilibrium terms-of-trade motives of individual countries offset each other at the world level: an improvement in the terms of trade of one country necessarily implies an equivalent terms-of-trade worsening of the other. Thus, terms-of-trade effects are a pure beggar-thy-neighbor incentive and hence the world policy maker disregards them. We show that in the one-sector version of the model production efficiency is always guaranteed and the world-policy-maker solution corresponds to the (Pareto-optimal) free-trade allocation.

[3] Dhingra and Morrow (2019) establish efficiency of the market allocation in a closed-economy one-sector version of Melitz under CES preferences.

By contrast, in the presence of multiple sectors the world policy maker closes the production-efficiency wedge and implements the planner allocation using sector-specific labor subsidies.

Next, we study trade and domestic policies from the individual-country perspective. Our welfare decomposition makes apparent that in the one-sector version of the model unilateral policies are driven unequivocally by a trade-off between improving domestic terms of trade and creating consumption wedges, while production efficiency is automatically guaranteed. By contrast, in the multi-sector version of the model, closing the domestic production-efficiency wedge becomes an additional motive for individual-country policy makers.

As a preparation for the case of strategic interaction, we study how unilateral policy deviations in individual instruments affect the terms of trade and production efficiency. We show that in the one-sector model, where labor supply is completely inelastic, a small unilateral import tariff is welfare enhancing from the unilateral perspective compared to free trade. A tariff improves domestic terms of trade by increasing the relative wage. In the presence of firm heterogeneity there are two additional opposing effects: an improvement in the terms of trade stemming from an increase in the variable-profit share arising from exports and a terms-of-trade worsening from tougher selection into exporting. Differently, in the multi-sector model with a linear outside good labor supply is perfectly elastic and a tariff worsens domestic terms of trade by increasing labor demand for the differentiated bundles and triggering entry into that sector. This reduces the relative price of exportables via the extensive margin. In the presence of firm heterogeneity there are again two additional opposing effects: changes in the variable-profit share arising from exports and changes in selection into exporting. At the same time, when starting from the free-trade allocation, a tariff improves production efficiency by increasing the amount of labor allocated to the differentiated sector. Similarly, a labor or export subsidy also improves production efficiency, while worsening the terms-of-trade. Thus, using any individual policy instrument gives rise to a trade-off between reducing the production-efficiency wedge and worsening the terms of trade.

We also investigate the role of firm heterogeneity in shaping unilateral policies in the multi-sector model. With homogeneous firms a small tariff is always welfare improving from the unilateral perspective because the increase in production efficiency always dominates the negative terms-of-trade effect. By contrast, in the presence of heterogeneous firms and selection into exporting the sign of the welfare effects stemming from unilateral policy changes – and thus whether a tariff or an import subsidy is unilaterally beneficial – depends on the average variable profit share from sales in the domestic market. Intuitively, if the bulk of variable profits are made domestically, increases in domestic production efficiency dominate negative terms-of-trade effects, while the opposite is the case if the larger part of profits arises from exporting. Thus, these results for unilateral policy changes suggest that firm heterogeneity can affect trade policy qualitatively.

We then return to the question if domestic policies provide an additional reason for signing free-trade agreements beyond terms-of-trade externalities. This would be the case if uncoordinated policy makers set them inefficiently, thereby imposing externalities on the other country. We thus characterize the Nash equilibrium of the policy game in the multi-sector model where individual-country policy makers set both trade and domestic policies simultaneously. We show that the equilibrium policies consist of the first-best level of labor subsidies that close production-efficiency wedges and inefficient import subsidies and export taxes that aim at improving the terms of trade. This implies that domestic policies do not create any additional motive for trade agreements since production inefficiencies are completely internalized by individual-country policy makers.[4]. Here, we show that it carries over to trade models with monopolistic competition and heterogeneous firms. Moreover –in contrast to unilateral policies – the sign of the Nash-equilibrium trade policies (import subsidies and export taxes) does not depend on firm heterogeneity.

Finally, we turn to the design of trade agreements in the presence of domestic policies. A globally efficient outcome requires cooperation on trade *and* domestic policies. We show that when trade agreements only limit the strategic use of trade-policy instruments but do not require coordination of domestic policies, individual-country policy makers use domestic policies both to increase production efficiency and to manipulate the terms of trade. Specifically, we study the Nash equilibrium of a policy game, where only domestic policies can be set strategically, while trade policy instruments are not available. In this situation, firm heterogeneity has a qualitative impact on the Nash policies: the Nash policy outcome depends on the variable-profit share from domestic sales. When at least half of the profits of the average active firm are made in the domestic market or when firms are homogeneous, the production-efficiency effect dominates and the Nash equilibrium features (inefficiently low) labor subsidies. In this case a trade agreement that prohibits the use of both trade and domestic policies provides lower welfare than an agreement that eliminates trade taxes but allows countries to choose domestic policies strategically. By contrast, when more than half of the average firm's profits arise from exports, the terms-of-trade effect dominates and the Nash equilibrium features labor taxes. Intuitively, the smaller the profit share from domestic sales, the more open the economy is and thus the larger the incentive to exploit international externalities. In this case a trade agreement that prohibits the use of both trade and domestic policies fares better in welfare terms than one that eliminates trade taxes but allows countries to set domestic policies strategically. Finally, we show that when variable or fixed physical trade costs fall and therefore the profit share from exporting rises, welfare gains from integrating cooperation on domestic policies into trade agreements become proportionally larger.Thus, the case for deep trade agreements that require coordination of domestic regulation becomes stronger when physical trade barriers are lower.

The rest of the paper is structured as follows. In the next subsection we briefly discuss the related literature. In Section 2 we describe a standard multi-sector Melitz (2003) model expressed in terms of macro bundles. In Section 3 we then set up and solve the problem of a planner who is concerned with maximizing world welfare. We separate it into a micro, a within-sector macro and a cross-sector macro stage and compare each stage to the market allocation in order to identify the relevant efficiency wedges. Next, in Section 4 we solve the problem of a world policy maker who is concerned with maximizing world welfare and disposes of trade and labor taxes. As an intermediate step of solving this problem, we derive a welfare decomposition that decomposes welfare effects of policy instruments into a consumption wedge, a production wedge and terms of trade effects. We then show that solving the world-policy-maker problem is equivalent to setting all wedges individually equal to zero.

[4] This result is an application of the targeting principle, which is known to hold in perfectly competitive trade models (Ederington, 2001)

In Section 5 we turn to the problem of individual-country policy makers, derive individual-country welfare and discuss welfare effects of unilateral policy deviations. Finally, we consider strategic trade and domestic policies. We first characterize the Nash equilibrium of the policy game where individual-country policy makers set both trade and labor taxes simultaneously and strategically. We then turn to the Nash equilibrium of the policy game when only labor taxes can be set strategically. Section 6 presents our conclusions.

1.1 Related literature

Several theoretical contributions have studied the incentives for trade policy in single and multiple-sector versions of the Krugman (1980) and Melitz (2003) models and have identified numerous mechanisms through which trade policy affects outcomes in these models.

Specifically, Gros (1987) and Helpman and Krugman (1989) examine the one-sector version of the Krugman (1980) model with homogeneous firms. They identify a terms-of-trade motive for strategic tariffs, which increase domestic factor prices. By contrast, studies investigating trade policy in the two-sector version of Krugman (1980) – which features a linear outside sector that pins down factor prices – typically find that strategic tariffs are due to a home market/production-relocation motive (Venables, 1987; Helpman and Krugman, 1989; Ossa, 2011). More recently, Campolmi, Fadinger and Forlati (2012), allow for the simultaneous choice of labor, import and export taxes in this model. They show that the tariff result found by previous studies is due to a missing instrument problem. Once policy makers dispose of enough instruments the strategic equilibrium is characterized by the first-best level of labor subsidies, import subsidies and export taxes that aim at improving domestic terms of trade (Campolmi, Fadinger and Forlati, 2014).

A small number of studies analyze trade policy in the Melitz (2003) model with a single sector. Demidova and Rodríguez-Clare (2009) and Haaland and Venables (2016) investigate optimal *unilateral* trade policy in a small-open-economy version of Melitz (2003) with Pareto-distributed productivity. While Demidova and Rodríguez-Clare (2009) identify a distortion in the relative price of imported varieties (markup distortion) and a distortion on the number of imported varieties (entry distortion) as motives for unilateral policy, Haaland and Venables (2016) single out terms-of-trade effects as the only motive for individual-country trade policy. Similarly, Felbermayr, Jung and Larch (2013), who consider strategic import taxes in a two-country version of this model, identify the same motives for tariffs as Demidova and Rodríguez-Clare (2009). Haaland and Venables (2016) also study unilateral policy in the two-sector small-open-economy variant of Melitz (2003) with Pareto-distributed productivities and identify terms-of-trade externalities and monopolistic distortions as drivers of unilateral policy.

Our contribution is to show that the welfare incentives for trade policy in the previous models can be understood using a common welfare decomposition in terms of macro aggregates. This approach makes clear that the terms-of-trade motive is the only externality that needs be addressed by trade agreements in this class of models.

Also closely related is Costinot et al. (2016), who consider unilateral trade policy in a generalized two-country version of Melitz (2003). Similarly to our approach, they consider the macro representation of the model and define the terms of trade in terms of aggregate bundles of importables and exportables. They investigate *unilaterally optimal* firm-specific and non-discriminatory policies and establish terms-of-trade effects as the motive for trade policy. Their main interest is to investigate how firm heterogeneity shapes trade policy. They show that, conditional on the elasticity of substitution and the share of expenditure on local goods abroad, firm heterogeneity reduces the optimal tariff if and only if it leads to non-convexities in the foreign production possibility frontier.

We consider our analysis as complementary to theirs. In particular, we show that the incentives for using trade and domestic policies in models with CES preferences and monopolistic competition can be analyzed with a common welfare decomposition. Their approach of using optimal tax formulas is convenient for evaluating the quantitative impact of firm heterogeneity but makes it difficult to isolate policy makers' welfare incentives, in particular when the set of policy instruments is insufficient to address all distortions separately. Most importantly, while their analysis is limited to *unilaterally optimal* policies we consider the case of *strategic* interaction and compare Nash equilibrium domestic and trade policies to the outcome of a world policy maker. Finally, while Costinot et al. (2016) find that firm heterogeneity potentially affects unilateral trade taxes both quantitatively and qualitatively compared to homogeneous-firm models, we uncover that the sign of the equilibrium strategic taxes is unaffected by the presence of firm heterogeneity as long as the set of instruments is sufficiently large.

This paper is also closely connected to the vast literature on trade policy in perfectly competitive trade models (Dixit, 1985). We show that many insights from this literature carry over to models with monopolistic competition and firm heterogeneity. Specifically, the result that trade agreements solve a terms-of-trade externality also applies in our context (Bagwell and Staiger, 2016). Moreover, also the Bhagwati-Johnson principle of targeting, which states that optimal policy should use the instrument that operates most effectively on the appropriate margin, remains valid.

Finally, there is also a close connection with the literature on trade and domestic policies in perfectly competitive models. Copeland (1990) discusses the idea that in the presence of trade agreements that limit the strategic use of tariffs individual-country policy makers have incentives to use domestic policies to manipulate their terms of trade. Ederington (2001) considers the optimal design of self-enforceable joint agreements on trade and domestic policies and establishes that such agreements should require full coordination of domestic policies but should allow countries to set positive levels of tariffs, in order to mitigate incentives to deviate from cooperation.

2 The Model

The setup follows Melitz and Redding (2015). The world economy consists of two countries i: Home (H) and Foreign (F). The only factor of production is labor which is supplied inelastically in amount L in each country, perfectly mobile across firms and sectors and immobile across countries. Each country has either one or two sectors. The first sector produces a continuum of differentiated goods under monopolistic competition with free entry. If present, the other sector is perfectly competitive and produces a homogeneous good. Labor markets are perfectly competitive. All goods are tradable but only the differentiated goods are subject to iceberg transport costs. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index corresponds to the location of consumption and the second sub-index to the location of production.

2.1 Technology and Market Structure

2.1.1 Differentiated sector

Firms in the differentiated sector operate under monopolistic competition with free entry. They pay a fixed cost in terms of labor, f_E, to enter the market and to pick a draw of productivity φ from a cumulative distribution $G(\varphi)$.[5] After observing their productivity draw, they decide whether to pay a fixed cost f in terms of domestic labor to become active and produce for the domestic market. Active firms then decide whether to pay an additional market access cost f_X (in terms of domestic labor) to export to the other country, or to produce only for the domestic market. Therefore, labor demand of firm φ located in market i for a variety sold in market j is given by:

$$l_{ji}(\varphi) = \frac{q_{ji}(\varphi)}{\varphi} + f_{ji}, \quad i, j = H, F \tag{1}$$

where

$$f_{ji} = \begin{cases} f & j = i \\ f_X & j \neq i \end{cases}$$

Here $q_{ji}(\varphi)$ is the production of a firm with productivity φ located in country i for market j. Varieties sold in the foreign market are subject to an iceberg transport cost $\tau > 1$. We thus define:

$$\tau_{ji} = \begin{cases} 1 & j = i \\ \tau & j \neq i \end{cases}$$

[5] We assume that φ has support $[0, \infty)$ and that $G(\varphi)$ is continuously differentiable with derivative $g(\varphi)$.

2.1.2 Homogenous sector

In case the homogeneous-good sector is present, labor demand L_{Zi} for the homogenous good Z, which is produced in both countries i with identical production technology, is given by:

$$L_{Zi} = Q_{Zi}, \tag{2}$$

where Q_{Zi} is the production of the homogeneous good. Since this good is sold in a perfectly competitive market without trade costs, its price is identical in both countries and equals the marginal cost of production W_i. We assume that the homogeneous good is always produced in both countries in equilibrium. This implies equalization of wages $W_i = W_j$ for $i \neq j$ (factor price equalization).

We also consider a version of the model without the homogeneous sector. In this case, wages across the two countries will not necessarily be equalized.

2.2 Preferences

Households' utility function is given by:

$$U_i \equiv \alpha \log C_i + (1-\alpha)\log Z_i, \quad i = H, F, \tag{3}$$

where C_i aggregates over the varieties of differentiated goods and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. When α is set to unity, we go back to a one-sector model (Melitz, 2003). Z_i represents consumption of the homogeneous good (Krugman, 1980). The differentiated varieties produced in the two countries are aggregated with a CES function given by:[6]

$$C_i = \left[\sum_{j \in H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i = H, F \tag{4}$$

$$C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H, F \tag{5}$$

Here, C_{ij} is the aggregate consumption bundle of country-i consumers of varieties produced in country j, $c_{ij}(\varphi)$ is consumption by country i consumers of a variety φ produced in country j, N_j is the measure of varieties produced by country j. φ_{ij} is the cutoff-productivity level, such that a country-j firm with this productivity level makes exactly zero profits when selling to country i, while firms with strictly larger productivity levels make positive profits from selling to this market, so that all country-j firms with $\varphi \geq \varphi_{ij}$ export to country

[6] Notice that we can index consumption of differentiated varieties by firms' productivity level φ since all firms with a given level of φ behave identically. Note also that our definitions of C_{ij} imply $C_i = \left[N_i \int_{\varphi_{ii}}^{\infty} c_{ii}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) + N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi)\right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e., the model is the standard one considered in the literature. However, it is convenient to define optimal consumption indices.

i. Finally, $\varepsilon > 1$ is the elasticity of substitution between domestic and foreign bundles and between different varieties.

2.3 Government

The government of each country disposes of three fiscal instruments. A labor tax/subsidy (τ_{Li}) on firms' fixed and marginal costs,[7] a tariff/subsidy on imports (τ_{Ii}) and a tax/subsidy on exports (τ_{Xi}). Note that τ_{mi} indicates a gross tax for $m \in \{L, I, X\}$, i.e., $\tau_{mi} < 1$ indicates a subsidy and $\tau_{mi} > 1$ indicates a tax. In what follows, we employ the word *tax* whenever we refer to a policy instrument without specifying whether τ_{mi} is smaller or larger than one and we use the following notation:

$$\tau_{Tij} = \begin{cases} 1 & i = j \\ \tau_{Ii}\tau_{Xj} & i \neq j \end{cases} \tag{6}$$

Moreover, we assume that taxes are paid directly by the firms[8] and that all government revenues are redistributed to consumers through a lump-sum transfer.

2.4 Equilibrium

Since the model is standard, we relegate a more detailed description of the setup and the derivation of the market equilibrium to Appendix A. In a market equilibrium, households choose consumption of the differentiated bundles and – when available –the homogeneous good in order to maximize utility subject to their budget constraint; firms in the differentiated sector choose quantities in order to maximize profits given their residual demand schedules and enter the differentiated sector until their expected profits – before productivity realizations are drawn – are zero; they produce for the domestic and export markets if their productivity draw is weakly above the market-specific survival-cutoff level at which they make exactly zero profits; if present, firms in the homogeneous-good sector price at marginal cost; governments run balanced budgets and all markets clear. We write the equilibrium in terms of sectoral aggregates. This approach follows Campolmi et al. (2012) and Costinot et al. (2016). Specifically, the one-sector model can be represented in terms of three aggregate goods: a good that is domestically produced and consumed; a domestic exportable good and a domestic importable good. The two-sector model additionally features a homogeneous good. This representation in terms of aggregate bundles (i) highlights that models with monopolistic competition and CES preferences have a common macro presentation and (ii) makes the connection to standard neoclassical trade models visible. It will also be useful for interpreting the wedges between the planner and the market allocations and for our welfare decomposition.

[7] We impose that the same labor taxes are levied on both fixed and marginal costs (including also the fixed entry cost f_E). This assumption is necessary to keep firm size unaffected by taxes, which turns out to be optimal, as we will show in Section 3.2.

[8] In particular, following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

Finally, the macro representation will make clear that the welfare-relevant terms of trade that policy makers consider in their objective should be defined in terms of ideal price indices of sectoral exportables relative to importables.

The market equilibrium can be described by the following conditions:

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})} \right]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \tag{7}$$

$$\delta_{ji} = \frac{f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}}{\sum\limits_{k=H,F} f_{ki}(1-G(\varphi_{ki}))\left(\frac{\widetilde{\varphi}_{ki}}{\varphi_{ki}}\right)^{\varepsilon-1}}, \quad i,j = H,F \tag{8}$$

$$\left(\frac{\varphi_{ii}}{\varphi_{ij}}\right) = \left(\frac{f_{ii}}{f_{ij}}\right)^{\frac{1}{\varepsilon-1}} \left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{W_i}{W_j}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ij}^{-1} \tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i,j = H,F, \quad i \neq j \tag{9}$$

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})), \quad i = H,F \tag{10}$$

$$C_{ij} = \left(\frac{\varepsilon-1}{\varepsilon}\right)(\varepsilon f_{ij})^{\frac{-1}{\varepsilon-1}} \tau_{ij}^{-1} \varphi_{ij} \left(\delta_{ij} L_{Cj}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{11}$$

$$P_{ij} = \left(\frac{\varepsilon}{\varepsilon-1}\right)(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{ij}\tau_{Tij}\tau_{Lj} W_j \varphi_{ij}^{-1} \left(\delta_{ij} L_{Cj}\right)^{\frac{-1}{\varepsilon-1}}, \quad i,j = H,F \tag{12}$$

$$L - L_{Ci} - \frac{(1-\alpha)}{\alpha} \sum_{k=H,F} (P_{ik}C_{ik}) + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{13}$$

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij} \tag{14}$$

$$Z_i = \frac{(1-\alpha)}{\alpha} \sum_{j=H,F} P_{ij} C_{ij} \quad i = H,F \tag{15}$$

Condition (7) defines the average productivity of country-i firms active in market j ($\tilde{\varphi}_{ji}$), given by the harmonic mean of productivity of those firms that operate in the respective market. Condition (8) defines δ_{ji}, the variable-profit share of a country-i firm with average productivity $\tilde{\varphi}_{ji}$ arising from sales in market j.[9] Equivalently, δ_{ji} is also the share of total labor used in the differentiated sector in country i that is allocated to production for market j. Condition (9) follows from dividing the zero-profit conditions defining the survival-productivity cutoffs – which imply zero profits for a country-i firm with the cutoff-productivity level φ_{ij} from selling in market j – for firms in their domestic market by the one for foreign firms that export to the domestic market. Condition (10) is the free-entry condition combined with the zero-profit conditions. In equilibrium, expected variable profits (left-hand side) have to equal the expected overall fixed cost bill (right-hand side).

Condition (11) can be interpreted as a sectoral aggregate production function $C_{ij} = Q_{Cij}(L_{Cj})$ in terms of aggregate labor allocated to the differentiated sector, L_{Cj}, measuring the amount of production of the aggregate bundle produced in country j for consumption in market i. Condition (12) defines the equilibrium consumer

[9] It can be shown that $f_{ji}(1-G(\varphi_{ji}))\left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}$ are variable profits of a the average country-i firm active in market j.

price index P_{ij} of the aggregate differentiated bundle produced in country j and sold in country i.[10]

Importantly, condition (13) defines the trade-balance condition that states that the value of net imports of the homogeneous good plus the value imports of the differentiated bundle (left-hand side) must equal the value of exports of the differentiated bundle (right-hand side). Note that imports and exports of differentiated bundles are evaluated at international prices (before tariffs are applied). The model-consistent definition of the terms of trade then follows directly from this equation.[11] The international price of imports $\tau_{Ii}^{-1} P_{ij}$ defines the inverse of the terms of trade of the differentiated importable bundle (relative to the homogeneous good), while the international price of exports $\tau_{Ij}^{-1} P_{ji}$ defines the terms of trade of the differentiated exportable bundle (relative to the homogeneous good). In addition, the terms of trade of the differentiated exportable relative to the importable bundle are given by $(\tau_{Ij}^{-1} P_{ji})/(\tau_{Ii}^{-1} P_{ij})$, which is the only relevant relative price when $\alpha = 1$. Given that terms of trade are defined in terms of sectoral ideal price indices of exportables relative to importables, they will be affected not only by changes in the prices of individual varieties but also by changes in the measure of exporters and importers and their average productivity levels. We will discuss this in detail in Section 5.2.

Finally, (14) is the the market-clearing condition for the homogeneous good[12] and condition (15) defines demand for the homogeneous good, presented here for future reference.

We normalize the foreign wage, W_i, $i = F$, to unity. Thus, we have a system of 24 equilibrium equations in 25 unknowns, namely δ_{ji}, φ_{ji}, $\widetilde{\varphi}_{ji}$, C_{ji}, P_{ij}, L_{Ci}, Z_i for $i, j = H, F$ and W_i for $i = H$. To close the model we note that if $\alpha < 1$, so that there exists a homogeneous sector, $W_i = 1$ for $i = H$, since factor prices must be equalized in equilibrium; by contrast, if $\alpha = 1$, so that there is only a single sector, $L_{Ci} = L$ for $i = F$, since the domestic labor market must clear.

Observe that when $\alpha < 1$ the equilibrium can be solved recursively. First, we can use conditions (7), (9) and (10) to implicitly determine the four productivity cutoffs φ_{ji} as well as the average productivity levels in the domestic and export market for both countries $\tilde{\varphi}_{ji}$, given the values of the policy instruments. Then, we can determine the variable profit shares of the average firm in each market δ_{ji}, since they are a function of the productivity cutoffs only. Finally, given both φ_{ji} and δ_{ji}, we can recover C_{ji} and P_{ij} and plug them into the trade-balance condition and the homogeneous-good-market-clearing condition to solve for the equilibrium levels of L_{Cj}.

Moreover, note that under some additional assumptions the equilibrium equations also nest the one-sector and the multi-sector versions of the Krugman (1980) model with homogeneous firms and exogenous productivity level $\varphi = 1$. A sufficient set of assumptions is that $f_{ji} = 0$ for $i, j = H, F$ and $G(\varphi)$ is degenerate at unity. In this case, conditions (7), (8) (9) and (10) need to be dropped from the set of equilibrium conditions and (11)

[10] More precisely, if $\alpha < 1$, P_{ij} should be interpreted as a relative aggregate price index in terms of the homogeneous good.

[11] This definition is also consistent with Campolmi et al. (2012) and Costinot et al. (2016), who also define terms of trade in terms of aggregate international price indices of exportables and importables.

[12] Alternatively if $\alpha = 1$ it states the domestic labor-market-clearing condition.

and (12) are replaced by

$$C_{ij} = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} \frac{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{-\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{Lj}^{-\varepsilon} \tau_{ij}^{-1} \tau_{Tji}^{\varepsilon} \tau_{Tij}^{-\varepsilon} \left(\frac{W_i}{W_j} \right)^{\varepsilon} \right]}{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \right]}, \quad i, j = H, F \tag{16}$$

$$P_{ij} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) (\varepsilon f_E)^{\frac{1}{\varepsilon-1}} (\tau_{ij} \tau_{Tij} \tau_{Lj}) W_j L_{Cj}^{\frac{-1}{\varepsilon-1}}, \quad i, j = H, F. \tag{17}$$

The remaining equilibrium conditions (13)-(15) remain valid.

3 The Planner Allocation

In this section we solve the problem of the world social planner who maximizes total world welfare[13] given the constraints imposed by the production technology in each sector and the aggregate labor resources available to each country. The solution to this problem provides a benchmark against which one can compare the market allocation, and the allocations implied by the world policy maker's and individual countries' policies. Moreover, and more importantly, it identifies the wedges between the market and the Pareto-efficient allocation which – as explained in Section 4 – will exactly correspond to the efficiency wedges in our welfare decomposition.

We solve the planner problem in three stages,[14] using the total-differential approach. First, we determine the consumption and labor allocated to each variety of the differentiated good consumed and produced in each location. Next, we solve for the optimal domestic and export productivity-cutoffs, the optimal allocation across varieties within sectors, and the measure of differentiated varieties given the allocation across sectors. Finally, in the third stage we find the optimal allocation of consumption and labor across aggregate sectors. There are two main advantages in following this three-stage approach: (i) it highlights the various trade-offs that the planner faces at the micro and the macro level; (ii) it allows comparing the planner and the market allocation in a transparent way by pointing to the specific wedges arising in the market allocation at each level of aggregation.

3.1 First stage: optimal production of individual varieties

At the first stage the planner chooses $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij} for $i, j = H, F$ by solving the following problem:[15]

[13] World welfare is defined as the unweighted sum of of individual countries' welfare. In this way we single out the symmetric Pareto-efficient allocation.

[14] Our approach is similar to the one of Costinot et al. (2016) for the unilateral optimal-policy problem.

[15] The results of this Section are derived in Appendix B.2.

$$\max u_{ij} \tag{18}$$
$$s.t. \quad c_{ij}(\varphi) = q_{ij}(l_{ij}(\varphi)), \quad i,j = H,F$$
$$L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i,j = H,F,$$

where $u_{ij} \equiv C_{ij}$, $C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi)\right]^{\frac{\varepsilon}{\varepsilon-1}}$, $q_{ij}(l_{ij}(\varphi)) = (l_{ij}(\varphi) - f_{ij})\frac{\varphi}{\tau_{ij}}$, and N_j and L_{Cij} – defining the amount of labor allocated in country j to produce differentiated goods consumed by country i – are taken as given since they are determined at the second stage. The optimality conditions of the problem (18) imply the equalization of the marginal value product (measured in terms of marginal utility) between any two varieties φ_1 and $\varphi_2 \in [\varphi_{ij}, \infty)$.[16]

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi_1)} \frac{\partial q_{ij}(\varphi_1)}{\partial l_{ij}(\varphi_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\varphi_2)} \frac{\partial q_{ij}(\varphi_2)}{\partial l_{ij}(\varphi_2)}, \quad i,j = H,F \tag{19}$$

The solution to this problem also determines the consumption of individual varieties $c_{ij}(\varphi)$, the amount of labor allocated to the production of each variety $l_{ij}(\varphi)$ and the optimal sectoral labor aggregator L_{Cij} and allows us to obtain the sectoral aggregate production function[17]

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1 - G(\varphi_{ij})]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij}[N_j(1 - G(\varphi_{ij})]^{\frac{\varepsilon}{\varepsilon-1}} \right\}, \quad i,j = H,F. \tag{20}$$

In addition, the optimality conditions of the first stage are satisfied in any market allocation and independent of policy instruments . This implies that the relative production levels of individual varieties are optimal in any market allocation.

3.2 Second Stage: optimal choice of aggregate bundles within sectors

At the second stage, the planner chooses C_{ij}, L_{Cij}, N_i and $\widetilde{\varphi}_{ij}$ for $i,j = H,F$ in order to solve the following problem:[18]

$$\max \sum_{i=H,F} u_i \tag{21}$$
$$s.t. \quad L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i = H,F$$
$$C_{ij} = Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij}), \quad i,j = H,F,$$

[16] Equivalently, this condition sets the marginal rate of substitution between any two varieties equal to their marginal rate of transformation.

[17] Note that condition (19) is also satisfied in the case of homogeneous firms. In this case equation (20) holds with $\tilde{\varphi}_{ij} = 1$, $(1 - G(\varphi_{ij})) = 1$ and $f_{ij} = 0$.

[18] The results of this Section are derived in Appendix B.3.

where $u_i = \log C_i$, C_i is given by (4) and $Q_{ij}(\tilde{\varphi}_{ij}, N_i, L_{Cij})$ is defined in (20). The first-order conditions of the above problem lead to the following conditions:

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i,j = H,F, \quad i \neq j \tag{22}$$

$$f_E = \sum_{j=H,F} \frac{\partial Q_{Cji}/\partial N_i}{\partial Q_{Cji}/\partial L_{Cji}}, \quad i = H,F \tag{23}$$

$$\frac{\partial Q_{Cji}}{\partial \tilde{\varphi}_{ji}} = 0, \quad i,j = H,F \tag{24}$$

Condition (22) states that the marginal value product of labor of the bundle produced and consumed domestically (measured in terms of domestic marginal utility) has to equal the marginal value product of labor of the domestic exportable bundle (measured in terms of foreign marginal utility).[19]

Condition (23) captures the trade-off between the extensive and intensive margins of production. Creating an additional variety (firm) requires f_E units of labor in terms of entry cost. This additional variety marginally increases output of the domestically produced and consumed and the exportable bundles at the extensive margin but comes at the opportunity cost of reducing the amount of production of existing varieties (intensive margin), since aggregate labor has to be withdrawn from these production activities.

Condition (24) reveals the trade-off between increasing average productivity and reducing the number of active firms. From the aggregate production function (20), an increase in $\tilde{\varphi}_{ji}$ on the one hand increases sectoral production by making the average firm more productive, on the other hand it decreases sectoral production by reducing the measure of firms that are above the cutoff-productivity level φ_{ji}. At the margin, these two effects have to offset each other exactly.

By combining (20), (23) and (24), we obtain a sectoral production function for Q_{Cji} in terms of aggregate labor L_{Ci}:

$$Q_{Cji}(L_{Ci}) = \left(\frac{\varepsilon - 1}{\varepsilon}\right) (\varepsilon f_{ji})^{\frac{-1}{\varepsilon-1}} \tau_{ji}^{-1} \varphi_{ji} \left(\delta_{ji} L_{Ci}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{25}$$

This equation corresponds exactly to condition (11) for the market allocation. Thus, consumption of the aggregate differentiated bundles is efficient in any market allocation conditional on the cutoffs φ_{ji} and the amount of aggregate labor allocated to the differentiated sector L_{Ci}.[20]

We now compare the planner's optimality conditions for the second stage with those of the market allocation. Even in a symmetric market allocation condition (22) is not satisfied, i.e., there is a wedge between the marginal

[19] Equivalently, this condition states that the marginal rate of substitution (in terms of home versus foreign utility) between the domestically produced and consumed bundle and the domestic exportable bundle has to equal the marginal rate of transformation of these bundles.

[20] Note that the planner's optimality conditions are also valid for the case of homogeneous firms with $\tilde{\varphi}_{ij} = 1$ exogenous, so that the first-order conditions are given by (22) and (23) only. Equation (25) also holds with $f_{ij} = f_E$, $\varphi_{ij} = 1$ and $\delta_{ji} = \left[1 + \tau_{ij}^{\varepsilon-1}\left(\frac{C_j}{C_i}\right)^{\varepsilon-1}\right]^{\frac{1-\varepsilon}{\varepsilon}}$.

value product of labor of the domestically produced and consumed bundle and the marginal value product of labor of the exportable bundle (both evaluated in terms of marginal utility of the consuming country) whenever $\tau_{Tji} \neq 1$:

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}} (\tau_{Tji})^{1-\varepsilon}, \quad i, j = H, F, \quad i \neq j. \tag{26}$$

Thus, a foreign tariff ($\tau_{Ij} > 1$) or a domestic export tax ($\tau_{Xi} > 1$) imply that the marginal value product on the right-hand side must increase relative to the one on the left-hand side, which happens when foreign consumers reduce imports and home consumers increase consumption of the domestically produced bundle. As a result, production and consumption is inefficiently tilted towards the domestically produced bundle.[21] By contrast, one can show that conditions (23)[22] and (24) are satisfied in any market allocation.

Alternatively, one can also compare the planner solution with the market outcome in terms of allocations. For the heterogeneous-firm model it turns out that efficiency of the cutoff-productivity levels φ_{ij} is sufficient for the market allocation to coincide with the planner solution for the second stage. Simultaneously, a distortion of the cutoffs implies that all equilibrium outcomes are distorted. Notice that according to conditions (7)-(10) distortions of the cutoffs are exclusively due to trade taxes. From (9), in a symmetric equilibrium, the cutoff-productivity levels in the market are determined by:

$$\left(\frac{\varphi_{ii}}{\varphi_{ji}}\right) = \left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ji}^{-1} \tau_{Tji}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i = H, F, \quad j \neq F$$

Thus, the cutoff-productivity levels φ_{ii} and φ_{ji} are efficient in the free-trade allocation, i.e. $\varphi_{ji}^{FT} = \varphi_{ji}^{FB}$, while there is a distortion induced on φ_{ji} whenever trade taxes are used (when $\tau_{Tji} = \tau_{Ij}\tau_{Xi} \neq 1$). In particular, $\tau_{Tji} > 1$ implies that $\varphi_{ii}/\varphi_{ji}$ is too small relative to the efficient level, so that the marginal exporter is too productive relative to the least productive domestic producer.

3.3 Third stage: allocation across sectors

The third stage is present only in the case of multiple sectors ($\alpha < 1$). In this stage, the planner chooses C_{ij} and Z_i for $i, j = H, F$, and the amount of aggregate labor allocated to the differentiated sector L_{Ci} to solve the

[21] Observe that this wedge is present independently of firm heterogeneity.

[22] This condition is satisfied in any market allocation as long as the same labor tax is charged on marginal and fixed costs.

following maximization problem:[23]

$$\max \sum_{i=H,F} U_i \tag{27}$$

$$\begin{aligned} s.t. \quad C_{ij} &= Q_{Cij}(L_{Cj}), \quad i,j = H,F \\ Q_{Zi} &= Q_{Zi}(L - L_{Ci}), \quad i = H,F \\ \sum_{i=H,F} Q_{Zi} &= \sum_{i=H,F} Z_i, \end{aligned}$$

where U_i is given by (3) and (4), $Q_{Zi}(L - L_{Ci}) = L - L_{Ci}$ and $Q_{Cij}(L_{Cj})$ is defined in (25).

The first-order conditions of this problem are given by:

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, j = F \tag{28}$$

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H,F \tag{29}$$

Condition (28) states that the marginal utility of the homogeneous good has to be equalized across countries, implying that $Z_i = Z_j$, so that there is no trade in the homogeneous good due to symmetry. Since $\partial Q_{Zi}/\partial L_{Ci} = -\partial Q_{Zi}/\partial L_{Zi}$, condition (29) states that the marginal value product of each country's aggregate labor,[24] evaluated with the marginal utility of the consuming country, has to be equalized across the three sectors.

Note that since the aggregate representation of the model does not depend on firm heterogeneity, the planner's optimality conditions for the third stage with homogeneous firms are identical to those above.

Condition (28) is satisfied in a symmetric market allocation, while (29) is – in general – violated by the market allocation unless (i) $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and (ii) $\tau_{Tij} = 1$ for $i,j = H,F$. Thus, the cross-sectoral allocation of labor is inefficient unless labor subsidies equal to the inverse of the monopolistic markup are set by both countries and trade taxes are not used.

Finally, we compare the planner solution of the third stage with the market allocation.[25]

$$L_{Ci}^{FT} = \alpha L < L_{Ci}^{FB} = \frac{\alpha \varepsilon L}{\alpha + \varepsilon - 1}, \quad i = H,F \tag{30}$$

$$N_i^{FT} = \frac{\alpha L}{\varepsilon \sum_{j=H,F} \left[f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} \right]} < N_i^{FB} = \frac{\alpha L}{(\alpha + \varepsilon - 1) \sum_{j=H,F} \left[f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} \right]}, \quad i = H,F$$

[23]We state the third stage of the planner problem as a choice between C_{ij} and Z_i (instead of a choice between C_i and Z_i) because this enables us to identify the efficiency wedges in the welfare decomposition, as will become clear below. The results of this Section are derived in Appendix B.4.

[24]By construction aggregate labor already incorporates the optimal split of labor in the differentiated sector between the domestically produced and consumed and the exportable bundles.

[25]In the case of homogeneous firms, $N_i^{FT} = \frac{\alpha L}{\varepsilon f} < N_i^{FB} = \frac{\alpha L}{(\alpha+\varepsilon-1)f}$.

According to condition (30), the wedge in the marginal value product of labor between the homogeneous and the differentiated sectors implies that the market allocates too little labor to the differentiated sector ($L_{Ci}^{FT} < L_{Ci}^{FB}$) and thus provides too little variety ($N_i^{FT} < N_i^{FB}$). This reflects the fact that the monopolistic markup in the differentiated sector depresses its relative demand.

3.4 Pareto efficiency and market equilibrium

The following Proposition summarizes all optimality conditions that need to be satisfied in order for the market allocation to be Pareto optimal.

Proposition 1 ***Relationship between the planner and the market allocation***

The market equilibrium coincides with the planner allocation if and only if:

(a)

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i,j = H,F \quad i \neq j$$

(b) and (for the multi-sector model only)

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, j = F$$

$$\sum_{j=H,F}\frac{\partial U_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i}\frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H,F$$

Proof See Appendix B.5. ■

Proposition 1 implies that the solution to the first stage of the planner problem always coincides with the market allocation, so that the relative production of individual firms is optimal in any market equilibrium. By contrast, for the market equilibrium to coincide with the solution to the second stage of the planner problem, trade taxes cannot be operative because this would distort the consumption choice of the importable bundles. Finally, the solution to the third stage of the planner problem (which is present only in the multi-sector model) coincides with the market solution only when the amount of labor allocated across the differentiated sector is efficient. This requires eliminating distortions from monopolistic markups with labor subsidies, as shown in the next section.

4 The World Policy Maker Problem and the Welfare Decomposition

In the previous section we have derived the symmetric Pareto-efficient allocation chosen by the world planner and have compared it with the market allocation. In this section, we solve the problem of the world policy maker who maximizes the sum of individual-country welfare and has all three policy instruments (labor, import and export taxes) at his disposal. We show that, by solving the world-policy-maker problem using a total-differential approach, we can obtain a welfare decomposition that: (i) incorporates all general-equilibrium effects of setting policy instruments under cooperation; (ii) and consequently allows separating policy makers' incentives driven by pure beggar-thy-neighbor motives from efficiency considerations.

The world policy maker sets domestic and foreign policy instruments τ_{Li}, τ_{Ii} and τ_{Xi} in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji},\varphi_{ji},\widetilde{\varphi}_{ji},C_{ji},W_i \\ P_{ij},L_{Ci},\tau_{Li},\tau_{Ii},\tau_{Xi}\}_{i,j=H,F}}} \sum_{i=H,F} U_i \qquad (31)$$

$$\textit{subject to conditions } (7)\text{-}(14).$$

where U_i is defined in (3), (4) and (15) with the additional restrictions that $\tau_{T,i}$ for $i = H, F$ is as defined in (6), $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$.[26]

As a first step, solving the world policy maker problem using the total-differential approach involves taking total differentials of (31) and the equilibrium equations. We then substitute the total differentials of the equilibrium equations into the objective to obtain the following welfare decomposition:

Proposition 2 ***Decomposition of world welfare***[27]

The total differential of world welfare in (31) *can be decomposed as follows:*

$$\sum_{i=H,F} dU_i = \qquad (32)$$

$$\underbrace{\sum_{i=H,F}\frac{(1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij}}{I_i}}_{\textit{consumption- efficiency wedge}} + \underbrace{\sum_{i=H,F}\frac{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci}}{I_i}}_{\textit{production- efficiency wedge}} + \underbrace{\sum_{i=H,F}\frac{C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}{I_i}}_{\textit{terms-of-trade effect}} \quad j \neq i$$

where $I_i = W_i L + T_i$ *is household income.*

Proof See Appendix C.1. ■

This decomposition implies that changes in world welfare are given by the sum of the differentials of individual countries' welfare and can be written as the sum of three terms: a *(i) consumption-efficiency wedge*; a *(ii) production-efficiency wedge* and *(iii) terms-of-trade effects*. Note that this decomposition is valid both with heterogeneous and homogeneous firms.

The following Proposition allows interpreting the terms of the welfare decomposition.

[26] In the case of homogeneous firms, conditions (7)-(10) need to be dropped and (11)-(12) are replaced by (16) and (17).

[27] A predecessor of this welfare decomposition can be found in chapter 1 of Helpman and Krugman (1989).

Proposition 3 ***Terms in the welfare decomposition***

(i) The consumption-efficiency wedge is present in the one-sector and in the multi-sector model. In the market equilibrium this term correspond to:

$$(1-\tau_{Xi})P_{ii} = \frac{\frac{\partial U_i}{\partial C_{ii}}}{\frac{\partial U_i}{\partial C_{ij}}} P_{ij} - \tau_{Xi} P_{ii}, \quad i = H, F \quad j \neq i \tag{33}$$

$$(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij} = \frac{\frac{\partial U_i}{\partial C_{ij}}}{\frac{\partial U_i}{\partial C_{ii}}} P_{ii} - \tau_{Ii}^{-1} P_{ij}, \quad i = H, F \quad j \neq i \tag{34}$$

i.e., it equals the wedges between the marginal rates of substitution of domestic and importable differentiated bundles and their corresponding international relative prices. Hence, this term measures the welfare effect of distortions in the relative choice of the domestically produced and consumed versus the imported differentiated bundle.

(ii) The production-efficiency wedge is present only in the multi-sector model ($\alpha < 1$). This term corresponds to:

$$\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1 = \tau_{Xi}P_{ii}\frac{\partial Q_{Cii}}{\partial L_{Ci}} + \tau_{Ij}^{-1}P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} + \frac{\partial Q_{Zi}}{\partial L_{Ci}} \quad i = H, F \quad j \neq i \tag{35}$$

i.e., it equals the wedge between the marginal value product of labor in the differentiated sector and the homogeneous-good sector evaluated at international prices. Hence, it measures welfare effects of distortions due to the mis-allocation of labor across sectors.

(iii) The terms-of-trade terms are present in the one-sector and in the multi-sector model. They measure the welfare effects due to changes in the terms of trade of both countries and sum to zero in any symmetric equilibrium .

Proof See Appendix C.2. ■

The *consumption-efficiency wedge* consist of the difference between the domestic and the international consumer price of the domestically consumed and produced bundle, $(1-\tau_{Xi})P_{ii}$, times the differential in its consumption plus the difference between the domestic and the international price of the imported bundle, $(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}$, times the differential in its consumption. Proposition 3 provides an economic interpretation for the *consumption-efficiency wedge* in (32). It represents the wedges between the marginal rates of substitution of the domestically produced and consumed and the importable differentiated bundles and their corresponding international relative prices. In order to avoid consumption wedges, the world policy maker should abstain from using trade taxes, which distort consumption of the domestically produced and consumed versus the importable bundle relative to their first-best levels. For instance – since consumers always set the marginal rate of substitution of C_{ii} and C_{ij} equal to the bundles' relative price P_{ii}/P_{ij} – in the presence of an export tax ($\tau_{Xi} > 1$) the first wedge is negative. This implies that consumers allocate too much consumption to the domestically produced and consumed bundle and too little to the imported one. Similarly, in the presence of a tariff ($\tau_{Ii} > 1$) the second wedge is positive, implying that the marginal rate of substitution of the importable and the domestically produced and consumed bundle is larger than their international price $\tau_{Ii}^{-1} P_{ij}$. Thus, consumers allocate too little consumption to the imported bundle and too much to the domestically produced one.

The *production-efficiency wedge* is present only in the case of multiple sectors ($\alpha < 1$). It consists of the difference between the international producer price of individual varieties and the international price of the homogeneous good times the differential of labor allocated to the differentiated sector. According to Proposition 3, these terms measure the difference of the marginal value product of labor between the differentiated and the homogeneous sector. This term determines whether the allocation of labor across sectors is efficient. To guarantee production efficiency, the world policy maker should set the labor subsidy τ_{Li} or the export subsidy τ_{Xi} equal to the inverse of the firms' mark up, $(\varepsilon - 1)/\varepsilon$.

Finally, the *terms-of-trade* terms consist of the differentiated exportable bundle times the differential of its international price minus the differentiated importable bundle times the differential of its international price. An increase in the price of exportables raises domestic welfare and decreases welfare in the other country, while an increase in the price of importables has the opposite effects. At the optimum the domestic and foreign terms-of-trade effects exactly compensate each other, so that the differential of world welfare consists exclusively of the *consumption efficiency* and the *production efficiency* terms.

Observe that the welfare decomposition in (32) holds independently of the number of instruments at the disposal of the world policy maker. However, as made clear by the next Proposition, if the world policy maker can set all three policy instruments at a time, setting all the terms in the welfare decomposition in (32) individually equal to zero is in fact the solution to the policy problem in (31). It identifies the optimal coordinated policy that implements the symmetric Pareto-efficient allocation as a market equilibrium.

Proposition 4 ***Optimal world policies and Pareto efficiency***

When labor, import and export taxes are available,

(a) Solving the world-policy-maker problem in (31) *by using the total-differential approach requires to set each of the wedges in* (32) *equal to zero.*

(b) As a result, under the optimal policy:

(i) $\tau_{Xi} = 1 \iff \frac{\frac{\partial U_i}{\partial C_{ii}}}{\frac{\partial U_i}{\partial C_{ij}}} P_{ij} = \tau_{Xi} P_{ii}$ *and* $\tau_{Ii} = 1 \iff \frac{\frac{\partial U_i}{\partial C_{ij}}}{\frac{\partial U_i}{\partial C_{ii}}} P_{ii} = \tau_{Ii} P_{ij}$ *for* $i = H, F$ *and* $j \neq i$, *i.e., the marginal rates of substitution of domestic and importable differentiated bundles are equal to their corresponding international relative prices.*

(ii) if $\alpha < 1$, $\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} = 1 \iff \tau_{Xi} P_{ii} \frac{\partial Q_{Cii}}{\partial L_{Ci}} + \tau_{Ij}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial Q_{Zi}}{\partial L_{Ci}}$ *for* $i = H, F$ *and* $j \neq i$, *i.e., the marginal value product of labor in the differentiated sector is equal to the marginal value of product in the homogeneous-good sector evaluated at international prices.*

(iii) if $\alpha = 1$, $\tau_{Xi} = \tau_{Ii} = \tau_{Li} = 1$, *i.e., in the one-sector model the free-trade allocation is optimal and if* $\alpha < 1$, $\tau_{Xi} = \tau_{Ii} = 1$ *and* $\tau_{Li} = \frac{\varepsilon - 1}{\varepsilon}$ *for* $i = H, F$ *i.e., in the presence of multiple sectors the world*

policy maker abstains from using trade taxes and uses the labor subsidy to offset the monopolistic distortion in the differentiated sector.

(c) The world policy maker implements the planner allocation given that:

(i) by (b) (i) the second-stage wedge is closed, i.e.:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i = H, F \quad j \neq i$$

(ii) in the presence of multiple sectors by combining (b) (i) and (b) (ii) also the third-stage wedges are closed, i.e.:

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, j = F$$
$$\sum_{j=H,F}\frac{\partial U_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i}\frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F$$

Proof See Appendix C.4. ∎

The interpretation of Proposition 4 is straightforward. In the one-sector model changes in world welfare are given exclusively by *consumption efficiency wedges*. *Production efficiency* is always guaranteed in any market allocation of the one-sector model: the monopolistic markup does not induce any cross-sectoral distortions in the allocation of labor because of inelastic labor supply. Note that for the case of heterogeneous firms implementing consumption efficiency is enough to ensure that all productivity cutoffs φ_{ij} are optimal, since this corresponds to closing the gap between the marginal rate of substitution and transformation within the differentiated sector in the second stage of the planner problem (equation (22)). This implies that the productivity cutoffs φ_{ij} are optimal in the free-trade allocation but are distorted whenever trade taxes are used. Consequently, L_{Ci} and N_i are also efficient. Hence, for the case of the one-sector model the free-trade allocation coincides with the planner allocation.

By contrast, in the market equilibrium of the multi-sector model, monopolistic markups do cause distortions in the allocation of labor across sectors: the size of the differentiated sector is too small, whereas the size of the homogeneous sector is too large. Consequently, the world policy maker can fully restore Pareto efficiency by providing a labor subsidy to the differentiated sector. Again, implementing consumption efficiency is enough to ensure that all productivity cutoffs φ_{ij} are optimal. This implies that the productivity cutoffs φ_{ij} are optimal in the free-trade allocation but are distorted whenever trade taxes are used. Finally, it follows that only the measure of firms that try to enter the differentiated sector, N_i, is too low in the free-trade allocation.

Proposition 4 confirms the message from the planner problem: achieving production and consumption efficiency requires abstaining from the use of trade taxes and, in the case of the multi-sector model, offsetting markups by subsidizing labor. Most importantly, a key result is contained in points (a) and (c) of Proposition 4: closing the wedges in the welfare decomposition (32) one by one is a necessary and sufficient condition for implementing the Pareto-efficient allocation, which is the allocation chosen by the world policy maker. This makes clear that the welfare decomposition in (32) internalizes all the equilibrium constraints binding the world policy maker's optimal decisions in problem (31). For this reason, this decomposition incorporates *all* general equilibrium effects and *all* relevant tradeoffs that the world policy maker faces when setting optimal policies. Finally, since the world policy maker is concerned with implementing an efficient allocation, the terms in the above welfare decomposition correspond unequivocally to consumption and production efficiency wedges.

5 Policy from the Individual Country Perspective

5.1 Individual Country Policy Maker Problem and the Welfare Decomposition

We now turn to the welfare incentives of policy makers that are concerned with maximizing the welfare of individual countries and have either all policy instruments (labor and trade taxes) or a subset of them available.

The individual-country policy maker sets domestic policy instruments $\mathcal{T}_i \in \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}\}_{i,j=H,F}, \mathcal{T}_i}} U_i \tag{36}$$

$$\textit{subject to conditions } (7)\text{-}(14).$$

where $\mathcal{T}_i \in \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ for $i = H, F$ and taking as given $\{\tau_{Lj}, \tau_{Ij}, \tau_{Xj}\}$, with $j \neq i$. U_i is defined in (3), (4) and (15) with the additional restrictions that $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$.[28]

Again, as a first step for solving the individual-country policy maker problem given foreign policy instruments, we take total differentials of the objective function and the constraints and substitute them into the differential of the objective in order to obtain the welfare decomposition for individual countries. We will then use this decomposition to solve for the Nash equilibrium of the policy game in Subsection 5.3.

Proposition 5 ***Decomposition of individual-country welfare***

[28] In the case of homogeneous firms, conditions (7)-(10) need to be dropped and (11)-(12) are replaced by (16) and (17).

The total differential of individual-country welfare in (36) *can be decomposed as follows:*

$$dU_i = \tag{37}$$

$$\underbrace{\frac{(1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij}}{I_i}}_{\text{consumption-efficiency wedge}} + \underbrace{\frac{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci}}{I_i}}_{\text{production-efficiency wedge}} + \underbrace{\frac{C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}{I_i}}_{\text{terms-of-trade effect}}, \quad j \neq i$$

where $I_i = W_i L + T_i$ *is household income.*

Proof See Appendix D.1. ■

Note that this welfare decomposition is valid both with homogeneous and heterogeneous firms and independently of the number of policy instruments that the domestic policy maker has at her disposal. Individual-country policy makers care about domestic consumption efficiency and production efficiency. Moreover, they also take into account the terms-of-trade effects of their policy choice, as these are not zero, even in the symmetric equilibrium. Comparing the welfare decomposition with the one of the world policy maker allows us to separate incentives aiming at achieving efficiency from beggar-thy-neighbor motives.

From Proposition 4 (b) (iii) we know that in the one-sector model the free-trade allocation is optimal from the perspective of the world policy maker. It is implemented by abstaining from the use of taxes, which allows setting the *consumption-efficiency* term equal to zero. However, for the individual-country policy maker, this allocation is not optimal because of the presence of terms-of-trade effects. It thus follows that: first, any deviation from the free-trade allocation must be due to terms-of-trade effects; second, any such deviation is a *pure beggar-thy-neighbor* policy, defined as an increase in domestic welfare that is compensated by an equal fall in the foreign one, i.e., a zero-sum game, because foreign terms-of-trade effects equal the opposite of their domestic counterpart.[29]

For the case of the multi-sector model, we know from Proposition 4 (b) (iii) that the world policy maker sets a labor subsidy in order to implement production efficiency. By contrast, the individual-country policy maker not only has the objective of achieving domestic production efficiency but also tries to manipulate domestic terms of trade in her favor. Thus, her incentives to deviate from free trade are due to a combination of *production efficiency* and *terms-of-trade* incentives. Consequently, their policy choices aim both at improving efficiency and at achieving *pure beggar-thy-neighbor* effects. Observe that policies aiming at improving efficiency may also induce externalities on the other country as a side effect.

[29] Costinot et al. (2016) also emphasize that in the one-sector heterogeneous-firm model terms-of-trade effects are the only externality driving the incentives of individual-country policy makers. Differently from us, they define the concept of efficiency from the individual-country perspective as the allocation that a planner concerned with maximizing individual-country welfare chooses. However, this allocation generally does not coincide with the allocation that is Pareto efficient and thus does not allow gaining insights into the purpose of trade agreements.

The following Corollary summarizes these observations:

Corollary 1 ***Individual-country incentives***

(a) In the one-sector model, any deviations of individual-country policy makers from free trade are due to terms-of-trade effects.

(b) In the multi-sector model, individual-country policy makers' deviations from free trade are driven by terms-of-trade and production-efficiency effects.

(c) Terms-of-trade effects are the only pure beggar-thy-neighbor effects.

5.2 How Policy Instruments affect the Terms of Trade and Production Efficiency

Before studying strategic policies in Section 5.3, we first analyze how unilateral policy choices affect the terms of trade and the efficiency wedges and thereby the welfare of individual countries. We are particularly interested in explaining the different channels through which policy instruments impact on the terms of trade and efficiency. As mentioned previously, the macro terms of trade can be influenced both through changes in the international prices of individual exportable and importable varieties and through changes in the measure of exporters and importers.

Note that, when starting from a symmetric allocation, the impact of a unilateral policy change on the terms of trade can be written as:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \tag{38}$$

$$\tau_{Ii}^{-1}P_{ij}C_{ij}\left[\frac{d\tau_{Li}}{\tau_{Li}} + \frac{d\tau_{Xi}}{\tau_{Xi}} + \underbrace{\left(\frac{dW_i}{W_i} - \frac{dW_j}{W_j}\right)}_{(i)} + \frac{1}{\varepsilon-1}\left(\underbrace{\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}}}_{(ii)} + \underbrace{\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}}_{(iii)}\right) + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)}\right],$$

where deviations are defined as $dX_i/X_i = \frac{\partial X_i}{\partial \tau_{Ii}}\frac{1}{X_i}d\tau_{Ii}$. We discuss the impact of tariffs (i.e., $d\tau_{Li} = d\tau_{Xi} = 0$) in more detail and then provide results for the other instruments. Thus, a domestic tariff influences the terms of trade (i) by changing the relative wage; (ii) by affecting the amount of labor allocated to the differentiated sector in both countries; (iii) by impacting on the average variable profit share of domestic and foreign firms in their respective export markets; (iv) by moving the cutoff productivity levels of domestic and foreign exporters. Here, (i) corresponds to a a change in the price of individual varieties, while (ii)-(iii) correpond to changes in the measure of exportables and importables. Finally, (iv), the change in the cutoff productivity levels, impacts both on the average price of individual varieties and the measure of domestic and foreign exporters.[30] In particular, an increase in the domestic relative factor price raises the price of exported varieties relative to imported ones

[30] An alternative decomposition splits the price index of exportables and importables into an extensive margin $[N_i(1 - G(\varphi_{ji}))]^{\frac{1}{1-\varepsilon}} = \left(\frac{\delta_{ji}L_{Ci}}{\varepsilon f_{ji}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\varphi_{ji}}{\tilde{\varphi}_{ji}}\right)$ and an intensive margin $\tau_{Ij}^{-1}p_{ji}(\tilde{\varphi}_{ji}) = \frac{\varepsilon}{\varepsilon-1}(\tau_{ji}\tau_{Xi}\tau_{Li})\left(\frac{W_i}{\tilde{\varphi}_{ji}}\right)$. Thus, δ_{ji} and L_{Ci} only impact on the extensive margin, and W_i only impacts on the intensive margin, while φ_{ji} affects both margins.

and improves the terms of trade. By contrast, an increase in the amount of labor allocated to the domestic differentiated sector worsens the terms of trade by reducing the price index of exportables via an increase in the number of varieties, while an increase in foreign labor allocated to the differentiated sector improves them by reducing the price index of importables. Domestic terms of trade worsen with an increment in the average variable-profit share of domestic firms from exports and improve in the foreign share by changing the measure of firms that export to each market. Finally, an increase in the domestic cutoff-productivity level for exports worsens the terms of trade both by making the average exportable variety cheaper and by affecting the measure of exporters, whereas an increase in the foreign productivity cutoff has the opposite effect.

We first discuss the impact of a small unilateral tariff (i.e., $d\tau_{Li} = d\tau_{Xi} = 0$) in the one-sector model (i.e., $dL_{Cj} = dL_{Ci} = 0$), starting from free trade. In the presence of a single sector, the terms-of-trade effects of a small tariff are positive and given by[31]

$$P_{ij}C_{ij}\left[\underbrace{\left(\frac{dW_i}{W_i}-\frac{dW_j}{W_j}\right)}_{(i)>0}+(\varepsilon-1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}}-\frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)>0}+\underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}}-\frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)<0}\right]>0 \tag{39}$$

A tariff raises home's demand for domestically produced varieties and thus, ceteris paribus, home firms' profits and the demand for domestic labor. Since labor supply is completely inelastic in the one-sector model, home's relative wage needs to adjust upward in response to a positive change in labor demand ((i)>0), thereby reducing equilibrium profits of domestic firms. Moreover, the increase in relative domestic income increases the share of variable profits of firms from both countries made in home's domestic market, which improves home's terms of trade ((iii)>0). Finally, the increase in the relative domestic wage leads to tougher selection into exporting at home and less selection in the other country, which negatively impacts on home's terms of trade ((iv)<0). In the absence of firm heterogeneity the tariff exclusively raises home's relative wage. Firm heterogeneity leads to two additional and opposing effects: if heterogeneity mostly affects the variable-profit share from exports, terms of trade respond more to tariffs compared to the case of homogeneous firms; by contrast, if selection effects are large firm heterogeneity tends to reduce the response of the terms of trade by reducing the average price of exported varieties relative to the one of imported varieties. Note also that in the one-sector model production efficiency is always guaranteed, so the only incentive to deviate from free trade by setting a tariff is the positive terms-of-trade effect.

Finally, we consider the impact of a unilateral export tax. Differently from a tariff, an export tax increases the international price of individual varieties directly but reduces the demand for domestic labor and thus home's relative wage. In the presence of homogeneous firms, the first effect dominates leading to a terms-of-trade improvement. By contrast, with heterogeneous firms the direct increase in the international price of individual varieties is completely offset by a drop in home's relative wage, while the impact on domestic and foreign

[31] In Appendix D.4 we sign the contribution of each component to the terms-of-trade effect for the one-sector model.

labor shares and export cutoffs is symmetric, so that these effects compensate each other. Thus, the total terms-of-trade-effect of an export tax is zero. The following Lemma summarizes these results.

Lemma 1 ***Unilateral deviations from free trade in the one-sector model***

Consider a marginal unilateral increase in each trade policy instrument at a time, starting from the free-trade equilibrium, i.e., with $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Then:

(a) the production-efficiency wedge is zero for all policy instruments.

(b) the consumption-efficiency wedge is zero for all policy instruments.

(c) the terms-of-trade effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

(d) the total welfare effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

Proof See Appendix D.5. ■

We now turn to the multi-sector model (i.e., $dW_i = dW_j = 0$). In this case the terms-of-trade effect of a small tariff starting from free trade is negative and given by:[32]

$$P_{ij}C_{ij}\left[(\varepsilon-1)^{-1}\underbrace{\left(\frac{dL_{Cj}}{L_{Cj}}-\frac{dL_{Ci}}{L_{Ci}}\right)}_{(ii)<0}+(\varepsilon-1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}}-\frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)<0\Leftrightarrow\delta_{ii}>1/2}+\underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}}-\frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)>0\Leftrightarrow\delta_{ii}>1/2}\right]<0 \quad (40)$$

A small tariff increases home's demand for domestically produced varieties and thus, ceteris paribus, the profits of home firms and the demand for domestic labor. Since wages are pinned down by the linear outside sector and workers can freely move across sectors, labor supply is perfectly elastic. Therefore, home labor supplied to the differentiated sector surges in response to the positive change in labor demand, raising the measure of domestic firms and reducing their equilibrium profits. At the same time, foreign firms experience a drop in demand and profits, leading to a reduction in foreign labor employed in the differentiated sector. These effects impact negatively on home's terms of trade ((ii)<0). Moreover, in the presence of heterogeneous firms there are two additional effects, the sign of which depends on whether firms make the larger fraction of variable profits in the domestic ($\delta_{ii} > 1/2$) or in the export market ($\delta_{ii} < 1/2$). In the first case, the tariff increases the variable-profit share of home firms and decreases the variable-profit share of foreign firm made in their respective export markets, which worsens home's terms of trade ((iii)<0) (more home exporters and less foreign exporters). In addition, the tariff leads to less selection into exporting at home and more selection in the other country, which positively impacts on home's terms of trade ((iv)>0). When $\delta_{ii} < 1/2$ the signs of the last two effects switch,

[32] In Appendix D.4 we sign the contribution of each component to the terms-of-trade effect for the multi-sector model.

but the overall terms-of-trade effect of a small tariff deviation from free trade remains negative.

Because the tariff increases the amount of labor allocated to the differentiated sector, it induces a positive production-efficiency effect when starting from free trade and thus creates a trade-off between increasing production efficiency and worsening the terms of trade. Which of the two effects dominates in welfare terms depends again on δ_{ii}: when δ_{ii} is larger than one half, so that the domestic market is more important in terms of profits, production-efficiency effects are dominant. Intuitively, when firms sell mostly to the domestic market, welfare gains from improving the terms of trade are small and policy makers care mostly about domestic production efficiency.

Analogous results hold for export and labor taxes: they improve domestic terms of trade by shifting labor away from the differentiated sector, which simultaneously worsens domestic production efficiency. Again, the total welfare effect depends on the magnitude of δ_{ii}. The following Lemma summarizes these findings.

Lemma 2 ***Unilateral deviations from free trade in the multi-sector model*** *Consider a marginal unilateral increase in each policy instrument at a time starting from the free-trade equilibrium, i.e., with* $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$*. Then:*

(a) the production-efficiency wedge is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li}*.*

(b) the consumption-efficiency wedge is zero for all policy instruments.

(c) the terms-of-trade effect is negative for τ_{Ii} *and positive for* τ_{Xi} *and* τ_{Li}*.*

(d) the total welfare effect is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li} *if and only if* $\delta_{ii} \in \left(\frac{1}{2}, 1\right)$ *or when firms are homogeneous.*

Proof See Appendix D.6. ■

To summarize, the direction in which a particular policy instrument moves the terms of trade depends crucially on the elasticity of labor supply. Moreover, when considering unilateral deviations in the multi-sector model the qualitative impact of policy instruments on welfare also depends on firm heterogeneity. In particular, whether unilateral policy makers would like to set a tariff or an import subsidy depends on the variable profit share from domestic sales (analogous statements hold for the other policy instruments). However, below we show that optimal strategic policies are qualitatively independent of firm heterogeneity as long as policy makers dispose of sufficiently many instruments.

5.3 Strategic Trade and Domestic Policies

After having analyzed individual-country incentives to set taxes in the absence of retaliation, we now turn to the case of strategic interaction and let individual-country policy makers set policy instruments simultaneously and non-cooperatively. Since we already know from the welfare decomposition that in the one-sector model deviations from free trade are exclusively due to terms-of-trade effects, we focus here on the more interesting case of the multi-sector model, where both production-efficiency and terms-of-trade motives are present. Moreover, for the moment we allow both domestic policies τ_{Li} and trade policies τ_{Ii}, τ_{Xi}, for $i = H, F$ to be used. Individual-country policy makers solve the problem described in (36). Similarly to the world policy maker problem, the welfare decomposition in (37) holds independently of the number of instruments at the disposal of the individual-country policy maker and corresponds to the policy maker's objective. After substituting additional equilibrium conditions, it can be written in terms of three terms that are individually equal to zero at the optimum. Proposition 6 states this more formally and characterizes the symmetric Nash equilibrium.

Proposition 6 ***Strategic trade and domestic policies***

When labor, import and export taxes are available,

(a) It is possible to rewrite (37) *as follows:*

$$dU_i = \frac{1}{I_i}\left[\Omega_{Cii} dC_{ii} + \Omega_{Cij} dC_{ij} + \Omega_{LCi} dL_{Ci}\right]$$

where Ω_{Cii}, Ω_{Cij} and Ω_{LCi} are defined in Appendix D.2.

(b) Solving the individual-country policy maker problem in (36) *by using the total-differential approach requires setting $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$.*

(c) As a result, any symmetric Nash equilibrium in the multi-sector model with homogeneous or heterogeneous firms when both countries can simultaneously set all policy instruments entails the first-best level of labor subsidies, and inefficient import subsidies and export taxes. Formally,

$$\tau_L^{Nash} = \tfrac{\varepsilon-1}{\varepsilon},\ \tau_I^{Nash} < 1 \text{ and } \tau_X^{Nash} > 1.$$

Proof See Appendix D.7 ■

Our welfare decomposition allow us to interpret the Nash policy outcome stated in Proposition 6. Domestic policies are set efficiently even under strategic interaction and do not cause any beggar-thy-neighbor effects, while trade policy instruments are used to try to manipulate the terms of trade. As made clear by the discussion in the last sub-section, an import subsidy or an export tax both aim at improving the terms of trade. Consequently, even in the presence of domestic policies terms-of-trade externalities remain the only motive for signing a trade agreement. Proposition 6 extends the result of Campolmi et al. (2014) – who find that in the two-sector model with homogeneous firms strategic trade policy consists of first-best labor subsidies and inefficient import subsidies and export taxes – to the case of heterogeneous firms.

The result that labor subsidies are set so as to completely offset monopolistic distortions is an application of the targeting principle in public economics (Dixit, 1985). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. If the policy maker disposes of sufficiently many instruments to deal with each incentive separately, she uses the labor subsidy to address production efficiency. The trade policy instruments are instead used to exploit the terms-of-trade effect, which is the only remaining incentive according to our welfare decomposition. Because there are two relative prices (the one of the differentiated exportable bundle and the one of the differentiated imporable bundle relative to the homogeneous good) two trade-policy instruments are necessary to target both.

This result implies that firm heterogeneity neither adds further motives for signing a trade agreement beyond the classical terms-of-trade effect nor changes the qualitative results (import subsidies and export taxes) of the equilibrium outcome compared to the case with homogeneous firms. This finding is different from the unilateral case (Lemma 2), where the welfare effects of unilateral deviations in trade taxes depend on parameter values, as summarized by the share of variable profits from domestic sales. It also differs from the results of Costinot et al. (2016), who find in a slightly more general model that the optimal unilateral policy may not only be affected quantitatively but also qualitatively by firm heterogeneity.

Our finding that domestic policies do not introduce new motives for trade policy coordination is closely related to the conclusion of Bagwell and Staiger (1999) and Bagwell and Staiger (2016), who uncover that in a large class of economic models terms-of-trade motives are the only reason for signing a trade agreement.Proposition 6 extends their result (i) to models with monopolistic competition and heterogeneous firms; and (ii) the presence of domestic policies.

Finally, we turn to the optimal design of trade agreements from the perspective of monopolistic competition models. Again, we focus on the multi-sector model, which features a clear motive for domestic policies.[33] Obviously, implementation of the first-best allocation requires a trade agreement that includes cooperation on both trade and domestic policies (see Proposition 4). However, most trade agreements – in particular, GATT-WTO – do not regulate domestic policies to the extent that they do not imply outright discrimination of foreign goods.[34] We therefore consider a situation where domestic policy is set non-cooperatively in the presence of a trade agreement that prevents countries from choosing trade policy instruments strategically. For simplicity, we focus on the case of full trade liberalization, as required, e.g. by a regional trade agreement under Article 24 of GATT-WTO, but one can also think of a multilateral agreement that prevents countries from using trade instruments strategically, such as GATT- WTO.[35] In the case where only domestic policies can be set strategically, individual-country policy makers face a missing-instrument problem and there is a trade off between increasing production efficiency (calling for a labor subsidy) and improving the terms of trade (calling for a labor tax). Thus, it is ex ante unclear which motive dominates in equilibrium and one has to characterize the Nash-equilibrium policies.

[33]In the one-sector model the free-trade allocation is Pareto optimal and individual-country policy makers' only incentive is to manipulate their terms of trade. Thus, the use of any type of policy instruments (both trade and labor taxes) should be restricted by a trade agreement.

[34] GATT Article III (1): The contracting parties recognize that internal taxes and other internal charges, and laws, regulations and requirements affecting the internal sale, offering for sale, purchase, transportation, distribution or use of products, and internal quantitative regulations requiring the mixture, processing or use of products in specified amounts or proportions, should not be applied to imported or domestic products so as to afford protection to domestic production. (...) (8 b) The provisions of this Article shall not prevent the payment of subsidies exclusively to domestic producers, including payments to domestic producers derived from the proceeds of internal taxes or charges applied consistently with the provisions of this Article and subsidies effected through governmental purchases of domestic products.

[35]GATT-WTO features tariff bindings and prohibition of export taxes.

Proposition 7 ***Strategic domestic policies in the presence of a trade agreement***

When only labor taxes are available,

(a) *It is possible to rewrite* (37) *as follows:*

$$dU_i = \frac{\Omega_i}{I_i} dL_{Ci} \tag{41}$$

where Ω_i is defined in Appendix D.8.

(b) *Solving the individual-country policy maker problem in* (36) *by using the total-differential approach when $\tau_{Ii} = \tau_{Xi} = 1$, $i = H, F$ requires setting $\Omega_i = 0$.*

(c) *As a result, the symmetric Nash equilibrium of the multi-sector model when trade taxes are not available and both countries can simultaneously set labor taxes is characterized as follows: it exists, is unique and entails positive but inefficiently low labor subsidies when the average variable-profit share from sales in the domestic market, δ_{ii}, is larger or equal than 1/2 or when firms are homogeneous. Otherwise, the Nash equilibrium generically entails positive labor taxes. Formally:*

- *If $\delta_{ii} \geq \frac{1}{2}$ then $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^{Nash} \leq 1$;*
- *If $\delta_{ii} < \frac{1}{2}$ and $\varepsilon \geq \frac{3-\alpha}{2}$ then $\tau_L^{Nash} > 1$;*
- *If $\delta_{ii} \in \left[\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}, \frac{1}{2}\right)$ and $\varepsilon < \frac{3-\alpha}{2}$ then $\tau_L^{Nash} > 1$;*
- *If $\delta_{ii} \in \left[0, \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}\right)$ and $\varepsilon < \frac{3-\alpha}{2}$ all of the following cases may occur: non-existence of a Nash equilibrium; existence of a unique Nash equilibrium with $\tau_L^{Nash} > 1$; existence of two Nash equilibria.*

Proof See Appendix D.8. ■

Proposition 7 extends the result of Campolmi et al. (2014) to firm heterogeneity. Thus, if the average variable-profit share stemming from domestic sales accounts for the larger part of profits, strategic domestic policies ~~still~~ feature positive subsidies. However, these subsidies are inefficiently low, reflecting the trade off between closing the production-efficiency wedge and improving the terms of trade. Thus – while an efficient agreement requires coordination of trade *and* domestic policies – in this case a trade agreement that prohibits the use of both trade and domestic policies fares worse in welfare terms than one that allows countries to choose domestic policies strategically. By contrast, when the variable-profit share from domestic sales falls below one half, strategic domestic policies feature a labor tax, which worsens the allocation compared to the laissez-faire free-trade allocation. In this case a trade agreement that prohibits the use of trade and domestic policies fares better in welfare terms than a trade agreement that allows countries to choose domestic policies strategically. Moreover, one can show that δ_{ii} is increasing in τ_{ij} and f_{ij} for $j \neq i$. Thus, as physical trade barriers fall, the variable-profit share from domestic sales falls and may even become smaller than one half. Therefore, with low physical trade barriers and uncoordinated domestic policies countries are more likely to end up in a situation that is worse than the laissez-faire free-trade equilibrium. Consequently, the proportional welfare gains from coordinating domestic policies in the presence of a trade agreement rise as physical trade barriers fall. These insights on the welfare effects of trade agreements are summarized by the following Proposition and the associated Corollary.

Proposition 8 ***Welfare effects of strategic domestic policies in the presence of a trade agreement***

Assume that $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and let the average variable-profit share from sales in the domestic market be given by δ_{ii}.

(a) When $\delta_{ii} \geq \frac{1}{2}$ or in the presence of homogeneous firms, the symmetric Nash equilibrium when countries can only set domestic policies strategically welfare-dominates the laisez-faire free-trade allocation with $\tau_{Li} = 1$, $i = H, F$.

(b) When $\delta_{ii} < \frac{1}{2}$ the symmetric Nash equilibrium when countries can only set domestic policies strategically is welfare-dominated by the free-trade allocation with $\tau_{Li} = 1$, $i = H, F$.

(c) δ_{ii} is increasing in τ_{ij} and f_{ij}, $j \neq i$.

Proof See Appendix D.9. ■

Corollary 2 ***The design of trade agreements in the presence of domestic policies***

(a) Implementing the first-best allocation requires a joint agreement on trade and domestic policies.

(b) When $\delta_{ii} \geq \frac{1}{2}$ or in the presence of homogeneous firms, an agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely welfare dominates an agreement that forbids countries to use domestic and trade policies.

(c) When $\delta_{ii} < \frac{1}{2}$ an agreement that forbids countries to use domestic and trade policies welfare dominates an agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely.

(d) The proportional welfare gains from a joint agreement on trade and domestic policies compared to an agreement that only prohibits trade taxes but allows countries to set domestic policies freely are larger when physical trade barriers are small compared to when they are large.

6 Conclusion

Trade models with monopolistic competition and heterogeneous firm (Melitz, 2003) have become the workhorse for positive analysis in trade theory. Surprisingly, so far relatively little research has been dedicated to the normative implications of this framework. In particular, the question which international externalities are solved by trade agreements in multi-sector models with heterogeneous firms and how to design trade agreements from their perspective has so far not been addressed by the literature. This is reflected in the review article by Bagwell and Staiger (2016), who limit their discussion of the design of trade agreements to homogeneous-firm models.

In this paper we have made progress on several fronts. Starting with the observation that trade models with CES preferences and monopolistic competition have a common macro representation, we have shown that this class of models also has common welfare incentives for trade and domestic policies. Solving the problem of a world policy maker, we have derived a welfare decomposition that decomposes world welfare changes induced by trade and domestic policies into changes in consumption wedges and production wedges. As long as the world policy maker disposes of a sufficient set of instruments, she closes these wedges one by one and implements the first-best allocation. In the multi-sector model this requires using labor subsidies to offset monopolistic markups.

From the individual-country perspective, welfare incentives for trade and domestic policies are additionally governed by terms-of-trade incentives. This makes clear that terms-of-trade effects are the only pure beggar-thy-neighbor externality in this class of models.

Then we have discussed how individual policy instruments affect the terms of trade. We have shown that firm heterogeneity matters here in two respects: first, through selection into exporting, by affecting the cutoff productivity of exporters; second, via the impact on the variable-profit share arising from sales in each market.

Finally, we have studied strategic trade and domestic policies within the multi-sector heterogeneous-firm model. We have shown that when all policy instruments can be set strategically, the Nash equilibrium entails the first-best level of labor subsidies and inefficient import subsidies and export taxes that aim at improving the terms of trade. This result is qualitatively independent of firm heterogeneity. Thus, even in the presence of firm heterogeneity and domestic policies terms-of-trade motives remain the only reason for signing a trade agreement. Moreover, when a trade agreement prevents countries from use trade policy strategically, domestic policies are set to balance a trade off between improving the terms of trade and increasing production efficiency. Nash-equilibrium domestic policies – and thus, whether the strategic use of domestic policies should be proscribed by a trade agreement – depends on the variable profit-share on domestic sales. When this number is larger than one half, the Nash equilibrium features positive (albeit inefficiently low) labor subsidies. By contrast, when this number is smaller than one half, the Nash equilibrium is characterized by positive labor taxes.

To conclude we point out some limitations of our work and discuss directions for future research. The main restriction of the CES framework with monopolistic competition is that markups are constant and independent of the level of competition. Empirically, markups tend to be higher for more productive firms and pass through is incomplete (Hottman, Redding and Weinstein, 2016). Relaxing the constant-markup assumption requires either to dispense of CES preferences or to introduce oligopolistic firms (Atkeson and Burstein, 2012). See Bagwell and Lee (2015) for an analysis of strategic trade policy in monopolistic competition models with linear demand. In the presence of variable markups achieving production efficiency necessitates firm-specific subsidies and in their absence policy makers operate in a second-best environment. A further potential extension of our study is to analyze optimal trade and domestic policies in the presence of trade in intermediate inputs and global production chains. Here, Blanchard, Bown and Johnson (2017) provide some first insights.

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APPENDIX

A The Model

A.1 Households

Given the Dixit-Stiglitz structure of preferences in (4), the households' maximization problem can be solved in three stages. At the first two stages, households choose how much to consume of each domestically produced and foreign produced variety, and how to allocate consumption between the domestic and the foreign bundles. The optimality conditions imply the following demand functions and price indices:

$$c_{ij}(\varphi) = \left[\frac{p_{ij}(\varphi)}{P_{ij}}\right]^{-\varepsilon} C_{ij}, \quad C_{ij} = \left[\frac{P_{ij}}{P_i}\right]^{-\varepsilon} C_i, \quad i,j = H,F \tag{A-1}$$

$$P_i = \left[\sum_{j\in H,F} P_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \quad P_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} p_{ij}(\varphi)^{1-\varepsilon} dG(\varphi)\right]^{\frac{1}{1-\varepsilon}}, \quad i,j = H,F \tag{A-2}$$

Here P_i is the price index of the differentiated bundle in country i, P_{ij} is the country-i price index of the bundle of differentiated varieties produced in country j, and $p_{ij}(\varphi)$ is the country-i consumer price of variety φ produced by country j.

In the last stage, households choose how to allocate consumption between the homogeneous good and the differentiated bundle. Thus, they maximize (3) subject to the following budget constraint:

$$P_i C_i + p_{Zi} Z_i = I_i, \quad i = H,F$$

where $I_i = W_i L + T_i$ is total income and T_i is a lump sum transfer which depends on the tax scheme adopted by the country-i government. The solution to the consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_i}{C_i} = \frac{P_i}{p_{Zi}}, \quad i = H,F \tag{A-3}$$

Then following Melitz and Redding (2015), we can rewrite the demand functions as

$$c_{ij}(\varphi) = p_{ij}(\varphi)^{-\varepsilon} A_i, \quad C_{ij} = P_{ij}^{-\varepsilon} A_i, \quad C_i = P_i^{-\varepsilon} A_i, \quad i = H,F, \tag{A-4}$$

where $A_i \equiv P_i^{\varepsilon-1}\alpha I_i$. A_i can be interpreted as an index of market (aggregate) demand.

A.2 Firms

A.2.1 Firms' behavior in the differentiated sector

Given the constant price elasticity of demand, optimal prices charged by country-i firms in their domestic market are a fixed markup over their perceived marginal cost ($\tau_{Li}\frac{W_i}{\varphi}$), and optimal prices charged to country-j consumers for varieties produced in country i equal country-i prices augmented by transport costs and trade taxes

$$p_{ji}(\varphi) = \tau_{ji}\tau_{Tji}\tau_{Li}\frac{\varepsilon}{\varepsilon-1}\frac{W_i}{\varphi}, \quad i,j = H,F \tag{A-5}$$

The optimal pricing rule implies the following firm revenues:

$$r_{ji}(\varphi) \equiv \tau_{Tji}^{-1} p_{ji}(\varphi) c_{ji}(\varphi) = \tau_{Tji}^{-1} p_{ji}(\varphi)^{1-\varepsilon} A_j = \varepsilon \tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon}\tau_{Li}^{1-\varepsilon} W_i^{1-\varepsilon}\varphi^{\varepsilon-1} B_j, \quad i = H,F, \tag{A-6}$$

where $B_i \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \frac{1}{\varepsilon} A_i$. Profits are given by:

$$\pi_{ji}(\varphi) \equiv B_j \left(\frac{\tau_{Li} W_i}{\varphi}\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} = \frac{r_{ji}(\varphi)}{\varepsilon} - \tau_{Li} W_i f_{ji}, \quad i = H, F \tag{A-7}$$

A.2.2 Zero-profit conditions

Firms choose to produce for the domestic (export) market only when this is profitable. Since profits are monotonically increasing in φ, we can determine the equilibrium productivity cutoffs for firms active in the domestic market and export market, φ_{ji}, by setting $\pi_{ji}(\varphi_{ji}) = 0$, namely:

$$\pi_{ji}(\varphi_{ji}) = 0 \Rightarrow \frac{r_{ji}(\varphi_{ji})}{\varepsilon} = \tau_{Li} W_i f_{ji} \tag{A-8}$$

As in Melitz (2003), we call these conditions the *zero profit* (**ZCP**) conditions. Using (A-7) we rewrite (A-8) as follows:

$$B_j = \tau_{ji}^{\varepsilon-1} \tau_{Li}^{\varepsilon} \tau_{Tji}^{\varepsilon} W_i^{\varepsilon} \varphi_{ji}^{1-\varepsilon} \quad i, j = H, F \tag{A-9}$$

A.2.3 Free-entry conditions (FE)

The *free entry* (**FE**) conditions require expected profits (before firms know the realization of their productivity) in each country to be zero in equilibrium:

$$\sum_{j \in H,F} \int_{\varphi_{ji}}^{\infty} \pi_{ji}(\varphi) dG(\varphi) = \tau_{Li} W_i f_E$$

Substituting optimal profits (A-7), we obtain

$$\sum_{j \in H,F} \int_{\varphi_{ji}}^{\infty} \left[B_j \left(\frac{\tau_{Li} W_i}{\varphi}\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li} W_i f_{ji} \right] dG(\varphi) = \tau_{Li} W_i f_E \tag{A-10}$$

A.2.4 Firms' behavior in the homogeneous sector

Since the homogeneous good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization in the presence of the homogeneous sector:

$$p_{Zi} = p_{Zj} = W_i = W_j = 1$$

A.3 Government

The government is assumed to run a balanced budget. Hence, country-i government's budget constraint is given by:

$$\begin{aligned} T_i = {} & (\tau_{Ii} - 1)\tau_{Ii}^{-1} P_{ij} C_{ij} + (\tau_{Xi} - 1)\tau_{Tji}^{-1} P_{ji} C_{ji} + \\ & + (\tau_{Li} - 1) N_i W_i \left[\sum_{k=H,F} \int_{\varphi_{ki}}^{\infty} \left(\frac{q_{ki}(\varphi)}{\varphi} + f_{ki} \right) dG(\varphi) + f_E \right] \quad i = H, F \quad j \neq i \end{aligned}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and foreign export taxes (thus, tariffs are charged on CIF values of foreign exports), export tax revenues charged on exports gross of transport costs, and labor tax revenues.

A.4 Equilibrium

Substituting **ZCP** (A-9) into **FE** (A-10), we obtain:

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})) \quad i=H,F, \tag{A-11}$$

where

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1}\frac{dG(\varphi)}{1-G(\varphi_{ji})}\right]^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F, \tag{A-12}$$

which correspond to (10) and (7) in the main text. Moreover, dividing the **ZCP** conditions (A-9), we obtain condition (9) in the main text:

$$\left(\frac{\varphi_{ii}}{\varphi_{ij}}\right) = \left(\frac{f_{ii}}{f_{ij}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{W_i}{W_j}\right)^{\frac{\varepsilon}{\varepsilon-1}}\tau_{ij}^{-1}\tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i,j=H,F \tag{A-13}$$

The remaining equilibrium equations are then given as follows:

Consumption sub-indices which can be determined using (A-4) jointly with (A-9):

$$C_{ij} = P_{ij}^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon-1}\varepsilon\tau_{Cj}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\tau_{Tij}^{\varepsilon}\varphi_{ij}^{1-\varepsilon}W_j^{\varepsilon}f_{ij}, \quad i,j=H,F \tag{A-14}$$

Price sub-indices which emerge from substituting (A-5) into (A-2):

$$P_{ij}^{1-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}N_j(1-G(\varphi_{ij}))(\tau_{ij}\tau_{Tij}\tau_{Lj})^{1-\varepsilon}\widetilde{\varphi}_{ij}^{\varepsilon-1}W_j^{1-\varepsilon}, \quad i,j=H,F \tag{A-15}$$

Aggregate profits Π_i are given by $\Pi_i = R_i - \tau_{Li}W_iL_{Ci} + \tau_{Li}W_iN_if_E$, where R_i are aggregate revenues, $R_i \equiv N_i\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty} r_{ji}(\varphi)dG(\varphi)$. From the **FE** condition (A-10) it then follows that $\Pi_i = \tau_{Li}W_iN_if_E$ and thus $R_i = \tau_{Li}W_iL_{Ci}$. Substituting the definition of optimal revenues (A-6) into the previous condition, we get

$$\tau_{Li}W_iL_{Ci} = \varepsilon N_i\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty} B_j\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon}\tau_{Li}^{1-\varepsilon}W_i^{1-\varepsilon}\varphi^{\varepsilon-1}dG(\varphi)$$

Combining this condition with (10) and (A-9), we obtain:

Labor market clearing in the differentiated sector

$$L_{Ci} = \varepsilon N_i\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})) + \varepsilon f_E N_i \quad i=H,F \tag{A-16}$$

This can be solved for the equilibrium level of N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) + \varepsilon f_E} \quad i = H, F \tag{A-17}$$

Combining this last condition with (10), plugging into (A-14) and (A-15) and taking into account the definition (7) allow us to recover (11) and (12) in the main text.

In the presence of the homogeneous sector, the trade-balance condition is given by:[36]

$$(Q_{Zi} - Z_i) + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij} \quad i = H, j = F$$

The left-hand side of this equation is the sum of the net export value of the homogeneous goods and the value of the differentiated exportable bundle (at CIF inclusive international prices), while the right-hand side is the value of the differentiated importable bundle (at CIF inclusive international prices). In the absence of the homogeneous sector the first term is zero and we can use this condition directly. By contrast, in presence of this sector, we can use the fact that $\sum_{j=H,F} P_{ij} C_{ij} = P_i C_i$ to rewrite (A-3) as:

$$Z_i = \frac{(1-\alpha)}{\alpha} \sum_{j=H,F} P_{ij} C_{ij} \quad i = H, F \tag{A-18}$$

We can combine this equation with the trade-balance condition above and the aggregate labor market clearing $L = L_{Ci} + L_{Zi}$ to obtain:

Trade-balance condition

$$L - L_{Ci} - \frac{(1-\alpha)}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F, \tag{A-19}$$

which corresponds to condition (13).

Finally, when the homogeneous sector is present, we also require equilibrium in the market for the homogenous good, i.e. $\sum_{i=H,F} Q_{Zi} = \sum_{i=H,F} Z_i$. Combining this condition with aggregate labor market clearing and demand for the homogeneous good (A-3) we obtain:

Homogeneous good market clearing condition

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij}, \tag{A-20}$$

which coincides with condition (14).

A.5 From Melitz to Krugman (1980)

In this section we show how the equilibrium equations (7) to (15) need to be modified to obtain the Krugman (1980) model.

To obtain the Krugman (1980) model with homogeneous firms, the following assumptions are needed:

1. Let $f_{ij} = 0$ for $i, j = H, F$ (there are no fixed market access costs).
2. Let $G(\varphi)$ be degenerate at unity.
3. Conditions (7), (8) (9) and (10) need to be dropped from the set of equilibrium conditions.

[36] Import taxes are collected directly by the governments at the border so they do not enter into this condition.

The free-entry conditions are given by

$$\sum_{j\in H,F}\pi_{ji}=\tau_{Li}W_i f_E,\quad i=H,F$$

In the absence of fixed market access costs profits are given by:

$$\pi_{ji}\equiv B_j\left(\frac{\tau_{Li}W_i}{\varphi}\right)^{1-\varepsilon}\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon},\quad i=H,F \tag{A-21}$$

From the free-entry conditions we can solve for B_i and B_j as function of W_i, W_j and the policy instruments:

$$B_i=\frac{f_E W_j^{\varepsilon}\left[\tau_{Lj}^{\varepsilon}-\tau_{Li}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\tau_{Tji}^{\varepsilon}\left(\frac{W_i}{W_j}\right)^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon}\tau_{ij}^{1-\varepsilon}-\tau_{Tji}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\right]},\quad i=H,F \tag{A-22}$$

Moreover, by substituting the optimal pricing decision into the definition of the price indices and observing that $N_j=L_{Cj}/(\varepsilon f_E)$:

$$P_{ij}=\left(\frac{\varepsilon}{\varepsilon-1}\right)(\varepsilon f_E)^{\frac{1}{\varepsilon-1}}(\tau_{ij}\tau_{Tij}\tau_{Lj})W_j L_{Cj}^{\frac{-1}{\varepsilon-1}},\quad i,j=H,F \tag{A-23}$$

From the definition of C_{ij}:

$$C_{ij}=P_{ij}^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon-1}\varepsilon B_i,\quad i,j=H,F \tag{A-24}$$

Substituting the expression for P_{ij} and B_i, we obtain

$$C_{ij}=\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1}L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}}(\varepsilon f_E)^{\frac{-1}{\varepsilon-1}}\frac{\left[\tau_{Tij}^{-\varepsilon}\tau_{ij}^{-\varepsilon}-\tau_{Li}^{\varepsilon}\tau_{Lj}^{-\varepsilon}\tau_{ij}^{-1}\tau_{Tji}^{\varepsilon}\tau_{Tij}^{-\varepsilon}\left(\frac{W_i}{W_j}\right)^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon}\tau_{ij}^{1-\varepsilon}-\tau_{Tji}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\right]} \tag{A-25}$$

Thus, if the homogeneous sector is present ($\alpha<1$), the equilibrium is given by equations (A-23) and (A-25) together with (13),(14) and (15)and the fact that $W_i=1$ for $i=H$. For the case of $\alpha=1$. $L_{Cj}=L$ for $j=H,F$.

A.6 System of equations for the one-sector model

In this section we show how the equilibrium conditions (7)-(15) simplify when $\alpha=1$, i.e., when there is no homogeneous sector. In this case $Z_i=0$ and $L_{Ci}=L$ for $i=H,F$. Remember also that $W_j=1$ for $j=F$, implying $dW_j=0$. Also, in the one sector model we do not allow for labor tax/subsidy as the free trade allocation is already efficient, i.e., we set $\tau_{Li}=1$ for $i=H,F$.

The equilibrium conditions (7)-(12) remain the same but for $\tau_{Li}=\tau_{Lj}=1$ for $i,j=H,F$. Imposing $Z_i=0$ into (15) gives us:

$$\sum_{j=H,F}P_{ij}C_{ij}=0\quad i=H,F$$

which, together with $L_{Ci}=L$ makes (14) an identity. Finally, (13) simplifies to:

$$\tau_{Ij}^{-1}P_{ji}C_{ji}=\tau_{Ii}^{-1}P_{ij}C_{ij},\quad i=H,\quad j=F \tag{A-26}$$

A.7 Solving for the free-trade allocation

Using equations (11) and (12), we find that

$$P_{ij}C_{ij} = \delta_{ij}L_{Cj}\tau_{Tij}\tau_{Lj}W_j, \quad i,j = H,F \tag{A-27}$$

Substituting into the trade-balance condition (13), we obtain:

$$L - L_{Ci} - \frac{(1-\alpha)}{\alpha}\sum_{k=H,F}\delta_{ik}L_{Ck}\tau_{Tik}\tau_{Lk}W_k + \delta_{ji}L_{Ci}\tau_{Xi}\tau_{Li}W_i = \delta_{ij}L_{Cj}\tau_{Xj}\tau_{Lj}W_j, \quad i = H, j = F \tag{A-28}$$

In the free-trade allocation, $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H,F$. Since the countries are symmetric, the equilibrium is also symmetric and thus $L_{Ci} = L_{Cj}$, $W_i = W_j = 1$, $\delta_{ij} = \delta_{ji}$ for $i,j = H,F$.

Substituting these conditions, we find that

$$L_{Ci}^{FT} = \alpha L, \quad i = H,F \tag{A-29}$$

Using this result together with (A-16) and (A-11), we obtain

$$N_i^{FT} = \frac{\alpha L}{\varepsilon \sum_{j=H,F}\left[f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}\right]}, \quad i = H,F \tag{A-30}$$

A.8 Total differentials of the equilibrium equations

Since total differentials of the equilibrium equations are extensively used in the proofs, we present them here for future reference.

The total differential of (7) gives:

$$d\widetilde{\varphi}_{ji} = \frac{1}{\varepsilon-1}\frac{g(\varphi_{ji})}{[1-G(\varphi_{ji})]}\widetilde{\varphi}_{ji}\left[1-\left(\frac{\varphi_{ji}}{\widetilde{\varphi}_{ji}}\right)^{\varepsilon-1}\right]d\varphi_{ji}, \quad i,j = H,F \tag{A-31}$$

Taking the total differential of (10) and using (A-31) we get:

$$d\varphi_{ji} = -\frac{f_{ii}[1-G(\varphi_{ii})]\varphi_{ii}^{-\varepsilon}\widetilde{\varphi}_{ii}^{\varepsilon-1}}{f_{ji}[1-G(\varphi_{ji})]\varphi_{ji}^{-\varepsilon}\widetilde{\varphi}_{ji}^{\varepsilon-1}}d\varphi_{ii}, \quad i,j = H,F, \quad i \neq j \tag{A-32}$$

Using the definition of δ_{ii} obtained by combining (8) with (10), this can also be written in a more compact way:

$$d\varphi_{ji} = -\frac{\delta_{ii}}{1-\delta_{ii}}\frac{\varphi_{ji}}{\varphi_{ii}}d\varphi_{ii} \tag{A-33}$$

Combining (8) with (10), taking the total differential, and using (A-31) and (A-32), we get:

$$d\delta_{ji} = -\frac{\delta_{ji}}{\varphi_{ji}}\left(\Phi_i + (\varepsilon-1)\right)d\varphi_{ji}, \quad i,j = H,F \tag{A-34}$$

where $\Phi_i \equiv \delta_{ii}\frac{g(\varphi_{ji})\varphi_{ji}^{\varepsilon}\widetilde{\varphi}_{ji}^{1-\varepsilon}}{1-G(\varphi_{ji})} + \delta_{ji}\frac{g(\varphi_{ii})\varphi_{ii}^{\varepsilon}\widetilde{\varphi}_{ii}^{1-\varepsilon}}{1-G(\varphi_{ii})} > 0$, $i = H,F$.

Taking the total differential of (11) we obtain:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{C_{ij}} dC_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{\delta_{ij}} d\delta_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{L_{Cj}} dL_{Cj}, \quad i, j = H, F \tag{A-35}$$

which, using the symmetric condition of (A-34) to substitute out $d\delta_{ij}$, becomes:

$$d\varphi_{ij} = \frac{\varepsilon \varphi_{ij}}{L_{Cj}(\varepsilon - 1)\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_j\right)} dL_{Cj} - \frac{\varphi_{ij}}{C_{ij}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_j\right)} dC_{ij}, \quad i, j = H, F \tag{A-36}$$

For future use, we substitute the symmetric condition of (A-36) into (A-34):

$$d\delta_{ji} = \frac{\delta_{ji}(\varepsilon - 1 + \Phi_i)}{C_{ji}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} dC_{ji} - \frac{\delta_{ji}\varepsilon(\varepsilon - 1 + \Phi_i)}{L_{Ci}(\varepsilon - 1)\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} dL_{Ci}, \quad i, j = H, F \tag{A-37}$$

Taking the total differential of (9), we have:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}} d\varphi_{ii} + \frac{\varepsilon}{\varepsilon - 1}\varphi_{ij}\left[\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} + \frac{dW_j}{W_j} - \frac{dW_i}{W_i} + \frac{d\tau_{Tij}}{\tau_{Tij}}\right], \quad i, j = H, F, \quad i \neq j \tag{A-38}$$

where $d\tau_{Tji} = 0$ if $i = j$ while $d\tau_{Tji} = \tau_{Xi} d\tau_{Ij} + \tau_{Ij} d\tau_{Xi}$ if $i \neq j$.

B The Planner Allocation

In this Appendix, we derive the main results of Section 3. First, we set up the planner problem and solve it using a three-stage approach. Finally, we prove Proposition 1.

B.1 The Planner Problem

The full planner problem can be written as follows. The planner maximizes:

$$\sum_{i=H,F} U_i = \sum_{i=H,F}\left[\left(\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z_i^{1-\alpha}\right]$$

with respect to C_{ij},L_{Cij}, Z_i, N_i, $c_{ij}(\varphi)$, $l_{ij}(\varphi)$, φ_{ij}, for $i, j = H, F$ and subject to:

$$C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi)\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F$$

$$l_{ij}(\varphi) = \frac{\tau_{ij} c_{ij}(\varphi)}{\varphi} + f_{ij}, \quad i, j = H, F$$

$$L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i, j = H, F$$

$$L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i, j = H, F$$

$$\sum_{i=H,F} L_i = \sum_{i=H,F} L_{Ci} + \sum_{i=H,F} Z_i$$

Notice that by combining L_{Cij} and $l_{ij}(\varphi)$ we get:

$$L_{Cij} = \tau_{ij} N_j \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} dG(\varphi) + N_j f_{ij}(1 - G(\varphi_{ij})), \quad i, j = H, F$$

This problem can be split into three separate stages. The proof that this approach is equivalent to solving the full planner problem in a single stage is available on request.

B.2 First stage

Here we derive the results used in Section 3.1.

B.2.1 First-stage optimality conditions

At the first stage the planner solves the problem stated in (18). Taking total differentials with respect to $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij}:

$$du_{ij} = \int_{\varphi_{ij}}^{\infty} \frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} dc_{ij}(\varphi) dG(\varphi) + \frac{\partial u_{ij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{A-39}$$

$$dc_{ij}(\varphi) = \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} dl_{ij}(\varphi) \tag{A-40}$$

$$N_j \int_{\varphi_{ij}}^{\infty} dl_{ij}(\varphi) dG(\varphi) + \frac{\partial L_{Cij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{A-41}$$

By using (A-40) and (A-41) to substitute out $d\varphi_{ij}$ from (A-39) we get:

$$\int_{\varphi_{ij}}^{\infty} \left(\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} - \frac{\frac{\partial u_{ij}}{\partial \varphi_{ij}}}{\frac{\partial L_{Cij}}{\partial \varphi_{ij}}} \frac{N_j}{\frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)}} \right) dc_{ij}(\varphi) dG(\varphi) = 0 \tag{A-42}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} = \frac{\partial u_{ij}}{\partial L_{Cij}} N_j,$$

for all $\varphi \in [\varphi_{ij}, \infty)$. As a consequence:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi_1)} \frac{\partial q_{ij}(\varphi_1)}{\partial l_{ij}(\varphi_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\varphi_2)} \frac{\partial q_{ij}(\varphi_2)}{\partial l_{ij}(\varphi_2)},$$

for any $\varphi_1 \in [\varphi_{ij}, \infty)$ and $\varphi_2 \in [\varphi_{ij}, \infty)$ which coincides with condition (19) in the main text.

B.2.2 First-stage aggregate production function

Plugging in the functional forms into (A-42) we obtain:

$$\int_{\varphi_{ij}}^{\infty} \left(c_{ij}(\varphi)^{-1/\varepsilon} - \frac{\varepsilon}{\varepsilon - 1} \frac{\tau_{ij}}{\varphi} \frac{c_{ij}(\varphi_{ij})^{\frac{\varepsilon-1}{\varepsilon}}}{l_{ij}(\varphi_{ij})} \right) dc_{ij}(\varphi) dG(\varphi) = 0, \tag{A-43}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$c_{ij}(\varphi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon} \frac{c_{ij}(\varphi_{ij})^{1-\varepsilon}}{l_{ij}(\varphi_{ij})^{-\varepsilon}} \tau_{ij}^{-\varepsilon} \varphi^{\varepsilon} \tag{A-44}$$

$$\tag{A-45}$$

Substituting (A-44) into the definition of C_{ij}, using the definition of $\widetilde{\varphi}_{ij}$, and noting that $N_{ij} = [1 - G(\varphi_{ij})]N_j$, we get:

$$c_{ij}(\varphi_{ij})^{1-\varepsilon} = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\varepsilon} l_{ij}(\varphi_{ij})^{-\varepsilon} \tau_{ij}^{\varepsilon} \widetilde{\varphi}_{ij}^{-\varepsilon} \tag{A-46}$$

If we substitute this back into (A-44) we obtain:

$$c_{ij}(\varphi) = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi}\right)^{-\varepsilon} \tag{A-47}$$

Finally, we can aggregate the production function as follows:

$$\begin{aligned} L_{Cij} &= N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi) = \tau_{ij} N_{ij} \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} + f_{ij} N_{ij} \\ &= \tau_{ij} N_{ij}^{-\frac{1}{\varepsilon-1}} \frac{C_{ij}}{\widetilde{\varphi}_{ij}} + f_{ij} N_{ij} \end{aligned} \tag{A-48}$$

This leads to the aggregate production function (20) in the main text:

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1 - G(\varphi_{ij})]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij} [N_j(1 - G(\varphi_{ij})]^{\frac{\varepsilon}{\varepsilon-1}} \right\} \quad i, j = H, F,$$

where $Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) = C_{ij}$.

B.2.3 First-stage comparison between planner and market allocation

We want to verify that the consumption of individual varieties chosen by the planner coincides with the one of the market allocation conditional on C_{ij}, N_{ij} and $\tilde{\varphi}_{ij}$ being the same. Recall that the demand function is:

$$c_{ij}(\varphi) = \left(\frac{p_{ij}(\varphi)}{P_{ij}}\right)^{-\varepsilon} C_{ij}$$

Since the price index is given by

$$P_{ij} = N_{ij}^{\frac{1}{1-\varepsilon}} p_{ij}(\tilde{\varphi}_{ij}),$$

it follows that $\frac{p_{ij}(\varphi)}{p_{ij}(\tilde{\varphi}_{ij})} = \frac{\tilde{\varphi}_{ij}}{\varphi}$. Thus, we can conclude that in the market equilibrium:

$$c_{ij}(\varphi) = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi}\right)^{-\varepsilon}, \tag{A-49}$$

This coincides with (A-47).

B.2.4 First stage with homogeneous firms

In this case the problem stated in (18) simplifies to choosing $c_{ij}(\omega)$ and $l_{ij}(\omega)$ for $i,j = H,F$ by solving the following problem:

$$\begin{aligned} &\max u_{ij} \\ s.t.\quad & c_{ij}(\omega) = q_{ij}(\omega), \quad i,j = H,F \\ & L_{Cij} = \int_0^{N_j} l_{ij}(\omega) d\omega, \quad i,j = H,F, \end{aligned} \tag{A-50}$$

where $u_{ij} \equiv C_{ij}$, $C_{ij} = \left[\int_0^{N_j} c_{ij}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega\right]^{\frac{\varepsilon}{\varepsilon-1}}$, $q_{ij}(\omega) = l_{ij}(\omega)\frac{1}{\tau_{ij}}$, and N_j and L_{Cij} are taken as given since they are determined at the second stage. Solving this problem gives back the same condition condition with derived with heterogeneous firms:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\omega_1)} \frac{\partial q_{ij}(\omega_1)}{\partial l_{ij}(\omega_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\omega_2)} \frac{\partial q_{ij}(\omega_2)}{\partial l_{ij}(\omega_2)}$$

This implies that all firms will employ the same quantity of labor and produce the same amount of consumption good, i.e., $l_{ij}(\omega) = l_{ij}$ and $c_{ij}(\omega) = c_{ij} \forall \omega \in [0, N_j]$. Following the same steps as with heterogeneous firms we can derive the aggregate level of consumption:

$$C_{ij} = N_j^{\frac{\varepsilon}{\varepsilon-1}} c_{ij},$$

which coincides with the market equilibrium (A-49). Finally, the aggregate production now simplifies to:

$$C_{ij} = \frac{\varphi}{\tau_{ij}} N_j^{\frac{1}{\varepsilon-1}} L_{Cij}$$

B.3 Second Stage

Here we derive the results of Section 3.2.

B.3.1 Second-stage optimality conditions and aggregate production function

At the second stage the planner solves the problem described in (21). Taking total differentials:

$$\begin{aligned} &\sum_{i=H,F} \sum_{j=H,F} \frac{\partial u_i}{\partial C_{ij}} dC_{ij} = 0 \\ &dN_i = -\frac{1}{f_E} \sum_{j=H,F} dL_{Cji}, \quad i = H,F \\ &dC_{ij} = \frac{\partial Q_{Cij}}{\partial N_j} dN_j + \frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} d\tilde{\varphi}_{ij} + \frac{\partial Q_{Cij}}{\partial L_{Cij}} dL_{Cij}, \quad i,j = H,F \end{aligned}$$

Substituting the differentials of the constraints into the objective, we obtain:

$$\sum_{i=H,F} \sum_{j=H,F} \frac{\partial u_i}{\partial C_{ij}} \left[\frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} d\tilde{\varphi}_{ij} + \frac{\partial Q_{Cij}}{\partial L_{Cij}} dL_{Cij} - \frac{\partial Q_{Cij}}{\partial N_j} \frac{1}{f_E} \sum_{k=H,F} dL_{Ckj} \right] = 0$$

Collecting terms:

$$\sum_{j=H,F}\sum_{i=H,F}\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}d\tilde{\varphi}_{ij}+\sum_{j=H,F}\sum_{i=H,F}\left[\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}}-\sum_{k=H,F}\frac{\partial u_k}{\partial C_{kj}}\frac{\partial Q_{Ckj}}{\partial N_j}\frac{1}{f_E}\right]dL_{Cij}=0 \tag{A-51}$$

Since (A-51) should hold for any $d\tilde{\varphi}_{ij}$ and dL_{Cij} it follows that:

$$\frac{\partial Q_{Cij}}{\partial\tilde{\varphi}_{ij}}=0,\quad i,j=H,F \tag{A-52}$$
$$\sum_{k=H,F}\frac{\partial u_k}{\partial C_{kj}}\frac{\partial Q_{Ckj}}{\partial N_j}=f_E\frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}},\quad i,j=H,F,$$

which leads to conditions (22), (23) and (24) in the main text.

B.3.2 Second-stage aggregate production function

Using the functional forms, we obtain the following derivatives:

$$\frac{\partial u_i}{\partial C_{ij}}=\frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F}C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}}=\left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}}C_i^{-1},\quad i,j=H,F \tag{A-53}$$
$$\frac{\partial Q_{Cji}}{\partial N_i}=\frac{\tilde{\varphi}_{ij}}{\tau_{ij}}[N_i(1-G(\varphi_{ji}))]^{\frac{2-\varepsilon}{\varepsilon-1}}\frac{L_{Cji}}{(\varepsilon-1)}(1-G(\varphi_{ji}))-\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon}{\varepsilon-1}\right)(1-G(\varphi_{ji})),\quad i,j=H,F$$
$$\frac{\partial Q_{Cji}}{\partial\tilde{\varphi}_{ji}}=\frac{1}{\tau_{ji}}\left\{[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}}L_{Cji}-f_{ji}[N_i(1-G(\varphi_{ji})]^{\frac{\varepsilon}{\varepsilon-1}}\right\}-\frac{[N_i(1-G(\varphi_{ji})]^{\frac{2-\varepsilon}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1}-\varphi_{ij}^{\varepsilon-1})}L_{Cji}(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1}N_i$$
$$+\frac{f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1}-\varphi_{ji}^{\varepsilon-1})}\varepsilon(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1}N_i,\quad i,j=H,F$$
$$\frac{\partial Q_{Cji}}{\partial L_{Cji}}=\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}},\quad i,j=H,F$$

This can be substituted into (24) to obtain:

$$L_{Cji}\left(1-\frac{\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1}-\varphi_{ji}^{\varepsilon-1})}\right)=f_{ji}[N_i(1-G(\varphi_{ji}))]\left(1-\frac{\varepsilon\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1}-\varphi_{ji}^{\varepsilon-1})}\right),\quad i,j=H,F$$

It follows that:

$$L_{Cji}=f_{ji}N_i(1-G(\varphi_{ji}))\left(\frac{\varphi_{ji}^{\varepsilon-1}+(\varepsilon-1)\tilde{\varphi}_{ji}^{\varepsilon-1}}{\varphi_{ji}^{\varepsilon-1}}\right)\quad i,j=H,F \tag{A-54}$$

Moreover, combining the derivatives above with condition (23) we obtain:

$$f_E=\sum_{j=H,F}\left[\frac{L_{Cji}}{N_i(\varepsilon-1)}-\frac{\varepsilon}{(\varepsilon-1)}f_{ji}(1-G(\varphi_{ji}))\right]$$

This implies that

$$\varepsilon N_i f_E+\sum_{j=H,F}\varepsilon(1-G(\varphi_{ji}))N_i f_{ji}=f_E N_i+\sum_{j=H,F}L_{Cji},\quad i=H,F. \tag{A-55}$$

Using (A-54) and $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out L_{Cji} and $f_E N_i$ in (A-55), we find:

$$L_{Ci} = \sum_{j=H,F} \varepsilon f_{ji} N_i (1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon - 1}, \quad i = H, F$$

We use this last condition to solve for N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F} \left[f_{ji}(1 - G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}} \right)^{\varepsilon-1} \right]}, \quad i = H, F \tag{A-56}$$

We now substitute (A-54) and (A-64) into the definition (20) to obtain (25) in the main text.

B.3.3 Second-stage comparison between planner and market allocation

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation.

First, consider condition (22). Plugging the relevant derivatives in (A-53), we obtain:

$$\frac{1}{C_i} \left(\frac{C_{ii}}{C_i} \right)^{\frac{-1}{\varepsilon}} \frac{\tilde{\varphi}_{ii}}{\tau_{ii}} [N_i (1 - G(\varphi_{ii})]^{\frac{1}{\varepsilon - 1}} = \frac{1}{C_j} \left(\frac{C_{ji}}{C_j} \right)^{\frac{-1}{\varepsilon}} \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} [N_i (1 - G(\varphi_{ji})]^{\frac{1}{\varepsilon - 1}}, \quad i = H, F, \quad j \neq i$$

Now consider the market allocation. Using (8) jointly with (9), (11) and (A-17) after some manipulations we get:

$$\frac{1}{C_i} \left(\frac{C_{ii}}{C_i} \right)^{\frac{-1}{\varepsilon}} \frac{\tilde{\varphi}_{ii}}{\tau_{ii}} [N_i (1 - G(\varphi_{ii})]^{\frac{1}{\varepsilon - 1}} = \frac{1}{C_j} \left(\frac{C_{ji}}{C_j} \right)^{\frac{-1}{\varepsilon}} \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} [N_i (1 - G(\varphi_{ji})]^{\frac{1}{\varepsilon - 1}} \left(\frac{C_j}{C_i} \frac{\tau_{ji}}{\tau_{ii}} \frac{\varphi_{ii}}{\varphi_{ji}} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \left(\frac{f_{ii}}{f_{ji}} \right)^{\frac{-1}{\varepsilon}} i = H, F, j \neq i$$

Thus, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \Omega_{P2} \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i = H, F, \quad j \neq i,$$

where Ω_{P2} is the wedge between the planner and the market allocation. Under symmetry:

$$\Omega_{P2} = \left(\frac{\tau_{ji}}{\tau_{ii}} \frac{\varphi_{ii}}{\varphi_{ji}} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \left(\frac{f_{ii}}{f_{ji}} \right)^{\frac{-1}{\varepsilon}}, \quad i = H, F, \quad j \neq i$$

Using condition (9), this can be written as $\Omega_{P2} = \tau_{Tij}^{-1}$. This leads to condition (26).

Next, consider the planner's optimality condition (23). Using the functional forms from (A-53), this corresponds to (see (A-55)):

$$f_E = \sum_{j=H,F} L_{Cji} N_i^{1-\varepsilon} - f_{ji}(1 - G(\varphi_{ji})) \frac{\varepsilon}{\varepsilon - 1} \tag{A-57}$$

We now want to check if this condition is also fulfilled by the market allocation. Recalling the labor market clearing condition in (A-16) and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (A-57) and this proves that (23) is satisfied in any market allocation.

Finally, consider the planner's optimality condition (24). As shown in Section B.3.2, this condition can be rewritten as (A-54). Now consider the market allocation. Appendix B.2.3 shows that condition (A-47) holds in the market equilibrium. As a consequence, also condition (A-48) holds in the market equilibrium. We can then use (A-48) and substitute it in equation (11) to obtain (A-54). This confirms that this condition and then (24) always holds both in the planner and in the market allocation.

B.3.4 Second stage with homogeneous firms

In this case the problem stated in (21) simplifies to choosing C_{ij}, L_{Cij}, N_i for $i,j=H,F$ in order to solve:

$$\max \sum_{i=H,F} u_i \tag{A-58}$$
$$s.t. \quad L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i=H,F$$
$$C_{ij} = Q_{Cij}(N_j, L_{Cij}), \quad i,j=H,F,$$

The first-order conditions of this problem are given by:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i,j=H,F, \quad i\neq j \tag{A-59}$$
$$f_E = \sum_{j=H,F}\frac{\partial Q_{Cji}/\partial N_i}{\partial Q_{Cji}/\partial L_{Ci}}, \quad i=H,F \tag{A-60}$$

B.3.5 Second-stage aggregate production function with homogeneous firms

Using the functional forms, we obtain the following derivatives:

$$\frac{\partial u_i}{\partial C_{ij}} = \frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F} C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}} C_i^{-1}, \quad i,j=H,F \tag{A-61}$$
$$\frac{\partial Q_{Cji}}{\partial N_i} = \frac{\varphi}{\tau_{ij}} N_i^{\frac{2-\varepsilon}{\varepsilon-1}}\frac{L_{Cji}}{(\varepsilon-1)}, \quad i,j=H,F$$
$$\frac{\partial Q_{Cji}}{\partial L_{Cji}} = \frac{\varphi}{\tau_{ji}} N_i^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F$$

Substituting the functional forms into (A-59), we obtain:

$$C_{ji} = \tau_{ij}^{-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1} C_{ii}, \quad i,j=H,F \tag{A-62}$$

and

$$f_E = \sum_{j=H,F}\left[\frac{L_{Cji}}{N_i(\varepsilon-1)}\right], \quad i=H,F \tag{A-63}$$

Using (A-54) and $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out L_{Cji} and $f_E N_i$ in (A-55), we find:

$$N_i = \frac{L_{Ci}}{\varepsilon f_E}, \quad i=H,F \tag{A-64}$$

We can then substitute the production function $C_{ji} = L_{ji} N_i^{\frac{1}{\varepsilon-1}} \tau_{ji}^{-1}$ into (A-62) to get:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Cii}, \quad i=H,F, j\neq i \tag{A-65}$$

Substituting this into the labor market clearing $L_{Ci} = L_{Cii} + L_{Cji} + N_i f_E$ and using the definition of $N_i = L_{Ci}/(\varepsilon f_E)$, we find that:

$$L_{Cii} = L_{Ci}\left(\frac{\varepsilon-1}{\varepsilon}\right)\left[1+\tau_{ij}^{1-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1}\right]^{-1}, \quad i = H, F \tag{A-66}$$

$$L_{Cji} = \tau_{ji}^{1-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Ci}\left(\frac{\varepsilon-1}{\varepsilon}\right)\left[1+\tau_{ij}^{1-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1}\right]^{-1}, \quad i = H, F, \quad j \neq i \tag{A-67}$$

Using again the definition of the aggregate production function, we get

$$C_{ij} = \left(\frac{\varepsilon-1}{\varepsilon}\right)\tau_{ij}^{-\varepsilon}(\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{C_j}{C_i}\right)^{\varepsilon-1}\left[1+\tau_{ij}^{1-\varepsilon}\left(\frac{C_j}{C_i}\right)^{\varepsilon-1}\right]^{-1} \quad i = H, F, \quad j \neq i. \tag{A-68}$$

B.3.6 Second-stage comparison between planner and market allocation with homogeneous firms

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation.

First, consider condition (A-62), which can be written as:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{1}{\tau_{ji}}, \quad i = H, F, \quad j \neq i$$

Now consider the market allocation. From the demand functions we get

$$\frac{C_{ii}}{C_{ji}} = \left(\frac{P_{ii}}{P_{ji}}\right)^{-\varepsilon}\left(\frac{C_i}{C_j}\right)^{1-\varepsilon}\left(\frac{P_i C_i}{P_j C_j}\right)^{\varepsilon}, \quad i = H, F, \quad j \neq i$$

This can also be written as:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{1}{\tau_{ji}}\tau_{ji}\left(\frac{P_{ii}}{P_{ji}}\right)\left(\frac{P_j C_j}{P_i C_i}\right), \quad i = H, F, \quad j \neq i$$

In other words, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}\Omega_{P2}, \quad i = H, F, \quad j \neq i$$

where $\Omega_{P2} \equiv \tau_{ji}\left(\frac{P_{ii}}{P_{ji}}\right)\left(\frac{P_j C_j}{P_i C_i}\right)$ is the wedge between the planner and the market allocation. Under symmetry $\Omega_{P2} = \tau_{Tji}^{-1}$.

Next, consider the planner's optimality condition (A-63):

$$f_E = \sum_{j=H,F}\frac{L_{Cji}}{N_i(\varepsilon-1)}, \quad i = H, F \tag{A-69}$$

We now want to check if this condition is also fulfilled in the market allocation. Recalling the labor market clearing requires $L_{Ci} = \varepsilon f_E N_i$ and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (A-69) and this proves that (A-63) is satisfied in any market allocation.

B.4 Third stage

Here we derive the results from Section 3.3.

B.4.1 Third-stage optimality conditions

In the third stage, the planner chooses C_{ij}, Z_i and L_{Ci} for $i, j = H, F$ to solve the maximization problem in (27). Taking total differentials of the objective function and of the constraints, we get:

$$
\begin{aligned}
\sum_{i=H,F} dU_i &= \sum_{i=H,F} \sum_{j=H,F} \frac{\partial U_i}{\partial C_{ij}} dC_{ij} + \sum_{i=H,F} \frac{\partial U_i}{\partial Z_i} dZ_i \\
dC_{ij} &= \frac{\partial Q_{Cij}}{\partial L_{Cj}} dL_{Cj}, \quad i, j = H, F \\
dQ_{Zi} &= \frac{\partial Q_{Zi}}{\partial L_{Ci}} dL_{Ci}, \quad i = H, F \\
\sum_{i=H,F} dQ_{Zi} &= \sum_{i=H,F} dZ_i
\end{aligned}
$$

Substituting the total differential of the constraints into the total differential of the objective and rearranging:

$$
\sum_{k=H,F} dU_k = \sum_{k=H,F} \left[\sum_{l=H,F} \frac{\partial U_l}{\partial C_{lk}} \frac{\partial Q_{Clk}}{\partial L_{Ck}} + \frac{\partial U_k}{\partial Z_k} \frac{\partial Q_{Zk}}{\partial L_{Ck}} \right] dL_{Ck} + \left[\frac{\partial U_i}{\partial Z_i} - \frac{\partial U_j}{\partial Z_j} \right] dZ_j, \quad i = H, \quad j = F
$$

It follows that at the optimum each term needs to equal zero, which leads to conditions (28) and (29) in the main text.

B.4.2 Third-stage comparison between planner and market allocation

In this Section we compare the planner and the market allocation emerging from the third stage of the planner's problem. Using the functional forms, we obtain:

$$
\begin{aligned}
\frac{\partial U_i}{\partial Z_i} &= \frac{1-\alpha}{Z_i}, \quad i = H, F \\
\frac{\partial U_i}{\partial C_{ji}} &= \frac{\alpha C_{ji}^{\frac{-1}{\varepsilon}}}{\sum_{j=H,F} C_{ji}^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i, j = H, F \\
\frac{\partial Q_{Cji}}{\partial L_{Ci}} &= \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Ci}}, \quad i, j = H, F \\
\frac{\partial Q_{Zi}}{\partial L_{Ci}} &= -1, \quad i = H, F
\end{aligned}
\tag{A-70}
$$

First consider condition (28). Using the derivatives above we get that $(1-\alpha)Z_j = (1-\alpha)Z_i$. This condition is satisfied in any symmetric market allocation.

Next consider condition (29). Plugging derivatives above in this condition we get:

$$
\alpha \sum_{j=H,F} \frac{1}{C_j} \left(\frac{C_{ji}}{C_j} \right)^{\frac{-1}{\varepsilon}} \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Cj}} = \frac{1-\alpha}{Z_i}, \quad i = H, F
$$

From definition (4), it follows that:

$$Z_i \frac{\alpha}{1-\alpha} = L_{Ci} \frac{\varepsilon - 1}{\varepsilon}, \quad i = H, F \tag{A-71}$$

We now compare this with the market allocation. We know that in the market allocation the following holds:

$$Z_i \frac{\alpha}{1-\alpha} = P_i C_i = \sum_{j=H,F} P_{ij} C_{ij}, \quad i = H, F$$

Moreover, given (11) and (12),

$$P_{ij} C_{ij} = \frac{f_{ij}(1 - G(\varphi_{ij})) \left(\frac{\tilde{\varphi}_{ij}}{\varphi_{ij}}\right)^{\varepsilon-1}}{\sum_{k=H,F} f_{ik}(1 - G(\varphi_{ik})) \left(\frac{\tilde{\varphi}_{ik}}{\varphi_{ik}}\right)^{\varepsilon-1}} L_{Cj} W_j \tau_{Lj} \tau_{Tij}, \quad i, j = H, F \tag{A-72}$$

Hence:

$$Z_i \frac{\alpha}{1-\alpha} = L_{Ci} \frac{\varepsilon - 1}{\varepsilon} \sum_{j=H,F} \left[\frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} \tau_{Tij} W_j \frac{L_{Cj}}{L_{Ci}} \frac{f_{ij}(1 - G(\varphi_{ij})) \left(\frac{\tilde{\varphi}_{ij}}{\varphi_{ij}}\right)^{\varepsilon-1}}{\sum\limits_{k=H,F} f_{ik}(1 - G(\varphi_{ik})) \left(\frac{\tilde{\varphi}_{ik}}{\varphi_{ik}}\right)^{\varepsilon-1}} \right] = L_{Ci} \frac{\varepsilon - 1}{\varepsilon} \Omega_{3P}, \quad i = H, F,$$

where Ω_{3P} is the wedge between the planner and the market allocation. Notice that in symmetric allocations:

$$\Omega_{3P} = \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} \sum_{j=H,F} \left[\tau_{Tij} \frac{f_{ij}(1 - G(\varphi_{ij})) \left(\frac{\tilde{\varphi}_{ij}}{\varphi_{ij}}\right)^{\varepsilon-1}}{\sum_{k=H,F} f_{ik}(1 - G(\varphi_{ik})) \left(\frac{\tilde{\varphi}_{ik}}{\varphi_{ik}}\right)^{\varepsilon-1}} \right],$$

which implies that $\Omega_{3P} = 1$ if $\tau_{Lj} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij} = 1$.

B.4.3 Third stage with homogeneous firms

In the third stage, the planner chooses C_{ij}, Z_i and L_{Ci} for $i, j = H, F$ to solve:

$$\begin{aligned} &\max \sum_{i=H,F} U_i \\ s.t. \quad & C_{ij} = Q_{Cij}(L_{Ci}, L_{Cj}), \quad i, j = H, F \\ & Q_{Zi} = Q_{Zi}(L - L_{Ci}), \quad i = H, F \\ \sum_{i=H,F} & Q_{Zi} = \sum_{i=H,F} Z_i, \end{aligned} \tag{A-73}$$

where U_i is given by (3) and (4), $Q_{Zi}(L - L_{Ci}) = L - L_{Ci}$ and $Q_{Cij}(L_{Ci}, L_{Cj})$ is implicitly defined in (A-68).

B.4.4 Third-stage optimality conditions with homogeneous firms

Taking total differentials of the objective function and of the constraints, we get:

$$\begin{aligned}
\sum_{i=H,F} dU_i &= \sum_{i=H,F}\sum_{j=H,F}\frac{\partial U_i}{\partial C_{ij}}dC_{ij} + \sum_{i=H,F}\frac{\partial U_i}{\partial Z_i}dZ_i \\
dC_{ij} &= \frac{\partial Q_{Cij}}{\partial L_{Ci}}dL_{Ci} + \frac{\partial Q_{Cij}}{\partial L_{Cj}}dL_{Cj}, \quad i,j=H,F \\
dQ_{Zi} &= \frac{\partial Q_{Zi}}{\partial L_{Ci}}dL_{Ci}, \quad i=H,F \\
\sum_{i=H,F} dQ_{Zi} &= \sum_{i=H,F} dZ_i
\end{aligned}$$

Note that

$$C_{ij} = \left(\frac{\varepsilon-1}{\varepsilon}\right)\tau_{ij}^{-1}(\varepsilon f_E)^{\frac{-1}{\varepsilon-1}}L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}}\left[1+\tau_{ij}^{\varepsilon-1}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1}\right]^{-1}, \quad i=H,F, \quad j\neq i$$

Taking total differentials:

$$dC_{ij} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ij}}{L_{Cj}}dL_{Cj} - C_{ij}\left[1+\tau_{ij}^{\varepsilon-1}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1}\right]^{-1}(\varepsilon-1)\tau_{ij}^{\varepsilon-1}\left(\frac{C_i}{C_j}\right)^{\varepsilon-2} d\left(\frac{C_i}{C_j}\right), \quad i=H,F, \quad j\neq i,$$

where

$$\begin{aligned}
d\left(\frac{C_i}{C_j}\right) &= \left(\frac{C_i}{C_j}\right)^{\frac{1}{\varepsilon}} C_j^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{ii}^{\frac{-1}{\varepsilon}}dC_{ii} + C_{ij}^{\frac{-1}{\varepsilon}}dC_{ij}\right) \\
&\quad - \left(\frac{C_i}{C_j}\right)^{\frac{2\varepsilon-1}{\varepsilon}} C_i^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{jj}^{\frac{-1}{\varepsilon}}dC_{ii} + C_{ji}^{\frac{-1}{\varepsilon}}dC_{ij}\right), \quad i=H,F, \quad j\neq i
\end{aligned}$$

Imposing symmetry and combining:

$$\begin{aligned}
dC_{ij} &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ij}}{L_{Cj}}dL_{Cj} - C_{ij}[1+\tau_{ij}^{1-\varepsilon}]^{-1}(\varepsilon-1)C_i^{\frac{1-\varepsilon}{\varepsilon}}\left[C_{ii}^{\frac{-1}{\varepsilon}}(1-1)dC_{ii} + C_{ij}^{\frac{-1}{\varepsilon}}(1-1)dC_{ij}\right] \\
&= \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ij}}{L_{Cj}}dL_{Cj}, \quad i=H,F, \quad j\neq i
\end{aligned}$$

Similarly,

$$C_{ii} = \left(\frac{\varepsilon-1}{\varepsilon}\right)(\varepsilon f_E)^{\frac{-1}{\varepsilon-1}}L_{Ci}^{\frac{\varepsilon}{\varepsilon-1}}\left[1+\tau_{ij}^{1-\varepsilon}\left(\frac{C_i}{C_j}\right)^{\varepsilon-1}\right]^{-1}, \quad i=H,F$$

It follows that under symmetry

$$dC_{ii} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ii}}{L_{Ci}}dL_{Ci}, \quad i=H,F$$

Imposing symmetry on the first-order conditions and substituting the total differential of the constraints into the total differential of the objective and rearranging we obtain the same first-order conditions as with heterogeneous firms.

B.4.5 Third-stage comparison between planner and market allocation with homogeneous firms

Finally, we compare the planner with the market allocation emerging from the third stage of the planner problem. Observe that the first-order conditions and the functional forms are equal to those of the case with heterogeneous firms (see (A-70)). Condition (28) is satified like in the case for heterogeneous firms, for condition (29) we have to compare the expression

$$Z_i \frac{\alpha}{1-\alpha} = L_{Ci} \frac{\varepsilon - 1}{\varepsilon}, \quad i = H, F$$

with the corresponding condition in the market allocation. We know that in the market allocation the following holds:

$$Z_i \frac{\alpha}{1-\alpha} = P_i C_i = \sum_{j=H,F} P_{ij} C_{ij}, \quad i = H, F$$

Moreover, from (A-23) and (A-25), we get:

$$P_{ij} C_{ij} = L_{Cj} W_j (\tau_{Lj} \tau_{Tij} \tau_{ij})^{1-\varepsilon} \frac{\left[\tau_{Lj}^{\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \tau_{Tji}^{\varepsilon} \left(\frac{W_i}{W_j}\right)^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1}\right]}, \quad i, j = H, F$$

Hence:

$$Z_i \frac{\alpha}{1-\alpha} = L_{Ci} \frac{\varepsilon - 1}{\varepsilon} \sum_{j=H,F} \frac{\varepsilon}{\varepsilon - 1} L_{Ci}^{-1} L_{Cj} W_j (\tau_{Lj} \tau_{Tij} \tau_{ij})^{1-\varepsilon} \frac{\left[\tau_{Lj}^{\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \tau_{Tji}^{\varepsilon} \left(\frac{W_i}{W_j}\right)^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1}\right]} = L_{Ci} \frac{\varepsilon - 1}{\varepsilon} \Omega_{3P}, \quad i = H, F,$$

where Ω_{3P} is the wedge between the planner and the market allocation. In any symmetric allocation:

$$\Omega_{3P} = \sum_{j=H,F} \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} (\tau_{Tij} \tau_{ij})^{1-\varepsilon} \frac{\left[1 - \tau_{ij}^{\varepsilon-1} \tau_{Tij}^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tij}^{\varepsilon} \tau_{ij}^{\varepsilon-1}\right]}$$

which implies that $\Omega_{3P} = 1$ if $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij} = 1$ for $i, j = H, F$ since:

$$\Omega_{3P} = \sum_{j=H,F} \frac{\varepsilon}{\varepsilon - 1} \frac{\varepsilon - 1}{\varepsilon} (\tau_{ij})^{1-\varepsilon} \frac{\left[1 - \tau_{ij}^{\varepsilon-1}\right]}{\left[\tau_{ij}^{1-\varepsilon} - \tau_{ij}^{\varepsilon-1}\right]} = \frac{(1 + \tau_{ij}^{1-\varepsilon})(1 - \tau_{ij}^{\varepsilon-1})}{\left[\tau_{ij}^{1-\varepsilon} - \tau_{ij}^{\varepsilon-1}\right]} = \frac{\left[\tau_{ij}^{1-\varepsilon} - \tau_{ij}^{\varepsilon-1}\right]}{\left[\tau_{ij}^{1-\varepsilon} - \tau_{ij}^{\varepsilon-1}\right]} = 1$$

B.4.6 The first-best allocation

In this section we show how to derive L_{Ci}^{FB} and N_i^{FB} reported in condition (30). Using condition (A-71) and the labor constraint, $L_{Ci} + L_{Zi} = L_{Ci} + Z_i = L$, we get:

$$L_{Ci}^{FB} = \frac{\alpha \varepsilon L}{\varepsilon + \alpha - 1}, \quad i = H, F,$$

which can then substituted in (A-64) to find N_i^{FB}. Since these expressions are identical in the homogeneous firm model, N_i^{FB} is also the same.

B.5 Proof of Proposition 1

Proof

First, we prove that conditions (22) and (28) and (29) (when $\alpha < 1$) are necessary conditions for the market equilibrium to coincide with the planner allocation. Suppose that in the market allocation (22) or (28) or (29) (when $\alpha < 1$) do not hold. Then, not all optimality conditions of the planner problem are satisfied in equilibrium and the market allocation cannot coincide with the planner allocation.

Second, we prove that if (22), (28) and (29)(when $\alpha < 1$) hold then the market allocation coincides with the planner allocation. If (22) holds then as shown in Appendices B.2.3 and B.3.3 for the heterogeneous-firm model and B.2.4 and B.3.6 for the homogeneous-firm model all the optimality conditions of the first and second stage of the planner problem are satisfied in the market equilibrium. Moreover, if for the case $\alpha < 1$ also conditions (28) and (29) are satisfied, then – as shown in Appendix B.4.2 and B.4.5 – all the optimality conditions of the third stage hold. As a consequence, the market equilibrium coincides with the planner allocation. ■

C The World Policy Maker Problem and the Welfare Decomposition

Here we prove the Lemmata and Propositions of Section 4. In order to do this, we first introduce a Lemma that will be useful for several proofs below.

Lemma 3 *In the market equilibrium:*

$$\frac{\tau_{Xi} P_{ii} C_{ii}}{L_{Ci}} + \frac{\tau_{Ij}^{-1} P_{ji} C_{ji}}{L_{Ci}} = \tau_{Xi} \tau_{Li} W_i, \quad i = H, F, \quad j \neq i \tag{A-74}$$

Proof In the case of heterogeneous firms, recalling (8) and using (A-72) we obtain:

$$\frac{P_{ji} C_{ji}}{L_{Ci}} = \tau_{Tji} \tau_{Li} \delta_{ji} W_i \quad i, j = H, F$$

which leads to Lemma 3.

In the case of homogeneous firms, using (16) and (17), we have that

$$\tau_{Xi} P_{ii} C_{ii} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Xi} \tau_{Li} W_i L_{Ci} [f_E^{-1} \tau_{Li}^{-\varepsilon} W_i^{-\varepsilon} B_i + f_E^{-1} \tau_{ij}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} \tau_{Li}^{-\varepsilon} W_i^{-\varepsilon} B_j]$$

where

$$B_i = \frac{f_E W_j^{\varepsilon} \left[\tau_{Lj}^{\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \tau_{Tji}^{\varepsilon} \left(\frac{W_i}{W_j} \right)^{\varepsilon} \right]}{\varphi^{\varepsilon-1} \left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \right]}$$

Substituting the expressions for B_i and B_j and multiplying and dividing the first term in square brackets by $\tau_{Tij}^{\varepsilon} \tau_{Tji}^{-\varepsilon}$, we find that the terms in square brackets can be written as:

$$\frac{\tau_{Li}^{-\varepsilon} \tau_{Lj}^{\varepsilon} \tau_{Tij}^{\varepsilon} \tau_{Tji}^{-\varepsilon} \left(\frac{W_i}{W_j} \right)^{-\varepsilon} - \tau_{ij}^{\varepsilon-1} \tau_{Tij}^{\varepsilon} + \tau_{ij}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} - \tau_{Li}^{-\varepsilon} \tau_{Lj}^{\varepsilon} \tau_{Tij}^{\varepsilon} \tau_{Tji}^{-\varepsilon} \left(\frac{W_i}{W_j} \right)^{-\varepsilon}}{-\tau_{ij}^{\varepsilon-1} \tau_{Tij}^{\varepsilon} + \tau_{ij}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon}} = 1$$

Therefore, $\tau_{Xi} P_{ii} C_{ii} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Xi} \tau_{Li} W_i L_{Ci}$ ■

Next, we prove Proposition 2.

C.1 Proof of Proposition 2

Proof The proof is organized in two steps. First, we derive the total differential of individual-country welfare by using the total differential of the trade-balance condition (13) and we show that this total differential leads to condition (32) if $\frac{\varepsilon}{\varepsilon-1} \tau_{Li} \tau_{Xi} dL_{Ci} = \tau_{Xi} P_{ii} dC_{ii} + \tau_{Ij}^{-1} P_{ji} dC_{ji}$ with $i = H, F$ and $j \neq i$. Second, we show that this condition holds in equilibrium.

Step 1 Substituting the definition of the consumption aggregator (4) into the utility function (3), we get:

$$\log U_i = \alpha \frac{\varepsilon}{\varepsilon - 1} \log \left[\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right] + (1-\alpha) \log Z_i, \quad i = H, F$$

Moreover, taking the total differential of this objective function we obtain:

$$dU_i = \alpha \frac{1}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} \sum_{j=H,F} C_{ij}^{-1/\varepsilon} dC_{ij} + (1-\alpha) \frac{1}{Z_i} dZ_i, \quad i = H, F \tag{A-75}$$

Note that $\frac{1-\alpha}{Z_i} = \frac{1}{I_i}$ and $\alpha \frac{C_{ii}^{-1/\varepsilon}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} = \frac{P_i C_i^{1/\varepsilon} C_{ii}^{-1/\varepsilon}}{I_i} = \frac{P_{ii}}{I_i}$ since $\left(\frac{C_i}{C_{ii}}\right)^{1/\varepsilon} = \frac{P_{ii}}{P_i}$ for $i = H, F$. Then, condition (A-75) can be rewritten as:

$$dU_i = \frac{1}{I_i} \sum_{j=H,F} P_{ij} dC_{ij} + \frac{1}{I_i} dZ_i, \quad i = H, F \tag{A-76}$$

Then, we can take the total differential of condition (13) and of its foreign counterpart[37] and use the fact that $Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij}$ to get:

$$-dZ_i - dL_{Ci} + d(\tau_{Ij}^{-1} P_{ji}) C_{ji} + \tau_{Ij}^{-1} P_{ji} dC_{ji} - C_{ij} d(\tau_{Ii}^{-1} P_{ij}) - (\tau_I^{-1} P_{ij}) dC_{ij} = 0, \quad i = H, F \quad j \neq i$$

Dividing this condition by I_i and adding it to (A-76) , we obtain:

$$\begin{aligned} dU_i &= \frac{P_{ii}}{I_i} dC_{ii} + \frac{P_{ij}}{I_i} dC_{ij} + \frac{1}{I_i} dZ_i - \frac{1}{I_i} dZ_i - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji} - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) - \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} \\ &= \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji}, \quad i = H, F \quad j \neq i \end{aligned}$$

Adding and subtracting terms, this can be rewritten as:

$$\begin{aligned} dU_i = & (1 - \tau_{Xi}) \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \tau_{Ii}^{-1} \frac{P_{ij}}{I_i} dC_{ij} + \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} - 1 \right) \frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) \\ & + \tau_{Xi} \frac{P_{ii}}{I_i} dC_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{dC_{ji}}{I_i} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} \frac{dL_{Ci}}{I_i}, \quad i = H, F \quad j \neq i \end{aligned}$$

Suppose the following condition holds:

$$\tau_{Xi} P_{ii} dC_{ii} + \tau_{Ij}^{-1} P_{ji} dC_{ji} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci} = 0, \quad i = H, F \quad j \neq i \tag{A-77}$$

If this is true, then:

$$dU_i = (1 - \tau_{Xi}) \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \tau_{Ii}^{-1} \frac{P_{ij}}{I_i} dC_{ij} + \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} - 1 \right) \frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}), \quad i = H, F \quad j \neq i$$

It is easy to show that this leads to condition (32) in the main text. Notice that if (A-77) holds, this last condition holds also in the case of the one-sector model in which $\alpha = 1$ and $dL_{Ci} = 0$.

Step 2 What remains to show is that in equilibrium (A-77) is always satisfied. First, notice that in the case of heterogeneous firms we have that (from equation (11)):

$$P_{ji} dC_{ji} = P_{ji} \frac{\partial C_{ji}}{\partial L_{Cji}} dL_{Ci} + P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji}, \quad i, j = H, F$$

[37] This condition can be recovered by combining (13) with (14).

Similarly, in the presence of homogeneous firms we have that (from (16))

$$P_{ji} dC_{ji} = P_{ji} \frac{\partial C_{ji}}{\partial L_{Cji}} dL_{Ci} + P_{ji} \frac{\partial C_{ji}}{\partial W_j} dW_j, \quad i, j = H, F$$

Then, showing that in equilibrium (A-77) is always satisfied is equivalent to showing that:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial L_{Ci}} dL_{Ci} + \tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \varphi_{ii}} d\varphi_{ii} + \tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial W_j} dW_j + \tag{A-78}$$
$$+ \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial L_{Ci}} dL_{Ci} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial W_j} dW_j = \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci}, \quad i = H, F \quad j \neq i$$

To see why this is the case, notice that by taking differentials of the equation in Lemma 3 and using the fact that by (11) (or (16) in the case of homogeneous firms) $\frac{\partial C_{ji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Ci}}$, we get:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial L_{Ci}} dL_{Ci} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial L_{Ci}} dL_{Ci} = \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci}, \quad i = H, F \quad j \neq i \tag{A-79}$$

Therefore, in order for (A-79) to hold for the case of heterogeneous firms, it must be that in equilibrium:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \varphi_{ii}} d\varphi_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} = 0, \quad i = H, F \quad j \neq i$$

To prove this result, first consider that by (A-33):

$$\frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} = \frac{C_{ji}}{\varphi_{ji}} \left[1 - \frac{\varepsilon}{\varepsilon - 1} \left(\Phi_i + (\varepsilon - 1) \right) \right] d\varphi_{ji}, \quad i, j = H, F \tag{A-80}$$

Hence

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \varphi_{ii}} d\varphi_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} = \left[1 - \frac{\varepsilon}{\varepsilon - 1} \left(\Phi_i + (\varepsilon - 1) \right) \right] \left(\tau_{Xi} P_{ii} \frac{C_{ii}}{\varphi_{ii}} d\varphi_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{C_{ji}}{\varphi_{ji}} d\varphi_{ji} \right), \quad i = H, F \quad j \neq i,$$

which, by (8) and (A-72), can be rewritten as:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \varphi_{ii}} d\varphi_{ii} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} = \tau_{Xi} \tau_{Li} W_i L_{Ci} \left[1 - \frac{\varepsilon}{\varepsilon - 1} \left(\Phi_i + (\varepsilon - 1) \right) \right] \left(\frac{\delta_{ii}}{\varphi_{ii}} d\varphi_{ii} + \frac{1 - \delta_{ii}}{\varphi_{ji}} d\varphi_{ji} \right)$$

Finally, recalling (A-32), we can conclude that, as postulated, this last condition is equal to zero in equilibrium for all $\alpha \in [0, 1]$. Instead, for (A-79) to hold for the case of homogeneous firms, it must be that in equilibrium:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial W_j} dW_j + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial W_j} dW_j = 0, \quad i = H, F \quad j \neq i$$

Note that by (16) and (17) we have that:

$$\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial W_j} = \frac{\varepsilon \tau_{Xi} \tau_{Lj}^{\varepsilon} \tau_{Li}^{1-\varepsilon} W_j^{\varepsilon - 1} L_{Ci}}{\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau Tji^{\varepsilon} \tau_{ij}^{\varepsilon - 1}}, \quad i = H, F \quad j \neq i \tag{A-81}$$

Moreover:

$$\tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial W_j} = - \frac{\varepsilon \tau_{Xi} \tau_{Lj}^{\varepsilon} \tau_{Li}^{1-\varepsilon} W_j^{\varepsilon - 1} L_{Ci}}{\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau Tji^{\varepsilon} \tau_{ij}^{\varepsilon - 1}}, \quad i = H, F \quad j \neq i \tag{A-82}$$

This proves that the terms in (A-81) and (A-82) sum to zero. ■

C.2 Proof of Proposition 3

Proof We prove Proposition 3 point by point.

(a) Recalling (A-1) and (A-70), we obtain the following condition:

$$\frac{\frac{\partial U_i}{\partial C_{ii}}}{\frac{\partial U_i}{\partial C_{ij}}} = \frac{P_{ii}}{P_{ij}}, \quad i = H, F, \quad j \neq i,$$

which, once they are used to substitute for either P_{ii} or P_{ij} in the consumption efficiency-term in condition (32), leads to the conditions postulated in (33).

(b) If there is is only one sector, then $dL_{Ci} = 0$ and the production-efficiency effect is absent. In addition, to see why condition (35) holds it is sufficient to substitute the derivatives in (A-70) into condition (3) and to take into account that in the two-sector model $W_i = 1$.

(c) The proof of this point is straightforward. ■

Before moving to the proof of Proposition 4 we recall how a constrained optimization problem can be solved by using the total- differential approach. This will prove to be useful in multiple instances below.

C.3 On the Total Differential Approach

In general an optimization problem in n variables and m constraints with $n > m$ can be solved using the total differential approach following the next steps.

Step 1 Take the total differential of the objective function and the constraints.

Step 2 Use the total differential of the constraints to solve for m total differentials as a function of the $n - m$ other total differentials.

Step 3 Substitute the solution of the m total differentials into the total differential of the objective function. Only then we can claim that the total differential of the objective function must be zero for *any* of the $n - m$ total differentials (i.e., for any *arbitrary* perturbation of the $n - m$ relevant variables) and find the $n - m$ conditions that need to be zero at the optimum.

Step 4 The $n - m$ conditions found in Step 3 jointly with the m constraints determine the solution of the n variables.

C.4 Proof of Proposition 4

Proof We prove Proposition 4 point by point.

(a) The constrained problem in (31) can be reduced to a maximization problem in 28 variables (22 endogenous variables plus 6 policy instruments) subject to the equilibrium conditions (8)-(14). As a first step we take the total differential of the objective of the world policy maker. In Appendix C.1 we showed how – once combined with the total differential of other equilibrium conditions – this total differential can be rewritten as in (32). Then, notice that condition (32) is a function of 6 total differentials only, namely $\{dC_{ij}, dL_{Ci}\}_{i,j=H,F}$. At the same time 6 is also the number of policy instruments available to the world policy maker. This implies that at the optimum condition (32) must be equal to zero for any arbitrary perturbation of $\{C_{ij}, L_{Ci}\}_{i,j=H,F}$ since for any arbitrary $\{dC_{ij}, dL_{Ci}\}_{i,j=H,F}$ the total differential of the 22 equilibrium conditions allows determining all the total differentials of the other 22 variables. Put differently, at the optimum all the wedges in (32) must be zero.

(b) Points b (i) and b (ii) follow directly from the proof of Lemma 3 in Appendix C.2. To show point b (iii) it is sufficient to notice that if $\tau_{Ii} = \tau_{Xi} = 1$ and $\frac{\varepsilon}{\varepsilon-1}\tau_{Xi}\tau_{Li} = 1$ for $i = H, F$ then $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$.

(c) Since by point b (i) $\tau_{Xi} = \tau_{Ii} = 1$ for $i = H, F$ and the allocation is symmetric, then by (26) the condition in point c (i) must be satisfied too so that $\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}$ for $i \neq j$. To prove point c (ii) observe that if $\alpha < 1$ – using the derivatives from (A-70) and condition (15) – it follows that in equilibrium:

$$\frac{\frac{\partial U_i}{\partial C_{ij}}}{\frac{\partial U_i}{\partial Z_i}} = \frac{\alpha}{1-\alpha}\frac{P_{ij}Z_i}{\sum_{k=H,F} P_{ik}C_{ik}} = P_{ij} \quad i,j = H,F$$

Moreover, substituting this condition into the condition in point b (i) and taking into account that $\tau_{Xi} = \tau_{Ii} = 1$ for $i = H, F$ we get the next condition:

$$\frac{\frac{\partial U_i}{\partial C_{ii}}}{\frac{\partial U_i}{\partial Z_i}}\frac{\partial Q_{Cii}}{\partial L_{Ci}} + \frac{\frac{\partial U_j}{\partial C_{ji}}}{\frac{\partial U_j}{\partial Z_j}}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial Q_{Zi}}{\partial L_{Ci}} \quad i = H,F \quad j \neq i$$

From this – given the symmetry of the allocation implemented by the world policy maker – the second condition in point c (ii) follows directly. ■

D Unilateral and Strategic Policies

Here we prove the Propositions and Corollaries of Section 5.

D.1 Proof of Proposition 5

Proof Please refer to Proof C.1 which goes through even for the individual-country policy maker. ■

D.2 Preliminary steps to study both unilateral policy changes as well as the Nash problem in the multi-sector model

We study the Nash problem for the multi-sector case, i.e., $\alpha < 1$ and $W_i = W_j = 1$ for $i, j = H, F$. Similarly to what we did to solve the world policy maker problem, we apply the total differential approach described in C.3 to solve the Nash problem. Therefore, to set up the Nash problem we proceed in three steps:

(1) First, we need the total differentials of the equilibrium equations (7)-(14) as computed in section A.8. We then impose $W_i = W_j = 1$, symmetry of the initial conditions, as well as $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, and combine the equations so as to be left with 3 equations which are linear functions of 6 differentials: dL_{Ci}, dC_{ii}, dC_{ij}, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. We can thus use the 3 equations to express 3 differentials as functions of the remaining 3. For the Nash problem we use the 3 equations to write the differentials of the tax instruments as functions of the other three differentials: $d\tau_{Li} = A(dL_{Ci}, dC_{ii}, dC_{ij})$, $d\tau_{Ii} = B(dL_{Ci}, dC_{ii}, dC_{ij})$, and $d\tau_{Xi} = C(dL_{Ci}, dC_{ii}, dC_{ij})$. To study the unilateral deviations in Section 5.2, we solve instead for dL_{Ci}, dC_{ii} and dC_{ij} as functions of the deviations of the policy instruments: $dL_{Ci} = D(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi})$, $dC_{ii} = E(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi})$, and $dC_{ij} = F(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi})$. Then, we allow only one policy instrument to vary at a time, while setting the deviations on the others two to zero.

(2) We use the differentials of the equilibrium conditions as well as the solutions for $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ derived in step 1, to write the differential of the terms-of trade effect in (37) in terms of only dL_{Ci}, dC_{ii}, dC_{ij}:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci}$$

(3) Finally, using the new expression for the terms-of-trade found in step 2, we are able to write (37) as follows:

$$\begin{aligned} dU_i =& \frac{1}{I_i}\left[(1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})\right] \\ =& \frac{1}{I_i}\left[E_{Cii}dC_{ii} + E_{Cij}dC_{ij} + E_{LCi}dLC_i + \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci}\right] \\ =& \frac{1}{I_i}\left[\Omega_{Cii}dC_{ii} + \Omega_{Cij}dC_{ij} + \Omega_{LCi}dL_{Ci}\right] \end{aligned} \tag{A-83}$$

where $E_{Cii} \equiv (1-\tau_{Xi})P_{ii}$, $E_{Cij} \equiv (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}$, $E_{LCi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1$, $\Omega_{Cii} \equiv E_{Cii} + \Sigma_{Cii}$, $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$, and $\Omega_{LCi} \equiv E_{LCi} + \Sigma_{LCi}$.

D.2.1 Step 1

We impose $W_i = W_j = 1$, symmetry of the initial conditions, as well as $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$. In terms of notation, after imposing symmetry all equations are to be considered valid for $i = H, F$ and $j \neq i$. We will thus omit to specify that for every equation. It is then useful to combine some of the differentials derived in A.8 and express them as functions of dL_{Ci}, dC_{ii}, dC_{ij}, and the differentials of the tax instruments only. Taking the symmetric condition of (A-38), using (A-33) to substitute out $d\varphi_{ji}$, solving for $d\varphi_{jj}$ and finally using (A-36) to substitute out $d\varphi_{ii}$, we obtain:

$$\begin{aligned} d\varphi_{jj} = & - \frac{\varphi_{jj}}{(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i)}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right) \\ & - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) \end{aligned} \tag{A-84}$$

Using (A-33) to substitute out $d\varphi_{jj}$ from (A-84) we find the following expression for $d\varphi_{ij}$:

$$d\varphi_{ij} = -\frac{\delta_{jj}\varphi_{ij}}{1-\delta_{jj}}\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Tji}}{\tau_{Tji}}\right) - \frac{\delta_{ii}}{1-\delta_{ii}}\frac{1}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right)\right] \tag{A-85}$$

Using (A-34) to find an expression for $d\delta_{jj}$ and combining it with (A-84) we have:

$$\begin{aligned} d\delta_{jj} = \delta_{jj}(\varepsilon-1+\Phi_j) & \left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right)\right. \\ & \left. - \frac{1}{\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right)\right] \end{aligned} \tag{A-86}$$

Then we proceed as follows. First, combine (11) and (12):

$$P_{ij}C_{ij} = L_{Cj}\delta_{ij}\tau_{Tij}\tau_{Lj} \tag{A-87}$$

Second, use (A-87) to rewrite (13) as follows:

$$L_{Cj} = \frac{\alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{(1-\delta_{jj})\tau_{Lj}\tau_{Xj}(\alpha + (1-\alpha)\tau_{Ii})} \tag{A-88}$$

Finally, take the total differential of (A-88) and use (A-37) to eliminate $d\delta_{ii}$, and (A-86) to eliminate $d\delta_{jj}$:

$$\begin{aligned}
\frac{dL_{Cj}}{L_{Cj}} = & - \frac{d\tau_{Xj}}{\tau_{Xj}} + \frac{\delta_{jj}}{1-\delta_{jj}}\frac{\varepsilon(\varepsilon-1+\Phi_j)}{\varepsilon-1}\frac{d\tau_{Ij}}{\tau_{Ij}} - \left(1 + \frac{\delta_{jj}}{1-\delta_{jj}}\frac{\varepsilon(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right)\frac{d\tau_{Lj}}{\tau_{Lj}} \\
& - \frac{1-\alpha}{\alpha+(1-\alpha)\tau_{Ii}} d\tau_{Ii} - \left(\frac{L_{Ci}((1-\alpha)\delta_{ii} - \alpha(1-\delta_{ii})\tau_{Xi})}{\Lambda_i} - \frac{\delta_{jj}\varepsilon(\varepsilon-1+\Phi_j)}{(1-\delta_{jj})(\varepsilon-1)\tau_{Li}}\right) d\tau_{Li} \\
& + \left(\frac{L_{Ci}\alpha(1-\delta_{ii})\tau_{Li}}{\Lambda_i} + \frac{\delta_{jj}\varepsilon(\varepsilon-1+\Phi_j)}{(1-\delta_{jj})(\varepsilon-1)\tau_{Xi}}\right) d\tau_{Xi} \\
& - \frac{\delta_{ii}}{\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}}\left(\frac{L_{Ci}\tau_{Li}(1-\alpha+\alpha\tau_{Xi})(\varepsilon-1+\Phi_i)}{\Lambda_i} + \frac{\delta_{jj})(\varepsilon-1+\Phi_j)}{(1-\delta_{jj})(1-\delta_{ii})}\right)\frac{dC_{ii}}{C_{ii}} \\
& - \frac{L_{Ci}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{\Lambda_i}\frac{dL_{Ci}}{L_{Ci}} \\
& + \frac{\varepsilon\delta_{ii}}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(\frac{\delta_{jj}(\varepsilon-1+\Phi_j)}{(1-\delta_{jj})(1-\delta_{ii})} + \frac{L_{Ci}\tau_{Li}(1-\alpha+\alpha\tau_{Xi})(\varepsilon-1+\Phi_i)}{\Lambda_i}\right)\frac{dL_{Ci}}{L_{Ci}}
\end{aligned} \tag{A-89}$$

Substituting (A-87) into (14), taking the total differential, and then using (A-37) to eliminate $d\delta_{ii}$, and (A-86) to eliminate $d\delta_{jj}$, we have:

$$\begin{aligned}
- & \quad (1-\alpha)L_{Cj}\left(\delta_{jj} + (1-\delta_{jj})\tau_{Ii}\tau_{Xj} - \frac{\delta_{jj}\varepsilon(1-\tau_{Ii}\tau_{Xj})(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right) d\tau_{Lj} \\
- & \quad (1-\alpha)L_{Ci}\left(\delta_{ii} + (1-\delta_{ii})\tau_{Ij}\tau_{Xi} + \frac{L_{Cj}\tau_{Lj}}{L_{Ci}\tau_{Li}}\frac{\delta_{jj}\varepsilon(1-\tau_{Ii}\tau_{Xj})(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right) d\tau_{Li} \\
- & \quad (1-\alpha)\left(L_{Ci}(1-\delta_{ii})\tau_{Li} + \frac{L_{Cj}\delta_{jj}\varepsilon\tau_{Lj}(1-\tau_{Ii}\tau_{Xj})(\varepsilon-1+\Phi_j)}{(\varepsilon-1)\tau_{Ij}\tau_{Xi}}\right)(\tau_{Xi}d\tau_{Ij} + \tau_{Ij}d\tau_{Xi}) \\
- & \quad L_{Cj}(1-\alpha)(1-\delta_{jj})\tau_{Lj}(\tau_{Ii}d\tau_{Xj} + \tau_{Xj}d\tau_{Ii}) - (\alpha + (1-\alpha)\tau_{Lj}(\delta_{jj} + (1-\delta_{jj})\tau_{Ii}\tau_{Xj}))dL_{Cj} \\
+ & \quad \frac{(1-\alpha)\delta_{ii}}{\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}}\left(\frac{L_{Cj}\delta_{jj}\tau_{Lj}(1-\tau_{Ii}\tau_{Xj})(\varepsilon-1+\Phi_j)}{1-\delta_{ii}} - L_{Ci}\tau_{Li}(1-\tau_{Ij}\tau_{Xi})(\varepsilon-1+\Phi_i)\right)\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right) \\
- & \quad (\alpha + (1-\alpha)\tau_{Li}(\delta_{ii} + (1-\delta_{ii})\tau_{Ij}\tau_{Xi}))\, dL_{Ci} = 0
\end{aligned} \tag{A-90}$$

We use (A-85) to substitute out $d\varphi_{ij}$ from (A-38), and also (A-36) to substitute out $d\varphi_{ii}$:

$$-\frac{d\tau_{Tij}}{\tau_{Tij}}(1-\delta_{jj}) + \frac{d\tau_{Tji}}{\tau_{Tji}}\delta_{jj} + \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{1-\delta_{ii}-\delta_{jj}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0 \tag{A-91}$$

Recall that $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$. This means that $d\tau_{Tji} = \tau_{Ij}d\tau_{Xi}$ and $d\tau_{Tij} = \tau_{Xj}d\tau_{Ii}$. Imposing those restrictions as well as symmetry of the initial conditions, we can rewrite (A-91):

$$\frac{d\tau_{Li}}{\tau_{Li}} - (1-\delta_{ii})\frac{d\tau_{Ii}}{\tau_{Ii}} + \delta_{ii}\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{1-2\delta_{ii}}{(1-\delta_{ii})\left(\varepsilon-1+\Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0 \tag{A-92}$$

This is going to be the first out of the three equations used to solve for $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$.

Imposing symmetry of the initial conditions as well as $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, we can rewrite (A-89) as follows:

$$\begin{aligned}
\frac{dL_{Cj}}{L_{Ci}} = & \left(\frac{\alpha}{(1-\alpha)\tau_{Ii}+\alpha} + \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})\left(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}\right)}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dC_{ii}}{C_{ii}} \\
& - \frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})\tau_{Li}\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} + \frac{\varepsilon\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii}(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dL_{Ci}}{L_{Ci}} \\
& - \frac{1-\alpha}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} - \left(\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Li}}{\tau_{Li}} \qquad \text{(A-93)}
\end{aligned}$$

Imposing symmetry of the initial conditions as well as $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, and using (A-93) to substitute out $\frac{dL_{Cj}}{L_{Ci}}$, we can rewrite (A-90) as follows:

$$\begin{aligned}
& - \frac{(1-\alpha)(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} \\
& - \frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{d\tau_{Li}}{\tau_{Li}} \\
& + \left((1-\alpha)(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \frac{\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Li}}\right)d\tau_{Li} \\
& + \frac{\alpha(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}d\tau_{Xi} \\
& + \left((1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii} + \frac{\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Xi}}\right)d\tau_{Xi} \\
& + \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(-\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} + (1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(1-\delta_{ii})\right. \\
& \left. - \frac{(1-\alpha+\alpha\tau_{Xi})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)\frac{dC_{ii}}{C_{ii}} \\
& - \left[\left(\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}} - 1\right)(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\right. \\
& - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left((\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{1-\alpha+\alpha\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right. \\
& \left.\left. + \frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} - (1-\delta_{ii})(1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})\right)\right]\frac{dL_{Ci}}{L_{Ci}} = 0 \qquad \text{(A-94)}
\end{aligned}$$

This is going to be the second out of the three equations used to solve for $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$.

Next, use (11) to solve for φ_{ii}. Second, substitute the expression for φ_{ii} into (9) and solve for φ_{ij}. Finally, use this expression for φ_{ij} together with $\delta_{ij} = 1 - \delta_{ii}$, and (A-88) to rewrite (11) as follows:

$$C_{ij} = C_{ii}\left(\frac{L_{Ci}\delta_{ii}\tau_{Ii}(L\alpha - L_{Ci}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}))}{\tau_{Li}(\alpha+(1-\alpha)\tau_{Ii})}\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{(A-95)}$$

Taking the total differential of (A-95), using (A-37) to substitute out $d\delta_{ii}$ and (A-88) and (A-95) to define, respectively, L_{Cj} and C_{ij}, we have:

$$0 = \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \left(\frac{dC_{ii}}{C_{ii}}\frac{\varepsilon-1}{\varepsilon} - \frac{dL_{Ci}}{L_{Ci}}\right)\left(1 - \frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1+\frac{L_{Ci}\delta_{ii}\tau_{Li}}{\Lambda_i}(1-\alpha+\alpha\tau_{Xi})\right)\right)$$
$$+\frac{dL_{Ci}}{\Lambda_i}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}) - \frac{d\tau_{Ii}}{\tau_{Ii}}\frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}} - d\tau_{Xi}\alpha\frac{L_{Ci}(1-\delta_{ii})\tau_{Li}}{\Lambda_i}$$
$$+d\tau_{Li}\left(\frac{L_{Ci}}{\Lambda_i}((1-\alpha)\delta_{ii}-(1-\delta_{ii})\alpha\tau_{Xi})+\frac{1}{\tau_{Li}}\right)$$

Using (A-88) under symmetry we can rewrite the previous expression as follows:

$$0 = \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}}\left(\frac{d\tau_{Ii}}{\tau_{Ii}}+\frac{d\tau_{Xi}}{\tau_{Xi}}\right) + \left(1+\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})}\right)\frac{d\tau_{Li}}{\tau_{Li}}$$
$$-\left(1-\frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1+\frac{\delta_{ii}(1-\alpha+\alpha\tau_{Xi})}{(1-\delta_{ii})\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\right)\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right)$$
$$+\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Xi}\tau_{Li}}{(1-\delta_{ii})\tau_{Xi}\tau_{Li}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} \tag{A-96}$$

This is going to be the third of the three equations used to solve for $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$.

Jointly solving (A-92), (A-94), and (A-96) with respect to $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ gives us

$$\begin{aligned} d\tau_{Li} &= A(dL_{Ci}, dC_{ii}, dC_{ij}) \\ d\tau_{Ii} &= B(dL_{Ci}, dC_{ii}, dC_{ij}) \\ d\tau_{Xi} &= C(dL_{Ci}, dC_{ii}, dC_{ij}) \end{aligned} \tag{A-97}$$

To study the unilateral deviations in Section 5.2 we will instead jointly solve (A-92), (A-94), and (A-96) with respect to dL_{Ci}, dC_{ii}, and dC_{ij} to get

$$\begin{aligned} dL_{Ci} &= D(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi}) \\ dC_{ii} &= E(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi}) \\ dC_{ij} &= F(d\tau_{Li}, d\tau_{Ii}, d\tau_{Xi}) \end{aligned} \tag{A-98}$$

The expressions for the functions A, B, C, D, E, and F are available upon request.

D.2.2 Step 2

Using (11) and (12) together with $\delta_{ji} = 1-\delta_{ii}$, $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, (A-33), (A-36) and (A-37) we obtain:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji}) = L_{Ci}(1-\delta_{ii})(\tau_{Li}d\tau_{Xi}+\tau_{Xi}d\tau_{Li}) - \frac{(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{\varepsilon-1}dL_{Ci} - \frac{\delta_{ii}\tau_{Li}\tau_{Xi}}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\frac{\Phi_i}{\varepsilon-1}\left(\frac{\varepsilon}{\varepsilon-1}dL_{Ci} - \frac{L_{Ci}}{C_{ii}}d_{Cii}\right) \tag{A-99}$$

Using (11) and (12) together with $\delta(\varphi_{ij}) = 1-\delta(\varphi_{jj})$, $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, (A-85), (A-86), and (A-93), and imposing symmetry of the initial conditions, we obtain:

$$
\begin{aligned}
C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = {} & \frac{\tau_{Li}\tau_{Xi}}{\varepsilon-1}\left[\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})}+\frac{\delta_{ii}\varepsilon}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{(1-\alpha+\alpha\tau_{Xi})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}+\frac{\delta_{ii}}{1-\delta_{ii}}\right)\right]dL_{Ci} \\
& -\frac{L_{Ci}\tau_{Xi}}{\varepsilon-1}\left(\delta_{ii}\varepsilon-\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)d\tau_{Li} \\
& +\frac{L_{Ci}(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_{Ii})}d\tau_{Ii}-\frac{L_{Ci}\tau_{Li}}{\varepsilon-1}\left(\delta_{ii}\varepsilon+(1-\delta_{ii})\left(\frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}}\right)\right)d\tau_{Xi} \\
& +\frac{L_{Ci}\delta_{ii}\tau_{Li}\tau_{Xi}}{C_{ii}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\left(\frac{(1-\alpha+\alpha\tau_{Xi})(\varepsilon-1+\Phi_i)}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}(\varepsilon-1)}+\frac{\delta_{ii}}{1-\delta_{ii}}\right)dC_{ii} \qquad \text{(A-100)}
\end{aligned}
$$

Combining (A-99) with (A-100) we obtain:

$$
\begin{aligned}
& C_{ji}d(\tau_{Ij}^{-1}P_{ji})-C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = \frac{L_{Ci}\tau_{Xi}}{\varepsilon-1}\left(\varepsilon-1+\delta_{ii}-\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)d\tau_{Li}-\frac{L_{Ci}(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_{Ii})}d\tau_{Ii} \\
& +\frac{L_{Ci}\tau_{Li}}{\varepsilon-1}\left(\varepsilon-\frac{(1-\delta_{ii})(1-\alpha)\tau_{Ii}}{\alpha+(1-\alpha)\tau_{Ii}}\right)d\tau_{Xi}-\frac{L_{Ci}\delta_{ii}\tau_{Li}\tau_{Xi}}{C_{ii}\left(\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\left(\frac{(1-\alpha)(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_i)}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}(\varepsilon-1)}+\frac{1}{1-\delta_{ii}}\right)dC_{ii} \\
& -\frac{\tau_{Li}\tau_{Xi}}{\varepsilon-1}\left[\frac{\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi}+(1-\alpha)\delta_{ii}\tau_{Li}}{\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})}\right. \\
& \left.-\frac{\delta_{ii}\varepsilon}{\varepsilon-1+\frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{1}{1-\delta_{ii}}+\frac{\varepsilon-1+\Phi_i}{\varepsilon-1}\frac{(1-\alpha)(1-\tau_{ii}\tau_{Xi})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)\right]dL_{Ci} \qquad \text{(A-101)}
\end{aligned}
$$

Using (A-97) to substitute out $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$, we can write (A-101) as function only of dL_{Ci}, dC_{ii} and dC_{ij}:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji})-C_{ij}d(\tau_{Ii}^{-1}P_{ij})=\Sigma_{Cii}dC_{ii}+\Sigma_{Cij}dC_{ij}+\Sigma_{LCi}dL_{Ci} \qquad \text{(A-102)}$$

where Σ_{Cii}, Σ_{Cij}, and Σ_{LCi} have been simplified as much as possible using equations (8)-(14) and imposing symmetry of the initial conditions:

$$
\begin{aligned}
\Sigma_{Cii} = {} & -\left(\frac{f\varepsilon}{L_{Ci}\delta_{ii}}\right)^{\frac{1}{\varepsilon-1}}\frac{\tau_{Li}\tau_{Xi}}{\delta_{ii}(\varepsilon-1)^2} \\
& \frac{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})+\alpha\delta_{ii}(\varepsilon-1)+\alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}]+\delta_{ii}\varepsilon[\alpha+(1-\alpha)\tau_{Li}]\Phi_i}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}]-\frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}\delta_{ii}[\alpha+(1-\alpha)\tau_{Li}]} \\
\Sigma_{Cij} = {} & \frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}L_{Ci}(1-\delta_{ii})\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{\varepsilon}{\varepsilon-1}}\varphi_{ij}((\delta_{ii}H-\Pi)(\varepsilon-1)-\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i))} \\
& \left[(\varepsilon-1+\delta_{ii})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})(\alpha\varepsilon+(1-\alpha)(1-\delta_{ii})\tau_{Ii})\tau_{Li}\tau_{Xi}-\frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\varepsilon-1}((1-\alpha)\tau_{Li}+\alpha)\right] \\
\Sigma_{LCi} = {} & \frac{\tau_{Li}\tau_{Xi}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)} \\
& \left[(\varepsilon-\delta_{ii})\frac{1-\alpha}{\varepsilon-1}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})+\alpha\delta_{ii}+\alpha\frac{\varepsilon}{\varepsilon-1}(1-\delta_{ii})\tau_{Li}\tau_{Xi}+\delta_{ii}\frac{\varepsilon}{(\varepsilon-1)^2}(\alpha+(1-\alpha)\tau_{Li})\Phi_i\right]
\end{aligned}
$$

with $\Pi=(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}$ and $H=\alpha+(1-\alpha)\tau_{Li}[\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}]$.

D.2.3 Step 3

Using (12) and imposing symmetry of the initial conditions, one of the two consumption efficiency terms in (37) can be written as $(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} = E_{Cij}dC_{ij}$ where:

$$E_{Cij} = (\tau_{Ii}-1)\left(\frac{\varepsilon f_{ij}}{L_{Ci}(1-\delta_{ii})}\right)^{\frac{1}{\varepsilon-1}} \frac{\varepsilon\tau_{ij}\tau_{Li}\tau_{Xi}}{(\varepsilon-1)\varphi_{ij}}$$

Therefore,

$$\Omega_{Cij} \equiv \Sigma_{Cij} + E_{Cij} = \frac{\overline{\Omega}_{Cij}\tau_{ij}\tau_{Li}\tau_{Xi}(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}}{\varphi_{ij}(\varepsilon-1)(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\left[(\delta_{ii}H-\Pi)(\varepsilon-1)-\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)\right]}$$

where

$$\overline{\Omega}_{Cij} = (\varepsilon-1)((\varepsilon-1)(1-\delta_{ii})H + \varepsilon\tau_{Ii}(\delta_{ii}H-\Pi)) - \delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)((1-\alpha)\tau_{Li}+\alpha)(\varepsilon\tau_{Ii}-\varepsilon+1) \quad \text{(A-103)}$$

Using (12), the second consumption efficiency term in (37) can be written as $(\tau_{Xi}-1)P_{ii}dC_{ii} = E_{Cii}dC_{ii}$ where:

$$E_{Cii} = (\tau_{Xi}-1)\left(\frac{\varepsilon f_{ii}}{L_{Ci}\delta_{ii}}\right)^{\frac{1}{\varepsilon-1}} \frac{\varepsilon\tau_{Li}}{(\varepsilon-1)\varphi_{ii}}$$

Therefore,

$$\Omega_{Cii} \equiv \Sigma_{Cii} + E_{Cii} = \left(\frac{\varepsilon f_{ii}}{L_{Ci}\delta_{ii}}\right)^{\frac{1}{\varepsilon-1}} \frac{\tau_{Li}}{\varphi_{ii}(\varepsilon-1)^2} \frac{\overline{\Omega}_{Cii}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})}$$

where

$$\begin{aligned}\overline{\Omega}_{Cii} \equiv &(1-\tau_{Xi})[\varepsilon(\varepsilon-1)(\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}))\\ &-(\varepsilon-1+\Phi_i)\varepsilon^2\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})]\\ &-\tau_{Xi}[(\varepsilon-1)(\varepsilon(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) - (1-\alpha)\delta_{ii}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})\\ &+\alpha\delta_{ii}(\varepsilon-1)+\alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}) + \delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i] \quad \text{(A-104)}\end{aligned}$$

The production efficiency term in (37) is given by $\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dLC_i$. Let us define $E_{LCi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1$. Then,

$$\begin{aligned}\Omega_{LCi} &\equiv \Sigma_{LCi} + E_{LCi} =\\ &= \frac{\overline{\Omega}_{LCi}(\varepsilon-1)^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}\end{aligned}$$

where

$$\begin{aligned}\overline{\Omega}_{LCi} \equiv\ & \delta_{ii}(\varepsilon-1)\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) - \varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ -\ & (\varepsilon-1)[\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ -\ & (\tau_{Li}\tau_{Xi}-1)\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i \quad \text{(A-105)}\end{aligned}$$

D.3 Preliminary steps to study unilateral policy changes in the one-sector model

In the special case $\alpha = 1$, i.e., when there is no homogeneous sector, we can apply some simplifications to the total differentials defined in A.8. Indeed in this case $Z_i = 0$ and $L_{Ci} = L$ for $i = H, F$ so that $dZ_i = dL_{Ci} = d\tau_{Li} = 0$ for $i = H, F$. Also, $W_j = 1$ so that $dW_j = 0$ for $j = F$. Finally, we set $\tau_{Li} = 1$ for $i = H, F$ since the free-trade allocation is efficient in this case. After taking the differentials we also impose symmetry of the initial free-trade allocation ($\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$) as well as $d\tau_{Ij} = d\tau_{Xj} = 0$ for $j = F$.

First, consider the case of heterogeneous firms. Our objective is to retrieve 4 conditions as a function of $dW_i, dC_{ii}, dC_{ij}, d\tau_{Ii}$ and $d\tau_{Xi}$. We proceed in 4 steps.

(1) First, recall that (A-38) simplifies to:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}} d\varphi_{ii} - \frac{\varepsilon}{\varepsilon - 1} \varphi_{ij} dW_i + \frac{\varepsilon}{\varepsilon - 1} \varphi_{ij} d\tau_{Tij}, \quad i, j = H, F, \quad i \neq j \tag{A-106}$$

Second, from (A-36) we have:

$$d\varphi_{ii} = -\frac{\varphi_{ii}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} dC_{ii}, \quad i = H, F \tag{A-107}$$

Third, from (A-106) we have $d\varphi_{jj} = \frac{\varphi_{jj}}{\varphi_{ji}} d\varphi_{ji} - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj} d\tau_{Tji}$ which, using (A-33), (A-107), and $d\tau_{Tji} = d\tau_{Xi}$ when $i = H, j = F$, can be written as:

$$d\varphi_{jj} = \frac{\varphi_{jj}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1 - \delta_{ii}} dC_{ii} - \frac{\varepsilon}{\varepsilon - 1} \varphi_{jj} d\tau_{Xi}, \quad i = H, j = F \tag{A-108}$$

Finally, using (A-33) to express $d\varphi_{ij}$ together with (A-108) to substitute out $d\varphi_{jj}$ we have:

$$d\varphi_{ij} = \frac{\delta_{jj}}{1 - \delta_{jj}} \varphi_{ij} \left(\frac{1}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1 - \delta_{ii}} dC_{ii} + \frac{\varepsilon}{\varepsilon - 1} d\tau_{Xi} \right), \quad i = H, j = F \tag{A-109}$$

Using (A-107), (A-109), $\delta_{jj} = \delta_{ii}$, and $d\tau_{Tij} = d\tau_{Ii}$ when $i = H, j = F$, we can rewrite (A-106) as follows:

$$(1 - \delta_{ii}) dW_i - (1 - \delta_{ii}) d\tau_{Ii} + \delta_{ii} d\tau_{Xi} + \frac{1 - 2\delta_{ii}}{(1 - \delta_{ii})\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\varepsilon - 1}{\varepsilon} \frac{dC_{ii}}{C_{ii}} = 0, \quad i = H, j = F \tag{A-110}$$

(2) Recall that (13) simplifies to (A-26) which, using (11) and (12) together with $L_{Ci} = L$ for $i = H, F$, $\delta_{ji} = 1 - \delta_{ii}$ for $i, j = H, F$, $\tau_{Ij} = \tau_{Xj} = 1$ for $j = F$, and $\tau_{Li} = 1$ for $i = H, F$, can be rewritten as:

$$L(1 - \delta_{ii})\tau_{Xi} W_i - L(1 - \delta_{jj}) = 0, \quad i = H, j = F \tag{A-111}$$

Using (A-34) for $i = j$ together with (A-108) we can write:

$$d\delta_{jj} = \delta_{ii}(\varepsilon - 1 + \Phi_i) \left(\frac{\varepsilon}{\varepsilon - 1} d\tau_{Xi} - \frac{\delta_{ii}}{1 - \delta_{ii}} \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{dC_{ii}}{C_{ii}} \right) \tag{A-112}$$

Taking the total differential of (A-111) and using (A-112) to substitute out $d\delta_{jj}$, and (A-34) to substitute out $d\delta_{ii}$, we have:

$$dW_i + \left(1 + \frac{\delta_{ii}\varepsilon(\varepsilon - 1 + \Phi_i)}{(\varepsilon - 1)(1 - \delta_{ii})}\right) d\tau_{Xi} - \frac{\delta_{ii}(\varepsilon - 1 + \Phi_i)}{C_{ii}(1 - \delta_{ii})^2(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i)} dC_{ii} = 0, \quad i = H, j = F \tag{A-113}$$

(3) We can rewrite (A-35) by imposing $dL_{Cj} = 0$, using (A-109) to substitute out $d\varphi_{ij}$, and $\delta_{ij} = 1 - \delta_{jj}$ (implying $d\delta_{ij} = -d\delta_{jj}$) together with (A-112) to substitute out $d\delta_{ij}$ we obtain:

$$\frac{\varepsilon - 1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} + \frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon(\varepsilon - 1 + \Phi_i)}{\varepsilon - 1} - 1\right)d\tau_{Xi} + \frac{\delta_{ii}^2}{(1-\delta_{ii})^2}\frac{1}{\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon-1}{\varepsilon} - \varepsilon + 1 - \Phi_i\right)\frac{dC_{ii}}{C_{ii}} = 0, \quad i = H, j = F \tag{A-114}$$

(4) Our last equation is given by the total differential of the terms of trade. When evaluated at the symmetric free-trade allocation of the one-sector model (38) simplifies to:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = P_{ij}C_{ij}\left[d\tau_{Xi} + dW_i + \frac{1}{\varepsilon-1}\left(\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}\right) + \left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)\right] \tag{A-115}$$

Note that at the symmetric free-trade allocation $P_{ij}C_{ij} = L(1-\delta_{ii})$. We can use $d\delta_{ij} = -d\delta_{jj}$ and (A-112) to substitute out $d\delta_{ij}$. Similarly, we can use $d\delta_{ji} = -d\delta_{ii}$ together with (A-37) for $j = i$ and $dL_{Ci} = 0$ to substitute out $d\delta_{ji}$. With (A-109) we can substitute out $d\varphi_{ij}$. Finally, using (A-33) together with (A-36) we can substitute out $d\varphi_{ji}$ and rewrite (A-115) as follows:

$$\begin{aligned} &C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \\ &L(1-\delta_{ii})dW_i + L\left(1 - \delta_{ii} - \delta_{ii}\frac{\varepsilon}{(\varepsilon-1)^2}\Phi_i\right)d\tau_{Xi} + \frac{L\delta_{ii}\Phi_i}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)(1-\delta_{ii})(\varepsilon-1)}dC_{ii} \end{aligned} \tag{A-116}$$

Then, consider the case with homogeneous firms. In this case condition (A-115) can be written as:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = P_{ij}C_{ij}\left[d\tau_{Xi} + dW_i\right] \tag{A-117}$$

Moreover, we can substitute conditions (16) and (17) into the trade balance (A-26). Taking the total differential of this condition and evaluating it at the free-trade allocation we get:

$$dW_i = \frac{\varepsilon\tau^{\varepsilon}}{\tau + (2\varepsilon - 1)\tau^{\varepsilon}}d\tau_{Ii} - \frac{\tau + (\varepsilon-1)\tau^{\varepsilon}}{(\tau + (2\varepsilon-1)\tau^{\varepsilon})}d\tau_{Xi} \tag{A-118}$$

D.4 How Policy Instruments affect the Terms of Trade

Lemma 4 *Consider a marginal unilateral increase in each of the trade policy instruments at a time, starting from the free-trade equilibrium, i.e., with $\tau_{Li} = \tau_{Ii} = \tau_{Xi}$ for $i = H, F$. Then:*

(a) In the one-sector model deviating from the free-trade equilibrium induces:

(1) $\frac{dW_i}{W_i} - \frac{dW_j}{W_j} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{dW_i}{W_i} - \frac{dW_j}{W_j} < 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$

(2) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(3) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$.

(b) In the multi-sector model deviating from the free-trade equilibrium induces:

(1) $\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$;
$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} > 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$;

(2) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} < 0 \iff \delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0 \iff \delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*;*

(3) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} > 0 \iff \delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0 \iff \delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*.*

Proof

(a) (1) Combining conditions (A-114), (A-113) and (A-110) we find dC_{ii}, dW_i and dC_{ij} ad a function of the deviations of the policy instruments. In particular it can be shown that at free trade:

$$dW_i = a_w d\tau_{Ii} + b_w d\tau_{Xi}$$

where $a_w = \frac{\delta_{ii} d\tau_{Ii} \varepsilon(\Phi_i + \varepsilon - 1)}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} > 0$ since $\varepsilon > 1$ and $0 < \delta_{ii} < 1$. Moreover $b_w = -1$.

(2) Recall that $d\delta_{ji} = -d\delta_{ii}$ for $i, j = H, F$ and $j \neq i$. Then we can use (A-112) and its symmetric counterpart to obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = a_\delta d\tau_{Ii}$$

where $a_\delta = -\frac{\delta_{ii}\varepsilon\phi_{ij}(\varepsilon-1+\Phi_i)}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$ for $\varepsilon > 1$ and $0 < \delta_{ii} < 1$.

(3) Using the solution for dC_{ii} found in point (a), condition (A-109) and their symmetric counterparts we obtain:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = a_\varphi d\tau_{Ii}$$

where $a_\varphi = -\frac{\delta_{ii}\varepsilon\phi_{ij}}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$ for $\varepsilon > 1$ and $0 < \delta_{ii} < 1$.

(b) (1) Using (A-93) together with (A-98), imposing symmetry of the initial conditions and $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$, and setting to zero $d\tau_{Li}$, $d\tau_{Lj}$, $d\tau_{Ij}$ and $d\tau_{Xj}$ we obtain:

$$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} = a_1 d\tau_{Ii} + a_2 d\tau_{Xi}$$

where $a_1 \equiv -\frac{(1-\delta_{ii})[(\varepsilon-1)(1-\alpha+2\delta_{ii}(\varepsilon-1+\alpha))+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} < 0$ and $a_2 \equiv \frac{(1-\delta_{ii})[(1+\alpha+2\delta_{ii}(\varepsilon-1-\alpha))(\varepsilon-1)+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} > 0$. To see that $a_2 > 0$ it is enough to notice that $a_3(\delta_{ii}) \equiv 1 + \alpha + 2\delta_{ii}(\varepsilon - 1 - \alpha)$ is linear in δ_{ii} and that $a_3(0) = 1 + \alpha > 0$ and $a_3(1) = 2\varepsilon - 1 - \alpha > 0$.

(2) Recall that $\delta_{ji} = 1 - \delta_{ii}$, implying that $d\delta_{ji} = -d\delta_{ii}$ and $d\delta_{ij} = -d\delta_{jj}$. Using (A-37) to compute $d\delta_{ii}$ and (A-86) to compute $d\delta_{jj}$, together with (A-98), imposing symmetry of the initial conditions and $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$, and setting to zero $d\tau_{Li}$, $d\tau_{Lj}$, $d\tau_{Ij}$ and $d\tau_{Xj}$ we obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = b_1 \left(d\tau_{Ii} - d\tau_{Xi}\right)$$

where $b_1 \equiv -\frac{(1-\delta_{ii})\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\delta_{ij}(\varepsilon-1)(2\delta_{ii}-1)} < 0 \iff \delta_{ii} > 1/2$;

(3) Using (A-33) and (A-36) to compute $d\varphi_{ji}$ and (A-85) to compute $d\varphi_{ij}$, together with (A-98), imposing symmetry of the initial conditions and $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$, and setting to zero $d\tau_{Li}$, $d\tau_{Lj}$, $d\tau_{Ij}$ and $d\tau_{Xj}$, we obtain:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = c_1 \left(d\tau_{Ii} - d\tau_{Xi}\right)$$

where $c_1 \equiv \frac{\delta_{ii}\varepsilon}{(2\delta_{ii}-1)(\varepsilon-1)} > 0 \iff \delta_{ii} > 1/2$.

■

D.5 Proof of Lemma 1

We prove Lemma 1 point by point.

(a) From Proposition 4 (b) (iii) we know that the free-trade allocation is Pareto optimal in the one-sector model, implying that the production-efficiency wedge is zero for all policy instruments. Formally, in the one-sector model $dL_{Ci} = 0$ in (37).

(b) When $\tau_{Ii} = \tau_{Xi} = 1$, the consumption-efficiency wedge in (37) is zero for any dC_{ii} and dC_{ij}.

(c) In the case of heterogeneous firms, we can use (A-116) to write the terms-of-trade effect in (37) as function of dW_i, dC_{ii}, and $d\tau_{Xi}$. The equations (A-110), (A-113), and (A-114) give us a linear system of equations that we can solve to express:

$$\begin{aligned} dC_{ii} &= A(d\tau_{Ii}, d\tau_{Xi}) \\ dW_i &= B(d\tau_{Ii}, d\tau_{Xi}) \\ dC_{ij} &= C(d\tau_{Ii}, d\tau_{Xi}) \end{aligned} \tag{A-119}$$

The expressions for A, B, and C are available upon request. Using (A-119) we can substitute out dW_i and dC_{ii} from (A-116), and express the terms-of-trade effect as function only of $d\tau_{Ii}$ and $d\tau_{Xi}$:

$$C_{ij}[d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})] = a\ d\tau_{Ii} + b\ d\tau_{Xi}$$

where

$$\begin{aligned} a &= \frac{L(1-\delta_{ii})\delta_{ii}\varepsilon((\varepsilon-1)^2 + \varepsilon\Phi_i)}{(\varepsilon-1)[(\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1)) + \varepsilon\delta_{ii}\Phi_i]} > 0 \\ b &= 0 \end{aligned}$$

This means that a small positive import tax generates a positive terms-of-trade effect while small changes in τ_{Xi} do not produce terms-of-trade effects.

In the case of homogeneous firms, if we substitute (A-118) into (A-117) we get:

$$C_{ij}[d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})] = c\ d\tau_{Ii} + d\ d\tau_{Xi}$$

where

$$\begin{aligned} c &= \frac{\varepsilon\tau^{\varepsilon}}{\tau + (2\varepsilon - 1)\tau^{\varepsilon}} > 0 \\ d &= \frac{\varepsilon\tau^{\varepsilon}}{\tau + (2\varepsilon - 1)\tau^{\varepsilon}} > 0 \end{aligned}$$

This means that the net terms-of-trade effect of a small tariff or a small export tax is positive.

(d) Follows from the previous points.

D.6 Proof of Lemma 2

We prove Lemma 2 point by point. Recall that:

- Production-efficiency wedge: $ProdWedge \equiv \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1\right) dL_{Ci}$
- Consumption-efficiency wedge: $ConsWedge \equiv (1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij}$
- Terms-of-trade effect: $TotEff \equiv \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci}$
- Total welfare change: $TotalWelfare = ProdWedge + ConsWedge + TotEff$.

Where we made use of (A-102) to write the terms-of-trade effect as function of dL_{Ci}, dC_{ii}, and dC_{ij}. Next, we use (A-98) to evaluate $ProdWedge$, $ConsWedge$, $TotEff$, and $TotalWelfare$ when the domestic country changes one instrument at the time. We evaluate those terms at the free-trade equilibrium i.e., $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$. For the proof it is useful to remember that: $\varepsilon > 1$, $0 < \alpha < 1$, $0 < \delta_{ii} < 1$, $\Phi_i > 0$, and $L_{Ci} > 0$.

(a1) When $d\tau_{Li} = d\tau_{Xi} = 0$ then $ProdWedge = \frac{L_{Ci}(1-\delta_{ii})\delta_{ii}}{(1-2\delta_{ii})^2(\varepsilon-1)^2}\left[(\varepsilon-1)((1-\alpha)(1-2\delta_{ii})+\varepsilon)+\varepsilon\Phi_i\right]d\tau_{Ii}$. Note that even when $\delta_{ii} = 1$, then $(1-\alpha)(1-2\delta_{ii})+\varepsilon = -(1-\alpha)+\varepsilon > 0$. Therefore, $ProdWedge > 0 \iff d\tau_{Ii} > 0$.

(a2) When $d\tau_{Li} = d\tau_{Ii} = 0$ then $ProdWedge = -\frac{L_{Ci}(1-\delta_{ii})}{(1-2\delta_{ii})^2(\varepsilon-1)^2}[(\varepsilon-1)(1+\delta_{ii}(\alpha+2\delta_{ii}(1-\alpha)+\varepsilon-3)) + \delta_{ii}\varepsilon\Phi_i]d\tau_{Xi}$. A sufficient condition for $ProdWedge < 0 \iff d\tau_{Xi} > 0$ is $Pol(\delta_{ii}) \equiv 1+\delta_{ii}(\alpha+2\delta_{ii}(1-\alpha)+\varepsilon-3) > 0$. In what follows we show that this is always the case i.e., $Pol(\delta_{ii}) > 0 \ \forall\ \delta_{ii} \in [0,1]$.

$Pol(\delta_{ii})$ is a quadratic function in δ_{ii} with:

- $Pol(0) = 1$;
- $Pol(1) = \varepsilon - \alpha > 0$;
- $\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=0} = \varepsilon + \alpha - 3 \lesseqgtr 0$;
- $\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=1} = \varepsilon + 1 - 3\alpha \lesseqgtr 0$;
- $\frac{\partial^2 Pol(\delta_{ii})}{\partial \delta_{ii}^2} = 4(1-\alpha) > 0$ i.e., the function as a minimum;
- $\min Pol(\delta_{ii}) \equiv MinPol(\varepsilon,\alpha) = -\frac{(1+\alpha)^2 - 2(3-\alpha)\varepsilon + \varepsilon^2}{8(1-\alpha)}$.

Note that $Pol(\delta_{ii}) > 0$ at both extremes of the interval $\delta_{ii} \in [0,1]$. This implies that if $\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=0} \geq 0$ ($\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=1} \leq 0$) then $Pol(\delta_{ii}) > 0$ has to be monotonically increasing (decreasing) and always positive in $\delta_{ii} \in [0,1]$. Therefore, two necessary conditions for $Pol(\delta_{ii}) < 0$ in $\delta_{ii} \in (0,1)$ are $\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=0} < 0$ and $\left.\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\right|_{\delta_{ii}=1} > 0$ i.e., $\max\{1, 3\alpha-1\} < \varepsilon < 3-\alpha$.

The last step is to show that $MinPol(\varepsilon,\alpha) > 0$ always when $\max\{1,3\alpha-1\} < \varepsilon < 3-\alpha$. Note that $\frac{\partial MinPol(\varepsilon,\alpha)}{\partial \varepsilon} = \frac{3-\varepsilon-\alpha}{4(1-\alpha)}$ decreases in ε and that at the maximum admissible range we have $\left.\frac{\partial MinPol(\varepsilon,\alpha)}{\partial \varepsilon}\right|_{\varepsilon=3-\alpha} = 0$, implying $MinPol(\varepsilon,\alpha)$ increases in ε. Our last step is therefore to evaluate the sign of $MinPol(\varepsilon,\alpha)$ at its minimum possible level i.e., substituting the minimum admissible range for ε. There are two cases. If $\alpha > \frac{2}{3}$ then $\varepsilon = \max\{1,3\alpha-1\} = 3\alpha-1$. If instead $\alpha < \frac{2}{3}$, then $\varepsilon = \max\{1,3\alpha-1\} = 1$. In the first case we have $MinPol(3\alpha-1,\alpha) = 2\alpha-1 > 0$ when $\alpha > \frac{2}{3}$, which is true. In the second case we have $MinPol(1,\alpha) = \frac{4-\alpha(4+\alpha)}{8(1-\alpha)}$. Note that $MinPol(1,\alpha) = 0$ for $\alpha_1 = -2(1+\sqrt{2}) < 0$ and $\alpha_2 = -2+2\sqrt{(2)} > \frac{2}{3}$. This means that $MinPol(1,\alpha)$ does not change sign in our relevant range $0 < \alpha < \frac{2}{3}$. Observing that $MinPol(1,0) = \frac{1}{2}$ shows that $MinPol(1,\alpha) > 0$ when $0 < \alpha < \frac{2}{3}$. We can thus conclude that when $Pol(\delta_{ii})$ has a minimum in $0 < \delta_{ii} < 1$, such minimum is always positive.

(a3) When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $ProdWedge = L_{Ci}\frac{(\varepsilon-1)[2\delta_{ii}^2(\varepsilon+\alpha-2)-1-\delta_{ii}(2\varepsilon+\alpha-4)]-2(1-\delta_{ii})\delta_{ii}\varepsilon\Phi_i}{(1-2\delta_{ii})^2(\varepsilon-1)^2}d\tau_{Li}$. A sufficient condition for $ProdWedge < 0 \iff d\tau_{Li} > 0$ is $Pol(\delta_{ii}) \equiv 2\delta_{ii}^2(\varepsilon+\alpha-2) - 1 - \delta_{ii}(2\varepsilon+\alpha-4) < 0$. In what follows we show that this is always the case i.e., $Pol(\delta_{ii}) < 0 \;\forall\; \delta_{ii} \in [0,1]$.

$Pol(\delta_{ii})$ is a quadratic function in δ_{ii} with:

- $Pol(0) = -1$;
- $Pol(1) = -1 + \alpha$;
- $\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\Big|_{\delta_{ii}=1} = -4 + 3\alpha + 2\varepsilon \lesseqgtr 0$;
- $\frac{\partial^2 Pol(\delta_{ii})}{\partial \delta_{ii}^2} = 4(\varepsilon - 2 + \alpha) \lesseqgtr 0$;
- $\max Pol(\delta_{ii}) \equiv MaxPol(\varepsilon,\alpha) = -\frac{\varepsilon}{2} - \frac{\alpha^2}{8(-2+\alpha+\varepsilon)}$.

Note that $Pol(0) < Pol(1) < 0$. This implies that when $\frac{\partial^2 Pol(\delta_{ii})}{\partial \delta_{ii}^2} > 0$, $Pol(\delta_{ii})$ has a minimum for $0 \leq \delta_{ii} < 1$ and it is always negative in this range. We thus have to show that $Pol(\delta_{ii}) < 0$ even when $\frac{\partial^2 Pol(\delta_{ii})}{\partial \delta_{ii}^2} < 0$ i.e., when the function has a maximum. This means we can restrict our analysis to the case $\varepsilon < 2 - \alpha$. Here there are two possible scenarios. When $\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\Big|_{\delta_{ii}=1} \geq 0$, the function is monotonically increasing, and thus always negative, for all $\delta_{ii} \in (0,1]$. This happens when $\varepsilon \geq 2 - \frac{3}{2}\alpha$. When instead $1 < \varepsilon < 2 - \frac{3}{2}\alpha$, then $Pol(\delta_{ii})$ has a maximum for $\delta_{ii} \in (0,1)$. The last step is to show that such maximum is always negative. Note that $MaxPol(\varepsilon,\alpha) = 0$ for $\varepsilon_1 = 1 - \sqrt{1-\alpha} - \frac{\alpha}{2}$ and $\varepsilon_2 = 1 + \sqrt{1-\alpha} - \frac{\alpha}{2}$. It is easy to see that $\varepsilon_1 < 1$ and that $\varepsilon_2 > 2 - \frac{3}{2}\alpha$, i.e., $MaxPol(\varepsilon,\alpha)$ never changes sign in $1 < \varepsilon < 2 - \frac{3}{2}\alpha$ and $0 < \alpha < 1$. To complete the proof it is then enough to show that $MaxPol(\varepsilon,\alpha) < 0$ at one point in our interval. For example, if $\alpha = 0.5$, $\varepsilon = 1.2 < 2 - \frac{3}{2}\alpha$ and $MaxPol(1.2, 0.5) = -0.49 < 0$.

(b) $ConsWedge = 0$ for all policy instruments when $\tau_{Ii} = \tau_{Xi} = 1$.

(c1) When $d\tau_{Li} = d\tau_{Xi} = 0$ then $TotEff = -\frac{L_{Ci}(1-\delta_{ii})[(1-\delta_{ii})(\varepsilon-1)(1-\alpha+(\varepsilon-1+\alpha)2\delta_{ii})+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}d\tau_{Ii}$. Therefore, $TotEff < 0 \iff d\tau_{Ii} > 0$.

(c2) When $d\tau_{Li} = d\tau_{Ii} = 0$ then $TotEff = \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-1)(\delta_{ii}+2\delta_{ii}^2(\varepsilon-1)+(\varepsilon+\alpha(1-\delta_{ii}))(1-2\delta_{ii}))+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}d\tau_{Xi}$. A sufficient condition for $TotEff > 0 \iff d\tau_{Xi} > 0$ is $Pol(\delta_{ii}) \equiv \delta_{ii} + 2\delta_{ii}^2(\varepsilon-1) + (\varepsilon+\alpha(1-\delta_{ii}))(1-2\delta_{ii}) > 0$. In what follows we show that this is always the case i.e., $Pol(\delta_{ii}) > 0 \;\forall\; \delta_{ii} \in [0,1]$.

$Pol(\delta_{ii})$ is a quadratic function in δ_{ii} with:

- $Pol(0) = \varepsilon + \alpha > \varepsilon - 1 = Pol(1) > 0$ i.e., the function is positive at both ends of the relevant interval;
- $\frac{\partial^2 Pol(\delta_{ii})}{\partial \delta_{ii}^2} = 4(\varepsilon - 1 + \alpha) > 0$ i.e., the function as a minimum;
- $\min Pol(\delta_{ii}) = \frac{4\varepsilon(\alpha+\varepsilon-1)-(1+\alpha)^2}{8(\varepsilon-1+\alpha)}$;
- $\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\Big|_{\delta_{ii}=1} = 2\varepsilon + \alpha - 3 \lesseqgtr 0$.

When $\varepsilon < \frac{3-\alpha}{2}$, then $\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\Big|_{\delta_{ii}=1} < 0$ implying $Pol(\delta_{ii})$ is monotonically decreasing and always positive for $\delta_{ii} \in [0,1]$. When $\varepsilon > \frac{3-\alpha}{2}$, then $\frac{\partial Pol(\delta_{ii})}{\partial \delta_{ii}}\Big|_{\delta_{ii}=1} > 0$ implying $Pol(\delta_{ii})$ reaches the minimum in $\delta_{ii} \in [0,1]$. Therefore, the last step we need to show is that $\min Pol(\delta_{ii}) > 0$ always when $\varepsilon > \frac{3-\alpha}{2}$.

This is indeed the case since $4\varepsilon(\alpha+\varepsilon-1) - (1+\alpha)^2 > 4\frac{3-\alpha}{2}(\alpha + \frac{3-\alpha}{2} - 1) - (1+\alpha)^2 = 2(1-\alpha^2) > 0$.

(c3) When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $TotEff = \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-\alpha(2\delta_{ii}-1))(\varepsilon-1)+\varepsilon 2\delta_{ii}\Phi_{ii}]}{(1-2\delta_{ii})^2(\varepsilon-1)^2} d\tau_{Li}$. Note that $\varepsilon - \alpha(2\delta_{ii} - 1) > 0$ even when $\delta_{ii} = 1$. Therefore, $TotEff > 0 \iff d\tau_{Li} > 0$.

(d1) When $d\tau_{Li} = d\tau_{Xi} = 0$ then $TotalWelfare = \frac{L_{Ci}(1-\delta_{ii})[\delta_{ii}\varepsilon-(2\delta_{ii}-1)(1-\alpha)]}{(2\delta_{ii}-1)(\varepsilon-1)} d\tau_{Ii}$. Note that $\delta_{ii} \leq 2\delta_{ii} - 1 \iff \delta_{ii} \leq 1$ which is always the case, implying that the numerator is always positive. Note also that at the denominator $2\delta_{ii} - 1 > 0 \iff \delta_{ii} > \frac{1}{2}$. Thus, $TotalWelfare > 0$ when $d\tau_{Ii} > 0 \iff \delta_{ii} > \frac{1}{2}$.

(d2) When $d\tau_{Li} = d\tau_{Ii} = 0$ then $TotalWelfare = \frac{L_{Ci}(1-\delta_{ii})[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} d\tau_{Xi}$. Note that $1 - 2\delta_{ii} < 1 - \delta_{ii}$ and $1 - \alpha < \varepsilon$, implying that $(1-2\delta_{ii})(1-\alpha) - (1-\delta_{ii})\varepsilon < 0$ i.e., the numerator is always negative. Note also that the denominator is positive $\iff \delta_{ii} > \frac{1}{2}$. Thus, $TotalWelfare < 0$ when $d\tau_{Xi} > 0 \iff \delta_{ii} > \frac{1}{2}$.

(d3) When $d\tau_{Ii} = d\tau_{Xi} = 0$ then $TotalWelfare = \frac{L_{Ci}[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} d\tau_{Li}$. Note that $1 - 2\delta_{ii} < 1 - \delta_{ii}$ and $1 - \alpha < \varepsilon$, implying that $(1-2\delta_{ii})(1-\alpha) - (1-\delta_{ii})\varepsilon < 0$ i.e., the numerator is always negative. Note also that the denominator is positive $\iff \delta_{ii} > \frac{1}{2}$. Therefore, $TotalWelfare < 0$ when $d\tau_{Li} > 0 \iff \delta_{ii} > \frac{1}{2}$.

D.7 Proof of Proposition 6

Proof We prove Proposition 6 point by point.

(a) and **(b)** The constrained problem in (36) can be reduced to a maximization problem in 25 variables (22 endogenous variables plus 3 policy instruments) subject to the equilibrium conditions (7)-(14). As a first step we take the total differential of the objective of the individual country policy maker. In Appendix D.1 we showed how – once combined with the total differential of other equilibrium conditions – this total differential can be rewritten as in (37). Next, in Appendix D.2 we showed how to rewrite (37) as (A-83) i.e., as a function of 3 total differentials only, namely $\{dC_{ii}, dC_{ij}, dL_{Ci}\}_{i \neq j}$. At the same time 3 is also the number of policy instruments available to the individual-country policy maker. This implies that at the optimum condition (A-83) must be equal to zero for any arbitrary perturbation of $\{C_{ii}, C_{ij}, L_{Ci}\}_{i \neq j}$ since for any arbitrary $\{dC_{ii}, dC_{ij}, dL_{Ci}\}_{i \neq j}$ the total differential of the 22 equilibrium conditions allows determining all the total differentials of the other 22 variables. Put differently, at the optimum all the wedges in (A-83) must be zero, i.e., in the Nash $\Omega_{LCi} = \Omega_{Cii} = \Omega_{Cij} = 0$. Finally, note that $\Omega_{LCi} = \Omega_{Cii} = \Omega_{Cij} = 0 \iff \overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = \overline{\Omega}_{Cij} = 0$. We thus use equations (A-103)-(A-105) to characterize the Nash equilibrium in point (b).

(c) For the model version with homogeneous firms see Campolmi et al. (2014).

For the case with firm heterogeneity, first, note that $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ are functions only of 8 variables: τ_{Li}, τ_{Ii}, τ_{Xi}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}.

Second, note that under symmetry of the initial conditions the equilibrium system of equations (7)-(14) gives us the following 5 equations, which allow us to solve for φ_{ii}, φ_{ji}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ji}$, and δ_{ii} given the 3 policy instruments:

$$\begin{aligned}
\widetilde{\varphi}_{ji} &= \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})}\right]^{\frac{1}{\varepsilon-1}} \\
\widetilde{\varphi}_{ii} &= \left[\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ii})}\right]^{\frac{1}{\varepsilon-1}} \\
\delta_{ii} &= \frac{f_{ii}(1-G(\varphi_{ii}))}{f_{ii}(1-G(\varphi_{ii}))+f_{ji}(1-G(\varphi_{ji}))+f_E} \left(\frac{\widetilde{\varphi}_{ii}}{\varphi_{ii}}\right)^{\varepsilon-1} \\
\frac{\varphi_{ii}}{\varphi_{ji}} &= \left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{1}{\varepsilon-1}} \left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ji}^{-1}(\tau_{Ii}\tau_{Xi})^{-\frac{\varepsilon}{\varepsilon-1}} \\
1 &= \frac{f_{ii}(1-G(\varphi_{ii}))}{f_{ii}(1-G(\varphi_{ii}))+f_{ji}(1-G(\varphi_{ji}))+f_E} \left(\frac{\widetilde{\varphi}_{ii}}{\varphi_{ii}}\right)^{\varepsilon-1} + \frac{f_{ji}(1-G(\varphi_{ji}))}{f_{ii}(1-G(\varphi_{ii}))+f_{ji}(1-G(\varphi_{ji}))+f_E} \left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}
\end{aligned} \tag{A-120}$$

where the last equation makes use of $\delta_{ji} = 1 - \delta_{ii}$.

Solving for the symmetric Nash problem simplifies to searching for $\{\tau_L^{Nash}, \tau_I^{Nash}, \tau_X^{Nash}\}$ such that

$$\overline{\Omega}_{LCi}(\tau_L^{Nash}, \tau_I^{Nash}, \tau_X^{Nash}) = \overline{\Omega}_{Cii}(\tau_L^{Nash}, \tau_I^{Nash}, \tau_X^{Nash}) = \overline{\Omega}_{Cij}(\tau_L^{Nash}, \tau_I^{Nash}, \tau_X^{Nash}) = 0$$

under the system in (A-120).

We then proceed in 3 steps. First, we show that in the Nash equilibrium it must be the case that $\tau_L^{Nash} = \frac{\varepsilon-1}{\varepsilon}$. Second, we show that $\overline{\Omega}_{LCi}(\tau_L^{Nash}, \tau_I, \tau_X) > 0$ always when $\tau_X < 1$. Therefore, when a Nash equilibrium exists it must be such that $\tau_X^{Nash} > 1$. Finally, we show that $\overline{\Omega}_{Cij}(\tau_L^{Nash}, \tau_I, \tau_X^{Nash}) < 0$ always when $\tau_I > 1$. Therefore, when a Nash equilibrium exists it must be such that $\tau_I^{Nash} < 1$.

(1) We use $\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = 0$ to solve for τ_L and τ_I and we obtain two set of solutions, (τ_L^1, τ_I^1) and (τ_L^2, τ_I^2):

$$\begin{aligned}
\tau_L^1 &= \frac{\varepsilon-1}{\varepsilon} \\
\tau_I^1 &= \frac{(1-\alpha)\delta_{ii}^2(\varepsilon(1-\tau_X)+\tau_X) - \alpha\varepsilon\tau_X + \delta_{ii}\varepsilon((\varepsilon-1+\alpha)\tau_X - \varepsilon)}{(1-\alpha)(1-\delta_{ii})\tau_X[\varepsilon(1-\delta_{ii}) + \delta_{ii}\tau_X(\varepsilon-1)]} \\
&\quad + \frac{\delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon(\tau_X-1)-\tau_X)\Phi_i}{(1-\alpha)(1-\delta_{ii})(\varepsilon-1)^2\tau_X[\varepsilon(1-\delta_{ii}) + \delta_{ii}\tau_X(\varepsilon-1)]} \\
\tau_L^2 &= -\alpha\frac{1+\varepsilon(\varepsilon-2+\Phi_i)}{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii}) + \alpha(1-\delta_{ii})\tau_X] + (1-\alpha)\varepsilon\Phi_i} \\
\tau_I^2 &= -\frac{\alpha}{1-\alpha}
\end{aligned}$$

Note that $\tau_I^2 < 0$ which is outside the admissible range for τ_I. Thus, the only possible solution is (τ_L^1, τ_I^1), implying that when a Nash solution exists, it must be that $\tau_L^{Nash} = \frac{\varepsilon-1}{\varepsilon}$. We can thus substitute τ_L^{Nash} into $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ obtaining $\overline{\Omega}_{LCi}^N$, $\overline{\Omega}_{Cii}^N$, and $\overline{\Omega}_{Cij}^N$, where the upper index N indicates that these are the expressions when $\tau_L = \tau_L^{Nash}$:

$$\begin{aligned}
\overline{\Omega}_{LCi}^N &= \overline{\Omega}_{LCi}^{N,1} + \overline{\Omega}_{LCi}^{N,2} \\
\overline{\Omega}_{Cii}^N &= -\frac{\overline{\Omega}_{LCi}^N}{\varepsilon} \\
\overline{\Omega}_{Cij}^N &= \overline{\Omega}_{Cij}^{N,1} + \overline{\Omega}_{Cij}^{N,2}
\end{aligned}$$

where

$$\overline{\Omega}_{LCi}^{N,1} \equiv (\varepsilon-1)^2[\delta_{ii}(\varepsilon-(\varepsilon-1)\tau_X)(\varepsilon-(1-\alpha)\delta_{ii}) + \delta_{ii}(\varepsilon-1)\tau_X((1-\alpha)(1-\delta_{ii})\tau_I\tau_X) \\ + \varepsilon((1-\delta_{ii})(\alpha+(1-\alpha)(1-\delta_{ii})\tau_I)\tau_X)]$$

$$\overline{\Omega}_{LCi}^{N,2} \equiv \delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon-(\varepsilon-1)\tau_X)\Phi_i$$

$$\overline{\Omega}_{Cij}^{N,1} \equiv (\varepsilon-1)\left[\delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1) + \delta_{ii}\tau_I(\alpha\varepsilon+\delta_{ii}(\varepsilon-1)(1-\alpha)) + (1-\delta_{ii})(\varepsilon-1)\left(\alpha+\frac{\delta_{ii}(1-\alpha)(\varepsilon-1)}{\varepsilon}\right)\right. \\ \left. +(1-\delta_{ii})(\varepsilon-1)\tau_I\tau_X\left(\frac{(1-\alpha)(\varepsilon-1)(1-\delta_{ii})}{\varepsilon}-\alpha-(1-\alpha)(1-\delta_{ii})\tau_I\right)\right]$$

$$\overline{\Omega}_{Cij}^{N,2} \equiv \delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1)\Phi_i$$

Note that $\overline{\Omega}_{Cii}^{N}$ and $\overline{\Omega}_{LCi}^{N}$ are collinear. In the next steps we can thus concentrate on $\overline{\Omega}_{LCi}^{N}$ and $\overline{\Omega}_{Cij}^{N}$ to characterize the Nash solution for the remaining two instruments, τ_X^{Nash} and τ_I^{Nash}.

(2) First, note that $\varepsilon-(\varepsilon-1)\tau_X > 0 \iff \tau_X < \frac{\varepsilon}{\varepsilon-1}$. This implies that when $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ we have both $\overline{\Omega}_{LCi}^{N,1} > 0$ and $\overline{\Omega}_{LCi}^{N,2} > 0$, Therefore, $\overline{\Omega}_{LCi}^{N} > 0 \forall \tau_X < \frac{\varepsilon}{\varepsilon-1}$, implying that there cannot be a Nash equilibrium in this region as it will never be the case that $\overline{\Omega}_{LCi}^{N} = 0$. Thus, in the Nash it must the case that $\tau_X^{Nash} > \frac{\varepsilon}{\varepsilon-1} > 1$.

(3) The last thing we need to show is that $\tau_I^{Nash} < 1$. We prove this by contradiction.

Assume $\tau_I^{Nash} > 1$. In the previous point we already showed that $\tau_X^{Nash} > 1$, thus we also have $\tau_I^{Nash}\tau_X^{Nash} > 1$. First, note that $\overline{\Omega}_{Cij}^{N,2} < 0$ when $\tau_I^{Nash} > 1$. Thus, a necessary condition for the Nash equilibrium to exist in the region $\tau_I > 1$ is that $\exists \tau_I > 1$ such that $\overline{\Omega}_{Cij}^{N,1} > 0$.

Next, note that $\overline{\Omega}_{Cij}^{N,1}$ is function of the two tax instruments, the endogenous variable δ_{ii}, and the parameters α and ε. Note also that δ_{ii} is an implicit function of the tax instruments, the fix costs f and f_x, ε, and the distribution function for firms' productivities $G(\varphi)$, but not of α, as can be seen from the system (A-120). Thus, $\overline{\Omega}_{Cij}^{N,1}$ is linear in α. Note that:

$$\left.\overline{\Omega}_{Cij}^{N,1}\right|_{\alpha=0} = (\varepsilon-1)^2\left[-\delta_{ii}(1-\delta_{ii}+\varepsilon(\tau_I-1)(\varepsilon-\delta_{ii})) - (1-\delta_{ii})^2(1+\varepsilon(\tau_I-1))\tau_I\tau_X\right] < 0$$

$$\left.\overline{\Omega}_{Cij}^{N,1}\right|_{\alpha=1} = -(\varepsilon-1)^2[(\tau_I\tau_X-1)(1-\delta_{ii}) + \delta_{ii}\varepsilon(\tau_I-1)] < 0$$

This implies that $\overline{\Omega}_{Cij}^{N,1} < 0$ for all $\tau_I > 1$. Therefore, $\overline{\Omega}_{Cij}^{N} < 0$ for all $\tau_I > 1$ which contradict our original hypothesis of a Nash equilibrium with $\tau_I^{Nash} > 1$. Thus, if a Nash equilibrium exists it must be such that $\tau_I^{Nash} < 1$. ■

D.8 Proof of Proposition 7

Proof

(a) When only labor taxes are available $\tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Ii} = d\tau_{Xi} = 0$ for $i = H, F$. Therefore, (37) simplifies to:

$$dU_i = \underbrace{\frac{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right) dL_{Ci}}{I_i}}_{\text{production-efficiency wedge}} + \underbrace{\frac{C_{ji}dP_{ji} - C_{ij}dP_{ij}}{I_i}}_{\text{terms-of-trade effect}} \quad j \neq i \tag{A-121}$$

In Appendix D.2 we derived a set of 3 equations ((A-92), (A-94), and (A-96)) in 6 variables ($d\tau_{Li}$, $d\tau_{Ii}$, $d\tau_{Xi}$, dL_{Ci}, dC_{ii}, dC_{ij}). We then used the 3 equations to express the differentials of the instruments as functions of the differentials of the other variables ((A-97)). Now we can impose $d\tau_{Ii} = d\tau_{Xi} = 0$ and search for the following solution:

$$\begin{aligned} d\tau_{Li} &= G(dL_{Ci}) \\ dC_{ii} &= H(dL_{Ci}) \\ dC_{ij} &= I(dL_{Ci}) \end{aligned} \tag{A-122}$$

The expressions for G, H, and I are available upon request.

Imposing $\tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Ii} = d\tau_{Xi} = 0$ in (A-101) we have a solution for the terms-of-trade effect ($C_{ji}dP_{ji} - C_{ij}dP_{ij}$) function only of $d\tau_{Li}$, dC_{ii}, and dLC_i. We can then use (A-122) to express the terms-of-trade effect as function of dL_{Ci} only:

$$C_{ji}dP_{ji} - C_{ij}dP_{ij} = \frac{(1-\delta_{ii})(\alpha + (1-\alpha)\tau_{Li})}{\varepsilon - 1} \frac{(\varepsilon-1)(\alpha(2\delta_{ii}-1)(1+\varepsilon(\tau_{Li}-1)) - \varepsilon\tau_{Li}) - 2\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i}{(\varepsilon-1)\left[(1-\delta_{ii})(1+2\delta_{ii}(\varepsilon-1))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1) - \delta_{ii}\tau_{Li})\right] + 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i} dL_{Ci} \tag{A-123}$$

Substituting (A-123) into (A-121) we obtain (41):

where

$$\Omega_i = \frac{\overline{\Omega}_i}{\varepsilon - 1} \frac{1}{(\varepsilon-1)\left[(1-\delta_{ii})(1+2\delta_{ii}(\varepsilon-1))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1) - \delta_{ii}\tau_{Li})\right] + 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i}$$

and

$$\begin{aligned} \overline{\Omega}_i \equiv\ & (\varepsilon-1)\left[(1+\varepsilon(\tau_{Li}-1))((1-\delta_{ii})(1-\alpha+2\delta_{ii}(\varepsilon-(1-\alpha)))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1) - \delta_{ii}\tau_{Li})) \right. \\ & \left. -(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})\varepsilon\tau_{Li}\right] + 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-(1-\alpha))(\tau_{Li}-1)\Phi_i \end{aligned}$$

(b) Characterizing the Nash problem when only labor taxes are available means solving the constrained problem in (36) imposing $\tau_{Ii} = \tau_{Xi} = 1$. The problem can be reduced to a maximization problem in 23 variables (22 endogenous variables plus 1 policy instrument) subject to the equilibrium conditions (7)-(14). In the previous point we showed how to rewrite the total differential of (36) as function of one total differential only, dL_{Ci}, 41. At the same time 1 is also the number of policy instrument available to the individual-country policy maker. This implies that at the optimum condition (41) must be equal to zero for any arbitrary perturbation of L_{Ci} since for any arbitrary dL_{Ci} the total differential of the 22 equilibrium conditions allows determining all the total differentials of the other 22 variables. Put differently, in the Nash $\Omega_i = 0$ i.e., $\overline{\Omega}_i = 0$.

(c) First, note that $\overline{\Omega}_i$ is function of 6 variables: τ_{Li}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}. Second, under symmetry of the initial conditions and when $\tau_{Ii} = \tau_{Xi} = 1$, the equilibrium system of equations (7)-(14) gives us the following 5 equations, which allow us to solve for φ_{ii}, φ_{ji}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ji}$, and δ_{ii} independently from τ_{Li}:

$$\begin{aligned}
\widetilde{\varphi}_{ji} &= \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})}\right]^{\frac{1}{\varepsilon-1}} \\
\widetilde{\varphi}_{ii} &= \left[\int_{\varphi_{ii}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ii})}\right]^{\frac{1}{\varepsilon-1}} \\
\delta_{ii} &= \frac{f_{ii}(1-G(\varphi_{ii}))}{f_{ii}(1-G(\varphi_{ii})) + f_{ji}(1-G(\varphi_{ji})) + f_E} \left(\frac{\widetilde{\varphi}_{ii}}{\varphi_{ii}}\right)^{\varepsilon-1} \\
\frac{\varphi_{ii}}{\varphi_{ji}} &= \left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ji}^{-1} \\
1 &= \frac{f_{ii}(1-G(\varphi_{ii}))}{f_{ii}(1-G(\varphi_{ii})) + f_{ji}(1-G(\varphi_{ji})) + f_E} \left(\frac{\widetilde{\varphi}_{ii}}{\varphi_{ii}}\right)^{\varepsilon-1} + \frac{f_{ji}(1-G(\varphi_{ji}))}{f_{ii}(1-G(\varphi_{ii})) + f_{ji}(1-G(\varphi_{ji})) + f_E} \left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}
\end{aligned} \tag{A-124}$$

This means that, differently from the Nash problem with all instruments, the symmetric Nash outcome will not alter firms' distribution between domestic and export sectors. Solving for the symmetric Nash equilibrium thus simplifies to searching for the τ_L^{Nash} such that $\overline{\Omega}_i(\tau_L^{Nash}) = 0$.

To study the Nash equilibrium first note the following:

- $\overline{\Omega}_i(\tau_{Li})$ is a quadratic function in τ_{Li}.
- $\overline{\Omega}_i(0) < 0$ for $\delta_{ii} \in (0,1]$ and $\overline{\Omega}_i(0) = 0$ when $\delta_{ii} = 0$.

  This is so since $\overline{\Omega}_i(0) = -(\varepsilon-1)^2\alpha\left[(1-\delta_{ii})(1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1)) - (1-2\delta_{ii})(1-\alpha)\right] - 2\alpha(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i$ and we have both $1-\delta_{ii} > 1-2\delta_{ii}$ and $1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1) > 1-\alpha$.
- $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$ for $\delta_{ii} \in [0,1)$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = 0$ when $\delta_{ii} = 1$.

  This is so since $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = -\frac{(1-\delta_{ii})(\alpha+\varepsilon-1)\left[(\varepsilon-1)^2+2\delta_{ii}(\alpha+\varepsilon-1)\Phi_i\right]}{\varepsilon}$.
- $\overline{\Omega}_i(1) \geq 0 \iff \delta_{ii} \geq \frac{1}{2}$.

  This is so since $\overline{\Omega}_i(1) = (2\delta_{ii}-1)(\varepsilon-1)\left[(1-\delta_{ii})(\varepsilon-1+\alpha) + \delta_{ii}(1-\alpha)\right]$.

We now proceed in steps.

(1) Consider the case $\delta_{ii} \geq \frac{1}{2}$.

Due to the fact that $\overline{\Omega}_i(\tau_{Li})$ is a quadratic function, $\overline{\Omega}_i(0) < 0$, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$, and $\overline{\Omega}_i(1) \geq 0$, whenever $\overline{\Omega}_i(\tau_{Li})$ is convex than the two solutions are such that $\tau_L^1 < 0$ and $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^2 \leq 1$. Given that our instrument has to be positive, this implies that there is only one Nash solution in the relevant parameter space: $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^{Nash} \leq 1$. In what follows we show that indeed $\overline{\Omega}_i(\tau_{Li})$ is always convex when $\delta_{ii} \geq \frac{1}{2}$. We start by computing the second derivative of $\overline{\Omega}_i(\tau_{Li})$: $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon\left[(\varepsilon-1)aa(\delta_{ii}) + 2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i\right]$ where $aa(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon) + 2\varepsilon + \alpha - 3$. Note that if $aa(\delta_{ii}) \geq 0$ when $\delta_{ii} \geq \frac{1}{2}$, then we are done. Note that:

- $aa(\delta_{ii})$ is linear in δ_{ii};
- $aa(0) = 2\varepsilon + \alpha - 3 \geq 0 \iff \varepsilon \geq \frac{3-\alpha}{2}$.
- $aa(\delta_{ii}) \geq 0$ when $\delta_{ii} \geq \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}$
- $aa(1) = 1 - \alpha > 0$.

When $\varepsilon \geq \frac{3-\alpha}{2}$ we have that $aa(\delta_{ii}) \geq 0 \forall \delta_{ii} \in [0,1]$ by linearity. When instead $\varepsilon < \frac{3-\alpha}{2}$ then $aa(\delta_{ii}) \geq 0 \forall \delta_{ii} \geq \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}$. Note however that $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ when $\varepsilon < \frac{3-\alpha}{2}$. Indeed, $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} <$

$\frac{1}{2} \iff \frac{2\varepsilon+\alpha-3}{\varepsilon+\alpha-2} < 1$ and $\varepsilon + \alpha - 2 < 0$ when $\varepsilon < \frac{3-\alpha}{2}$ therefore the inequality becomes $2\varepsilon + \alpha - 3 > \varepsilon + \alpha - 2 \iff \varepsilon > 1$ which is always true.

(2) Now consider the case $\delta_{ii} < \frac{1}{2}$ and $\varepsilon \geq \frac{3-\alpha}{2}$.

In the previous point we have already shown that $\overline{\Omega}_i(\tau_{Li})$ is convex when $\varepsilon \geq \frac{3-\alpha}{2}$. Due to the fact that $\overline{\Omega}_i(\tau_{Li})$ is a quadratic function, $\overline{\Omega}_i(0) \leq 0$, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$, and $\overline{\Omega}_i(1) < 0$, than the two solutions are such that $\tau_L^1 \leq 0$ and $\tau_L^2 > 1$. Given that our instrument has to be positive, this implies that there is only one Nash equilibrium in the relevant parameter space: $\tau_L^{Nash} > 1$.

(3) Next, consider the case $\delta_{ii} \in \left[\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}, \frac{1}{2}\right)$ and $\varepsilon < \frac{3-\alpha}{2}$. In this case we still have $aa(\delta_{ii}) \geq 0$ and thus again there is only one Nash equilibrium in the relevant parameter space: $\tau_L^{Nash} > 1$.

(4) Finally, consider the case $\delta_{ii} \in \left[0, \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}\right)$ and $\varepsilon < \frac{3-\alpha}{2}$. When this is the case we have $aa(\delta_{ii}) < 0$, and we cannot know whether $\overline{\Omega}_i(\tau_{Li})$ is convex or concave implying we cannot characterize the Nash equilibrium. Thus, all the following can happen: no Nash equilibrium, a unique Nash equilibrium with $\tau_L^{Nash} > 1$, two Nash equilibria. Note that $\frac{\partial\left(\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}\right)}{\partial\varepsilon} = -\frac{1-\alpha}{2(\varepsilon+\alpha-2)^2} < 0$ implying that the cutoff point for $aa(\delta_{ii}) < 0$ decreases with ε. Indeed, $\lim_{\varepsilon\to\frac{3-\alpha}{2}} \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} = 0$ and $\lim_{\varepsilon\to 1} \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} = \frac{1}{2}$.

■

D.9 Proof of Proposition 8

Proof I We prove Proposition 8 point by point.

(a) According to Proposition 7, when $\delta_{ii} \geq \frac{1}{2}$ and only domestic policies are available any symmetric Nash equilibrium – if it exists – is such that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^{Nash} \leq 1$. Hence, a sufficient condition for the Nash allocation to entail higher welfare than the free-trade allocation is that in a symmetric equilibrium individual-country welfare is monotonically decreasing in τ_{Li}. In other words, we need to demonstrate that in the symmetric equilibrium $\frac{dU_i}{d\tau_{Li}} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. To show this result, first notice that $\frac{dU_i}{d\tau_{Li}} = \frac{dU_i}{dL_{Ci}}\frac{dL_{Ci}}{d\tau_i}$. Second, consider that the total differential of utility (3) can be written as in condition (A-76). Then, if we combine this total differential with the total differential of (13) and (14) departing from a symmetric allocation we get:

$$dU_i = -\frac{P_{ii}}{I_i}dC_{ii} - \frac{P_{ij}}{I_i}dC_{ij} + -\frac{1}{I_i}dL_{Ci}$$

Moreover, it can be shown[38] that under symmetry $dC_{ij} = \frac{C_{ij}}{L_{Ci}}\frac{\varepsilon}{(\varepsilon-1)}dL_{Ci}$ for $i, j = H, F$. By substituting these conditions into the differential above and taking into account conditions (11) and (12) we obtain:

$$dU_i = \frac{1}{I_i}\left(\frac{\tau_{Li}\varepsilon}{\varepsilon-1} - 1\right)dL_{Ci} \qquad \text{(A-125)}$$

This last result follows directly from the fact that symmetric deviations of the labor subsidy from a symmetric allocation do not have an impact on the cut offs φ_{ij} and on the market shares. Since we are starting from a symmetric allocation where import tariffs and export taxes are absent, changes in welfare are equal to the production-efficiency wedge of condition (37) implying that consumption wedges and terms of trade effects are zero. Finally, it can be shown that:

$$\frac{dL_{Ci}}{d\tau_{Li}} = -\frac{(1-\alpha)L_{Ci}}{\alpha + \tau_{Li}(1-\alpha)} < 0$$

This allows us to conclude that $\frac{dU_i}{d\tau_{Li}} = -\frac{L_{Ci}}{I_i}\left(\frac{\tau_{Li}\varepsilon}{\varepsilon-1} - 1\right)\frac{(1-\alpha)}{\alpha+\tau_{Li}(1-\alpha)} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$.

(b) In point (a), we just showed that $\frac{dU_i}{d\tau_{Li}} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$ and independently of δ_{ii}. This implies that since when $\delta_{ii} < \frac{1}{2}$, the symmetric Nash equilibrium is welfare dominated by the free-trade allocation

[38] The proof is available on request.

because of the result from Proposition 8. It is easy to show that when starting from a symmetric allocation condition (A-125) holds also when firms are homogeneous. Moreover, Campolmi et al. (2014) have already proved that in this case $dL_{Ci}/d\tau_{Li} = dN_i/d\tau_{Li} < 0$.

(c) By taking the the differential of conditions (7), (8) and (9) with respect to f_{ij} and τ_{ij}, it can be shown that:

$$d\delta_{ii} = \frac{(\varepsilon - 1 + \Phi_i)\delta_{ii}(1-\delta_{ii})}{\tau_{ij}} d\tau_{ij} + \frac{\Phi_i \delta_{ii}(1-\delta_{ii})\tilde{\varphi}_{ij}^{1-\varepsilon}\varphi_{ij}^{\varepsilon-1}}{(\varepsilon-1)f_{ij}} df_{ij}$$

which confirms that δ_{ii} is monotonically increasing in both τ_{ij} and f_{ij}.

■